8.4 Unifying View

Machine Learning 1: Foundations

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Unifying View

Recall our unifying view

$$\min_{[W_\cdot]\ b,\mathbf{w}} \quad \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n}\ell(y_i(\langle\mathbf{w},\phi(\mathbf{x}_i)\rangle + b)) \quad \left[+\frac{1}{2}\sum_{l=1}^{L}\|W_l\|^2_{\text{Fro}}\right],$$

which comprises (linear and kernelized) SVM and LR, as well as ANN in just one equation.

Wouldn't it be nice to add also regression into this equation? :)

In order to do so, we slightly change our notation of the loss:

$$l(t,y) := \begin{cases} (t-y)^2 & \text{for} \quad \text{regression} \\ \ell(yt) & \text{for} \quad \text{classification} \end{cases}$$

We obtain...

Unifying View of Regression and Classification

Unifying formulation of linear, kernel, and neural classification and regression

$$\min_{[W,]\ b,\mathbf{w}} \quad \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n} l(\langle \mathbf{w}, \phi(\mathbf{x}_i)\rangle + b, y_i) \quad \left[+ \frac{1}{2}\sum_{l=1}^{L} \| W_l \|_{\text{Fro}}^2 \right],$$

where

- $l(t, y) := \max(0, 1 - yt)$ for SVM ("hinge loss")
- $l(t, y) := \ln(1 + \exp(-yt))$ for LR and ANN ("logistic loss")
- $l(t, y) := (t - y)^2$ for regression

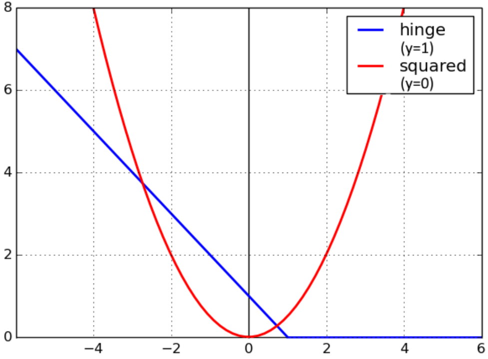
and

- $\phi :=$ id for linear SVM, linear LR, and RR
- $\phi := \phi_k$ for kernel SVM, kernel LR, and KRR
- $\phi := \phi_W$ for ANN and DR.

The terms in brackets apply only to ANN and DR.

Unifying View Reveals: Classification and Regression Differ only in the Loss

- Regression uses the squared loss: $l(t,y) = (t-y)^2$
- E.g., the SVM uses the hinge loss: $l(t,y) = \max(0, 1-yt)$



Can we derive a SVM-style regression method?

Support Vector Regression (SVR)

SVR uses the same regularizer as SVM, but the following loss:

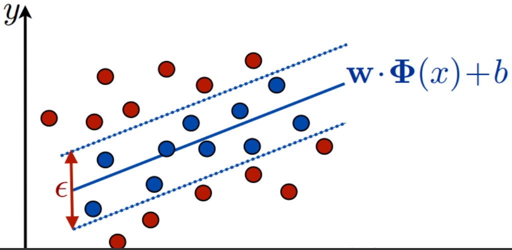
Definition

The **ϵ-insensitive loss** is defined as
$\ell(t, y) := \max(0, |y - t| - \epsilon)$

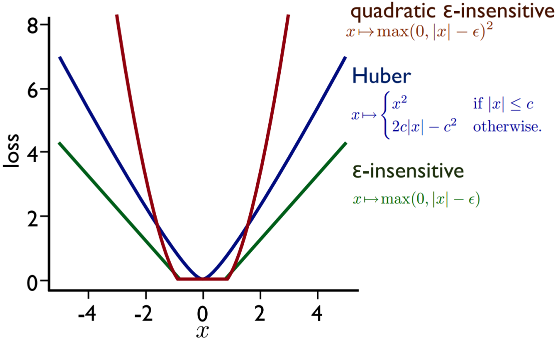
Support Vector Regression (SVR)

$$\mathbf{w}^*_{SVR} := \arg\min_{\mathbf{w}\in\mathcal{H}} \; \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n} \max\left(0, |y_i - \langle \mathbf{w}, \phi(\mathbf{x}_i)\rangle| - \epsilon\right)$$

Fit 'tube' with width ϵ to data.



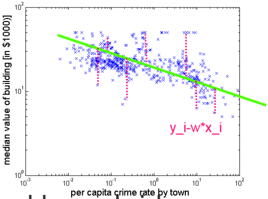
Outlook: More Losses



Conclusion

Regression

- Given $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^d$ and $y_1, \ldots y_n \in \boxed{\mathbb{R}}$, find f such that $f(\mathbf{x}) \approx y$ on new x and y



- Important example: ridge regression and kernel ridge regression

$$\min_{\mathbf{w} \in \mathbb{R}^d} \quad \frac{1}{2}\|\mathbf{w}\|^2 + C\left\|\mathbf{y} - X^\top \mathbf{w}\right\|^2$$

$$= \quad \min_{\boldsymbol{\alpha} \in \mathbb{R}^n} \quad \frac{1}{2}\|X\boldsymbol{\alpha}\|^2 + C\left\|\mathbf{y} - X^\top X\boldsymbol{\alpha}\right\|^2$$

- With analytic solutions:

$$\mathbf{w}_{\mathrm{RR}} = \left(XX^\top + \tfrac{1}{2C} I\right)^{-1} Xy, \qquad \boldsymbol{\alpha}^* = \left(K + \tfrac{1}{2C} I_{n\times n}\right)^{-1} y$$

Conclusion

LOOCV (= amazingly accurate validation) of (K)RR:

- comes (computationally) for free!

Deep regression:

$$\min_{\mathbf{w},W} \quad \frac{1}{2}\|\mathbf{w}\|^2 \;+\; \frac{1}{2}\sum_{l=1}^{L}\|W_l\|_{\text{Fro}}^2 \;+\; C\sum_{i=1}^{n}(\langle \mathbf{w}, \phi_W(\mathbf{x}_i)\rangle - y_i)^2$$

Unifying view:

$$\min_{[W,]\; b,\mathbf{w}} \quad \frac{1}{2}\|\mathbf{w}\|^2 \;+\; C\sum_{i=1}^{n} l(\langle \mathbf{w}, \phi(\mathbf{x}_i)\rangle + b, y_i) \quad \left[+\,\frac{1}{2}\sum_{l=1}^{L}\|W_l\|_{\text{Fro}}^2\right],$$