Machine Learning I: Foundations
Exercise Sheet 6

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Deadline: 09.06.2020

1) (MANDATORY) 10 Points
Let $M \in \mathbb{R}^{n \times n}$ be a symmetric square matrix. Prove that the following statements are equivalent.

- M is positive semi-definite, i.e. $\forall \mathbf{z} \in \mathbb{R}^n : \mathbf{z}^T M \mathbf{z}$.
- All eigenvalues of M are non-negative.

2) A square matrix M is called diagonalizable if and only if there exists an invertible matrix P such that $P^{-1}MP = D$ with D diagonal, i.e. $D = diag(\mathbf{v})$ for some $\mathbf{v}$.

a) Prove that M is diagonalizable if M has n distinct eigenvalues.

b) The converse of the above is not true. Give an example for this.

3) Are the following matrices diagonalizable? If yes, determine P and D as above. If no, give a reason why not.

a)

$$\begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

c)

$$\begin{bmatrix} 5 & 4 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 3 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$$

d)

$$\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

4) Solve programming task 6.