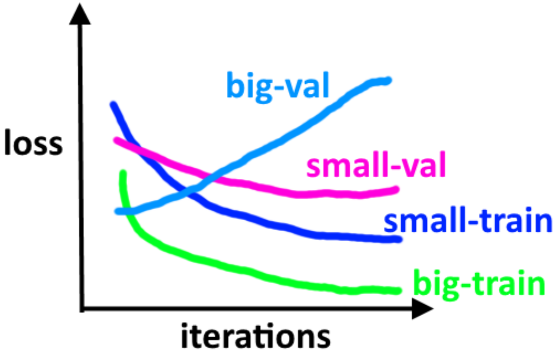
7.4 Regularization for Deep Learning

Machine Learning 1: Foundations

Marius Kloft *(TUK)*

Deep Neural Networks, Due to Their High Complexity, Are in Very High Danger of Overfitting



Adapted from `https://keras.rstudio.com/articles/tutorial_overfit_underfit.html`

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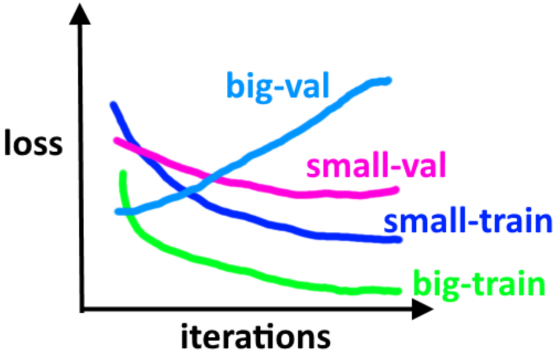

Deep learning requires specific regularization machinery—we will learn about some common strategies in the following

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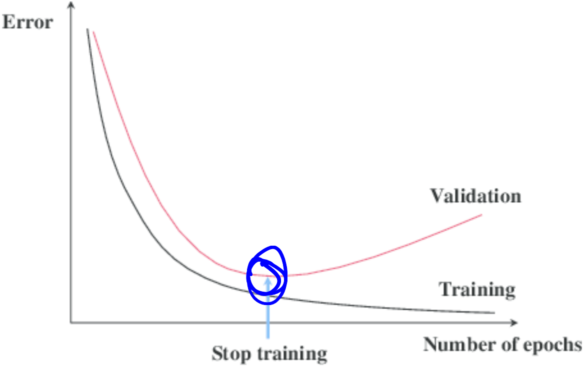
Trick 1 for DL Regularization: Early Stopping

Idea:

- Split training set into a new (smaller) training set and a validation set
- Monitor the validation error every couple of mini-batch SGD iterations
- Stop SGD optimization early, when validation error goes up



A colleague just gave me this data ...

$$X := (\mathbf{x}_1, \ldots, \mathbf{x}_n) = \begin{pmatrix} 230.23 & 625.43 & 709.25 & 434.39 \\ 439.11 & 153.00 & 248.52 & 834.76 \\ \boxed{12.04} & \boxed{12.83} & \boxed{12.09} & \boxed{11.80} \\ 244.59 & 935.98 & 710.26 & 437.63 \\ \vdots & & & \vdots \end{pmatrix}$$

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$w_1 x_1 + w_2 x_2 + w_3 x_3$

$+ \ldots$

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- Small-scale features $x_j \approx 0$ contribute to $f(\mathbf{x})$ only if their weight w_j is very large (to compensate for the small x_j)
- But w_j cannot become large because the regularizer $\frac{1}{2} \|\mathbf{w}\|^2$ promotes small w_js

Solution: Standardization

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Notation:

- For a feature f, denote by $f_1, \ldots, f_n \in \mathbb{R}$ its values for the data points $\mathbf{x}_1, \ldots, \mathbf{x}_n$
- Denote $\mu_f := \frac{1}{n} \sum_{i=1}^{n} f_i$ ("**mean**")
- and $\sigma_f := \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f_i - \mu_f)^2}$ ("**standard deviation**")

1 2 3

1.5 3 4

2.875

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1. Center each feature: $\forall i = 1, \ldots, n: \ f_i \leftarrow f_i - \mu_f$
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This motivates a form of regularization for deep learning

Trick 2 for DL Regularization: Batch Normalization

Batch normalization takes feature standardization to the next level and standardizes **every neuron**:

Batch Normalization

Denote, for a neuron f, its activation values on a mini-batch by $\mathbf{f} = (f_1, \ldots f_B)$. Then, for each mini-batch in the SGD optimization and every neuron in the ANN, do:

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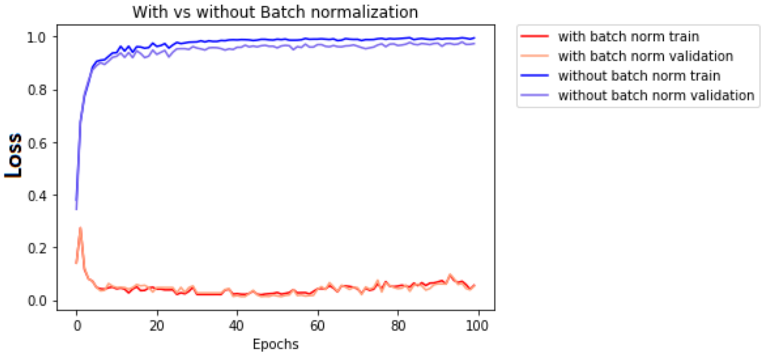
Overwrite the neuron's activation by:

- $\gamma \mathbf{f} + \beta$
- Here $\gamma \in \mathbb{R}_+$ and $\beta \in \mathbb{R}$ are parameters that are learned during optimization

- The last step ensures that we can obtain the same range of activations in the optimization

Does Batch Normalization Work?



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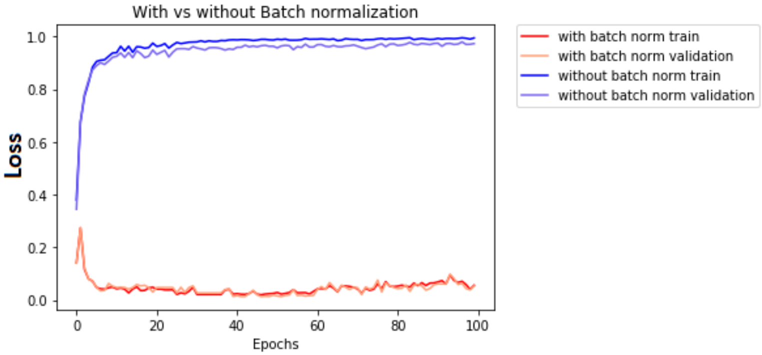

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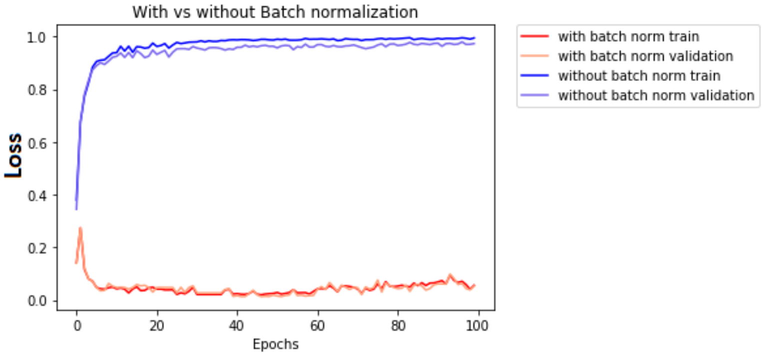

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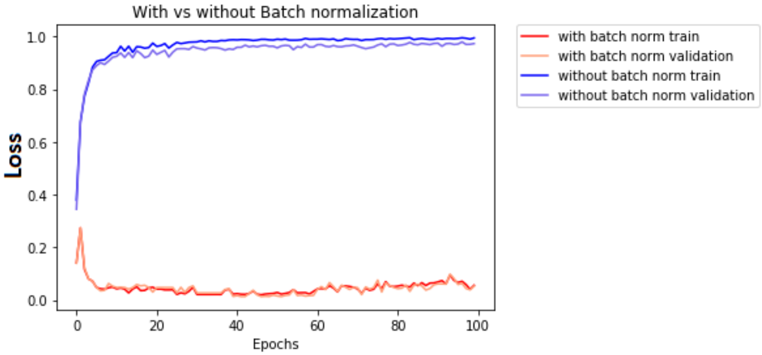

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Unclear why it works... this is a pressing research topic.

Netflix Price

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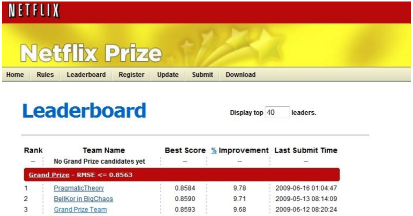

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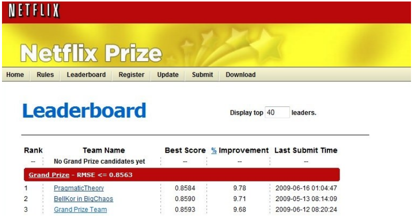

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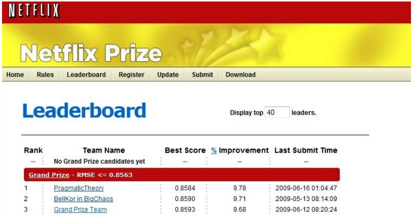

NETFLIX

Netflix Prize

Home | Rules | Leaderboard | Register | Update | Submit | Download

Leaderboard

Display top 40 leaders.

Rank	Team Name	Best Score	% Improvement	Last Submit Time
--	No Grand Prize candidates yet	--	--	--
Grand Prize - RMSE <= 0.8563				
1	PragmaticTheory	0.8584	9.78	2009-06-16 01:04:47
2	BellKor in BigChaos	0.8590	9.71	2009-05-13 08:14:09
3	Grand Prize Team	0.8593	9.68	2009-06-12 08:20:24

How did they win the challenge?

The Netflix challenge and many others have been won using a technique called **ensembling**.

Ensemble Methods

Ensemble methods are methods that aggregate the prediction from multiple predictors

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Example of a simple ensemble method:

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Problem: too costly to train multiple deep ANNs

Trick 3 for DL Regularization: Dropout

Dropout regularization

Dropout regularization randomly removes in each mini-batch SGD iteration a fixed percentage of randomly selected neurons of the network.

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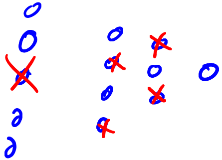
Common choice:

- input neurons are dropped out with probability $p = 0.2$
- hidden neurons are dropped out with probability $p = 0.5$



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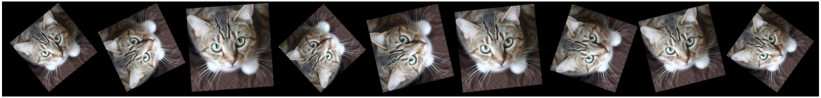

Idea:

- Say we wanna classify images, e.g., cats vs. dogs
- If we turn an image showing a cat upside down, it will still show a cat—just with the cat being upside down
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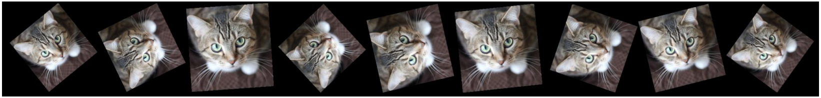

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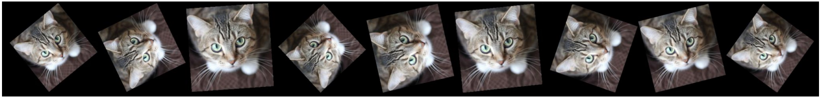

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This regularization strategy is known as **data augmentation**

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This regularization strategy is known as **transfer learning**—that is, transfering information into a learning problem from a related problem

Conclusion

The problem: overfitting dilemna

- do not know whether simple or complex model is adequate

The solution:

- choose complex model and use regularization

Regularization techniques:

- Norm regularization
  - makes solutions more smooth
- Batch normalization
  - standardize each neuron
- Dropout
  - randomly removing neurons from the architecture in each SGD iteration

Outlook:

- Another type of regularization: adversarial training