Machine Learning I: Foundations
Exercise Sheet 2

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05.05.2020
Deadline: 12.05.2020

1) (MANDATORY) 10 Points
In this exercise we will consider the soft-margin SVM

$$\min \ \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{n}\xi_i$$
$$\text{s.t. } 1 - \xi_i - y_i(\mathbf{w}^T\mathbf{x}_i + b) \leq 0, \ \ -\xi_i \leq 0, \ \forall i \in \{1, \ldots, n\}$$

Let $a \in (2, -1)$, consider the one-dimensional datapoints $(2, 1), (a, 1), (-2, -1)$. For each datapoint the second value is the label. This is a binary classification dataset. We will investigate the behavior of C.

a) Determine for (1) $w = \frac{1}{2}, b = 0$ and (2) $w = \frac{2}{a+2}, b = \frac{2-a}{a+2}$ the objective function depending on a and C.

b) For which value of C is (1) uniformly better than (2), i.e. $\forall a \in (2, -1)$. For which value of C is (2) uniformly better than (1)? **Hint:** Explicitly evaluating the intersections of functions is quite involved in this case. Consider using a plotting tool to compare the functions. Prove or argue whatever you find post hoc.

c) Conclude how C influences the optimization problem. Justify your conclusion using at most 5 sentences.

2) Let $f(a_1, a_2, \cdots, a_n) = \ln(e^{a_1} + e^{a_2} + \cdots + e^{a_n})$. Show that f is convex.

3) Using any techniques you have learned from class, determine the domain on which the function $f(x, y) = xy^3$ is convex, i.e. for which combination of x and y is $f(x, y)$ convex?

4) Solve programming task 2.