

# Agent Based Stochastic SIR Model

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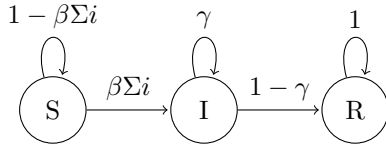
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## Introduction

Deterministic models are valid only at the thermodynamic limit: population  $n \rightarrow \infty$ . Stochastic models can more accurately describe outbreaks in their early stages. [1] provides an overview for Markov chain models among other stochastic models. Some other stochastic models: [2], [3]

We present a SIR based stochastic model.

## Formulation



where  $\Sigma i$  is the number of infected individuals adjacent to an individual. The state space is described by  $\mathbf{S}, \mathbf{I}$  both  $n$  dimensional vectors with elements 0 or 1, where  $n$  is the population size. The conditional probability of S to I state transition during a simulation period is  $\mathbf{P}_{S \rightarrow I} = \beta \mathbf{A}_{n \times n} \mathbf{I} \cdot \mathbf{S}$ , where  $\mathbf{A}$  is the adjacency matrix and  $\cdot \mathbf{S}$  is an element-wise multiplication, and  $\beta \in \mathbb{R}^+$  is the infection standard deviation.

Conditional probability of I to R state transition has mean at  $\gamma_m \in \mathbb{Z}^+$  simulation periods after infection with a standard deviation of  $\gamma \in (0, 1]$ . If  $\gamma = 1$ , recovery will always occur after  $\gamma_m$  periods.  $\mathbf{P}_{I \rightarrow R} = \gamma(\mathbf{I}_t - \gamma_m) \cdot \mathbf{I}$ . Number of periods after infection is tracked by vector  $\mathbf{I}_t \in \mathbb{Z}^n$  and is updated every cycle as  $\mathbf{I}_t = \mathbf{I}_t + \mathbf{I}$ .

Both transition probability vectors are compared element wise to a random vector whose elements are in the range  $[0, 1]$ . If the values are larger than the random number, the state for that person is transitioned. Initial condition is defined as a single infected individual. The susceptible vector is set as the complement of the infected vector. The program is run multiple iterations to show different paths.

```
for k=1:n/10 %number of runs
    I=zeros(n,1); %infected population , binary
    I(2)=1; %initial condition infected
    S=~I; %susceptible population , binary
    It=I; %time a person has been infected , natural number
    x=zeros(T,1); %plot number of infecteds over time
    for t=1:T
        %contraction probability given that individual is susceptible
        contract = (beta*A*I > rand(n,1)).*S;
        %recovery probability given that individual is infected
        recover = (gamma*(It-gamma_m) > rand(n,1)).*I;
        S += -contract;
        I += contract - recover;
```

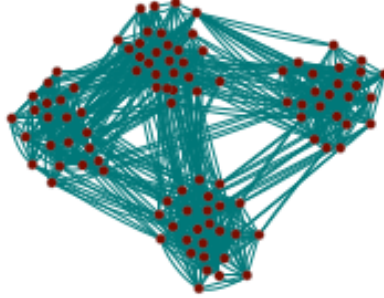


Figure 1: Random Modular Graph

```

    It += I;
    x(t) = sum(I);
end
plot(x)
hold on
end

```

## Graph Generation

Contact in a population are best described by "small-world graph" where contacts are clustered.[4] [5] is used to generate the adjacency matrix for a clustered undirected graph. A random graph with 100 nodes clustered into four groups is generated in figure 1. The ratio of inside-module links to outside-module links is set to 5.

```
[A, ~] = randomModularGraph(n, 4, 0.1, 5);
```

## Result

Using the algorithm and graph described, the number of infected persons are plotted over time. The program is run 10 times to show 10 trajectories. If the inter-modular links are reduced, the trajectories are more divergent run-to-run, see figure 3.

## References

[1] L. Allen, An Introduction to Stochastic Epidemic Models, [Online] Available: <https://www.math.mun.ca/~zhao/ARRMSschool/StochEpidModels.pdf>

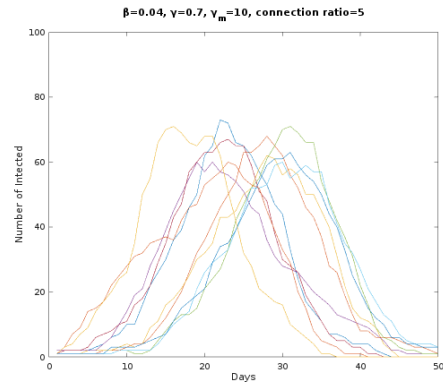


Figure 2: Infected Population, cluster connection ratio 5

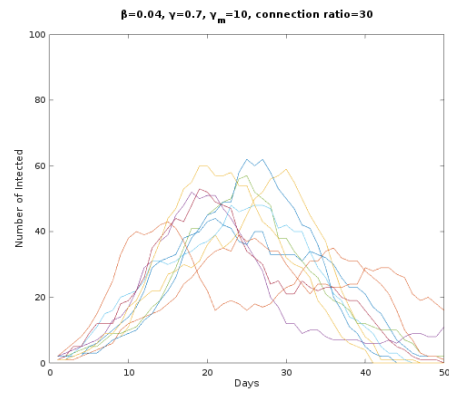


Figure 3: Infected Population, cluster connection ratio 30

- [2] R. Connell, P. Dawson, A. Skvortsov, "Comparison of an Agent-based Model of Disease Propagation with the Generalised SIR Epidemic Model," [Online] Available: <https://apps.dtic.mil/dtic/tr/fulltext/u2/a510899.pdf>
- [3] EpiModel "Mathematical Modeling of Infectious Disease Dynamics" [Online] Available: <http://www.epimodel.org/index.html>
- [4] M. Wolf "Differential Equation and Agent-Based Models in Epidemiology" page 36, [Online] Available: [https://www.sandia.gov/~mmwolf/presentations/CS591MH/CS591MH\\_20060215.pdf](https://www.sandia.gov/~mmwolf/presentations/CS591MH/CS591MH_20060215.pdf)
- [5] octave-networks-toolbox [Online] Available: <https://github.com/aeolianine/octave-networks-toolbox>