Let T be a tree with n vertices; that is, a connected simple graph on n vertices that contains no cycle. For every pair u, v of vertices, let $d(u, v)$ denote the distance between u and v, that is, the number of edges in the shortest path in T that connects u with v.

Consider the sums

$$W(T) = \sum_{\substack{\{u,v\} \subseteq V(T) \\ u \neq v}} d(u, v)$$

and

$$H(T) = \sum_{\substack{\{u,v\} \subseteq V(T) \\ u \neq v}} \frac{1}{d(u, v)}.$$

Prove that

$$W(T) \cdot H(T) \geq \frac{(n-1)^3(n+2)}{4}.$$