In the linear space of all real $n \times n$ matrices, find the maximum possible dimension of a linear subspace V such that

$$\forall X, Y \in V : \mathrm{trace}(XY) = 0.$$

(The trace of a matrix is the sum of the diagonal entries.)