Let A and B be $n \times n$ real matrices such that

$$\operatorname{rk}(AB - BA + I) = 1$$

where I is the $n \times n$ identity matrix.
Prove that

$$\operatorname{tr}(ABAB) - \operatorname{tr}(A^2B^2) = \frac{1}{2}n(n-1).$$

($\operatorname{rk}(M)$ denotes the rank of matrix M, i.e., the maximum number of linearly independent columns in M. $\operatorname{tr}(M)$ denotes the trace of M, that is the sum of diagonal elements in M.)