

Let  $p$  be a polynomial with integer coefficients and let  $a_1 < a_2 < \cdots < a_k$  be integers.

- a) Prove that there exists  $a \in \mathbb{Z}$  such that  $p(a_i)$  divides  $p(a)$  for all  $i = 1, 2, \dots, k$ .
- b) Does there exist an  $a \in \mathbb{Z}$  such that the product  $p(a_1) \cdot p(a_2) \cdot \dots \cdot p(a_k)$  divides  $p(a)$ ?