a) For each $1 < p < \infty$ find a constant $c_p < \infty$ for which the following statement holds: If $f : [-1, 1] \to \mathbb{R}$ is a continuously differentiable function satisfying $f(1) > f(-1)$ and $|f'(y)| \le 1$ for all $y \in [-1, 1]$, then there is an $x \in [-1, 1]$ such that $f'(x) > 0$ and $|f(y) - f(x)| \le c_p (f'(x))^{1/p} |y - x|$ for all $y \in [-1, 1]$.

b) Does such a constant also exist for $p = 1$?