

Define the sequence a_0, a_1, \dots inductively by $a_0 = 1$, $a_1 = \frac{1}{2}$ and

$$a_{n+1} = \frac{na_n^2}{1 + (n+1)a_n} \quad \text{for } n \geq 1.$$

Show that the series $\sum_{k=0}^{\infty} \frac{a_{k+1}}{a_k}$ converges and determine its value.