Let S_n denote the set of permutations of the sequence $(1, 2, \dots, n)$. For every permutation $\pi = (\pi_1, \dots, \pi_n) \in S_n$, let $\operatorname{inv}(\pi)$ be the number of pairs $1 \le i < j \le n$ with $\pi_i > \pi_j$; i.e. the number of inversions in π. Denote by $f(n)$ the number of permutations $\pi \in S_n$ for which $\operatorname{inv}(\pi)$ is divisible by $n + 1$.

Prove that there exist infinitely many primes p such that $f(p-1) > \dfrac{(p-1)!}{p}$, and infinitely many primes p such that $f(p-1) < \dfrac{(p-1)!}{p}$.