a) Let $f \in C[0,b]$, $g \in C(\mathbb{R})$ and let g be periodic with period b. Prove that $\int_0^b f(x)g(nx)\,\mathrm{d}x$ has a limit as $n \to \infty$ and

$$\lim_{n\to\infty} \int_0^b f(x)g(nx)\,\mathrm{d}x = \frac{1}{b}\int_0^b f(x)\,\mathrm{d}x \cdot \int_0^b g(x)\,\mathrm{d}x.$$

b) Find

$$\lim_{n\to\infty} \int_0^\pi \frac{\sin(x)}{1+3\cos^2 nx}\,\mathrm{d}x.$$