Let $f(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ be a polynomial with real coefficients. Prove that if all roots of f lie in the left half-plane $\{z \in \mathbb{C} : \Re z < 0\}$ then

$$a_k a_{k+3} < a_{k+1} a_{k+2}$$

holds for every $k = 0, 1, \ldots, n - 3$.