Today, Ivan the Confessor prefers continuous functions $f : [0, 1] \to \mathbb{R}$ satisfying $f(x) + f(y) \geq |x - y|$ for all pairs $x, y \in [0, 1]$. Find the minimum of $\int_0^1 f$ over all preferred functions.