For every positive integer n, let $f(n), g(n)$ be the minimal positive integers such that

$$1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!} = \frac{f(n)}{g(n)}.$$

Determine whether there exists a positive integer n for which $g(n) > n^{0.999\, n}$.