For each positive integer n, let  $f_n(\vartheta) = \sin \vartheta \cdot \sin(2\vartheta)$ .  $\sin(4\vartheta)\cdots\sin(2^n\vartheta)$ . For all real  $\vartheta$  and all n, prove that

$$\sin(4\vartheta)\cdots\sin(2^n\vartheta)$$
. For all real  $\vartheta$  and all  $n$ , prove that  $|f_n(\vartheta)| \leq \frac{2}{\sqrt{3}}|f_n(\pi/3)|$ .