

For every complex number  $z \notin \{0, 1\}$  define

$$f(z) := \sum (\log z)^{-4},$$

where the sum is over all branches of the complex logarithm.

- a) Show that there are two polynomials  $P$  and  $Q$  such that  $f(z) = P(z)/Q(z)$  for all  $z \in \mathbb{C} \setminus \{0, 1\}$ .
- b) Show that for all  $z \in \mathbb{C} \setminus \{0, 1\}$

$$f(z) = z \frac{z^2 + 4z + 1}{6(z - 1)^4}.$$