

A polynomial p with real coefficients satisfies the equation $p(x+1) - p(x) = x^{100}$ for all $x \in \mathbb{R}$. Prove that $p(1-t) \geq p(t)$ for $0 \leq t \leq 1/2$.