

Let  $(a_n)_{n \in \mathbb{N}}$  be the sequence defined by

$$a_0 = 1, \quad a_{n+1} = \frac{1}{n+1} \sum_{k=0}^n \frac{a_k}{n-k+2}.$$

Find the limit

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{a_k}{2^k},$$

if it exists.