Let $f(x) = 2x(1-x)$, $x \in \mathbb{R}$. Define

$$f_n = \underbrace{f \circ \cdots \circ f}_{n}.$$

a) Find $\lim_{n\to\infty} \int_0^1 f_n(x)\,\mathrm{d}x$.

b) Compute $\int_0^1 f_n(x)\,\mathrm{d}x$ for $n = 1, 2, \ldots$.