

Let f be a real-valued function with $n + 1$ derivatives at each point of \mathbb{R} . Show that for each pair of real numbers a, b , $a < b$, such that

$$\ln \left(\frac{f(b) + f'(b) + \cdots + f^{(n)}(b)}{f(a) + f'(a) + \cdots + f^{(n)}(a)} \right) = b - a$$

there is a number c in the open interval (a, b) for which

$$f^{(n+1)}(c) = f(c).$$

Note that \ln denotes the natural logarithm.