Let $\mathbb{R}$ be the set of real numbers. Prove that there is no function $f : \mathbb{R} \to \mathbb{R}$ with $f(0) > 0$, and such that

$$f(x + y) \geq f(x) + yf(f(x)) \qquad \text{for all } x, y \in \mathbb{R}.$$