Let n be a positive integer, and consider the matrix $A = (a_{ij})_{1 \le i,j \le n}$, where

$$a_{ij} = \begin{cases} 1 & \text{if } i+j \text{ is a prime number,} \\ 0 & \text{otherwise.} \end{cases}$$

Prove that $|\det A| = k^2$ for some integer k.