

Let V be a real vector space, and let f, f_1, f_2, \dots, f_k be linear maps from V to \mathbb{R} . Suppose that $f(x) = 0$ whenever $f_1(x) = f_2(x) = \dots = f_k(x) = 0$. Prove that f is a linear combination of f_1, f_2, \dots, f_k .