Let $f \neq 0$ be a polynomial with real coefficients. Define the sequence $f_0, f_1, f_2, \ldots$ of polynomials by $f_0 = f$ and $f_{n+1} = f_n + f_n'$ for every $n \geq 0$. Prove that there exists a number N such that for every $n \geq N$, all roots of f_n are real.