

Let A_i, B_i, S_i ($i = 1, 2, 3$) be invertible real 2×2 matrices such that

(1) not all A_i have a common real eigenvector;

(2) $A_i = S_i^{-1} B_i S_i$ for all $i = 1, 2, 3$;

(3) $A_1 A_2 A_3 = B_1 B_2 B_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Prove that there is an invertible real 2×2 matrix S such that $A_i = S^{-1} B_i S$ for all $i = 1, 2, 3$.