Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin(\log n)}{n^\alpha}$ converges if and only if $\alpha > 0$.