

Let $f : [0, 1] \rightarrow (0, \infty)$ be an integrable function such that $f(x) \cdot f(1 - x) = 1$ for all $x \in [0, 1]$. Prove that

$$\int_0^1 f(x) dx \geq 1.$$