

Let A_1, A_2, \dots, A_k be $n \times n$ idempotent complex matrices such that

$$A_i A_j = -A_j A_i \quad \text{for all } i \neq j.$$

Prove that at least one of the given matrices has $\text{rank} \leq \frac{n}{k}$.

(A matrix A is called idempotent if $A^2 = A$.)