Let A and B be real $n \times n$ matrices. Assume that there exist $n+1$ different real numbers $t_1, t_2, \ldots, t_{n+1}$ such that the matrices

$$C_i = A + t_i B, \quad i = 1, 2, \ldots, n+1,$$

are nilpotent (i.e. $C_i^n = 0$).
Show that both A and B are nilpotent.