Let A be an $n \times n$-matrix with integer entries and $b_1, \ldots, b_k$ be integers satisfying $\det A = b_1 \cdot \ldots \cdot b_k$. Prove that there exist $n \times n$-matrices $B_1, \ldots, B_k$ with integer entries such that $A = B_1 \cdot \ldots \cdot B_k$ and $\det B_i = b_i$ for all $i = 1, \ldots, k$.