a) Prove that every function of the form

$$f(x) = \frac{a_0}{2} + \cos x + \sum_{n=2}^{N} a_n \cos(nx)$$

with $|a_0| < 1$, has positive as well as negative values in the period $[0, 2\pi)$.

b) Prove that the function

$$F(x) = \sum_{n=1}^{100} \cos\left(n^{\frac{3}{2}} x\right)$$

has at least 40 zeros in the interval $(0, 1000)$.