

Let $0 < c < 1$ and

$$f(x) = \begin{cases} \frac{x}{c} & \text{for } x \in [0, c], \\ \frac{1-x}{1-c} & \text{for } x \in [c, 1]. \end{cases}$$

We say that p is an n -periodic point if

$$\underbrace{f(f(\dots f(p)))}_n = p$$

and n is the smallest number with this property. Prove that for every $n \geq 1$ the set of n -periodic points is non-empty and finite.