Let $p(x)$ be a nonconstant polynomial with real coefficients. For every positive integer n, let

$$q_n(x) = (x+1)^n p(x) + x^n p(x+1).$$

Prove that there are only finitely many numbers n such that all roots of $q_n(x)$ are real.