Let G be the subgroup of $\mathrm{GL}_2(\mathbb{R})$, generated by A and B, where

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Let H consist of those matrices $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ in G for which $a_{11} = a_{22} = 1$.

(a) Show that H is an abelian subgroup of G.

(b) Show that H is not finitely generated.