

(i) Prove that

$$\lim_{x \rightarrow +\infty} \sum_{n=1}^{\infty} \frac{nx}{(n^2 + x)^2} = \frac{1}{2}.$$

(ii) Prove that there is a positive constant  $c$  such that for every  $x \in [1, \infty)$  we have

$$\left| \sum_{n=1}^{\infty} \frac{nx}{(n^2 + x)^2} - \frac{1}{2} \right| \leq \frac{c}{x}.$$