

For a prime number p , let $\mathrm{GL}_2(\mathbb{Z}/p\mathbb{Z})$ be the group of invertible 2×2 matrices of residues modulo p , and let S_p be the symmetric group (the group of all permutations) on p elements. Show that there is no injective group homomorphism $\varphi : \mathrm{GL}_2(\mathbb{Z}/p\mathbb{Z}) \rightarrow S_p$.