Let f be a continuous function on $[0,1]$ such that for every $x \in [0,1]$ we have

$$\int_x^1 f(t)\,\mathrm{d}t \geq \frac{1-x^2}{2}.$$

Show that

$$\int_0^1 f^2(t)\,\mathrm{d}t \geq \frac{1}{3}.$$