Let $f, g : [a,b] \to [0,\infty)$ be continuous and non-decreasing functions such that for each $x \in [a,b]$ we have

$$\int_a^x \sqrt{f(t)}dt \leq \int_a^x \sqrt{g(t)}dt$$

and $\int_a^b \sqrt{f(t)}dt = \int_a^b \sqrt{g(t)}dt$.

Prove that $\int_a^b \sqrt{1+f(t)}dt \geq \int_a^b \sqrt{1+g(t)}dt$.