Let $g : [0, 1] \to \mathbb{R}$ be a continuous function and let $f_n : [0, 1] \to \mathbb{R}$ be a sequence of functions defined by $f_0(x) = g(x)$ and

$$f_{n+1}(x) = \frac{1}{x} \int_0^x f_n(t)dt \ (x \in (0, 1], n = 0, 1, 2, \dots).$$

Determine $\lim_{n\to\infty} f_n(x)$ for every $x \in (0, 1]$.