Let $f : (0, \infty) \to \mathbb{R}$ be a twice continuously differentiable function such that

$$|f''(x) + 2xf'(x) + (x^2 + 1)f(x)| \leq 1$$

for all x. Prove that $\lim_{x\to\infty} f(x) = 0$.