Let  $f \in C^1(a, b)$ ,  $\lim_{x \to a+} f(x) = +\infty$ ,  $\lim_{x \to b-} f(x) = -\infty$  and  $f'(x) + f^2(x) \ge -1$  for  $x \in (a, b)$ . Prove that  $b - a \ge \pi$  and give an example where  $b - a = \pi$ .