Consider the following sequence

$$(a_n)_{n=1}^{\infty} = (1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, 1, \dots).$$

Find all pairs  $(\alpha, \beta)$  of positive real numbers such

that 
$$\lim_{k \to 1} \frac{\sum_{k=1}^{\infty} a_k}{\sum_{k=1}^{\infty} a_k} = \beta$$
.