Let A be a $n \times n$ diagonal matrix with characteristic polynomial

$$(x - c_1)^{d_1} (x - c_2)^{d_2} \ldots (x - c_k)^{d_k},$$

where $c_1, c_2, \ldots, c_k$ are distinct (which means that c_1 appears d_1 times on the diagonal, c_2 appears d_2 times on the diagonal, etc. and $d_1 + d_2 + \cdots + d_k = n$). Let V be the space of all $n \times n$ matrices B such that $AB = BA$. Prove that the dimension of V is

$$d_1^2 + d_2^2 + \cdots + d_k^2.$$