For a positive integer n, let $f(n)$ be the number obtained by writing n in binary and replacing every 0 with 1 and vice versa. For example, $n = 23$ is 10111 in binary, so $f(n)$ is 1000 in binary, therefore $f(23) = 8$. Prove that

$$\sum_{k=1}^{n} f(k) \leq \frac{n^2}{4}.$$

When does equality hold?