Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function whose gradient $\nabla f = \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}\right)$ exists at every point of $\mathbb{R}^n$ and satisfies the condition

$$\exists L > 0 \forall x_1, x_2 \in \mathbb{R}^n : \\ ||\nabla f(x_1) - \nabla f(x_2)|| \leq L||x_1 - x_2||.$$

Prove that

$$\forall x_1, x_2 \in \mathbb{R}^n : ||\nabla f(x_1) - \nabla f(x_2)||^2 \\ \leq L\langle \nabla f(x_1) - \nabla f(x_2), x_1 - x_2 \rangle.$$

In this formula $\langle a, b\rangle$ denotes the scalar product of the vectors a and b.