Let $f(x) = x^2 + bx + c$, where b and c are real numbers, and let

$$M = \{x \in \mathbb{R} : |f(x)| < 1\}.$$

Clearly the set M is either empty or consists of disjoint open intervals. Denote the sum of their lengths by $|M|$. Prove that

$$|M| \leq 2\sqrt{2}.$$