Prove that for any function  $f: \mathbb{Q} \to \mathbb{Z}$ , there exist  $a, b, c \in \mathbb{Q}$  such that  $a < b < c, f(b) \ge f(a)$ , and

 $f(b) \ge f(c)$ .