

Compute

$$\lim_{n \rightarrow \infty} \frac{1}{\log \log n} \sum_{k=1}^n (-1)^k \binom{n}{k} \log k.$$

(Here \log denotes the natural logarithm.)