

For  $n \geq 1$  let  $M$  be an  $n \times n$  complex matrix with distinct eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_k$  with multiplicities  $m_1, m_2, \dots, m_k$ , respectively. Consider the linear operator  $L_M$  defined by  $L_M(X) = MX + XM^T$ , for any complex  $n \times n$  matrix  $X$ . Find its eigenvalues and their multiplicities. ( $M^T$  denotes the transpose of  $M$ ; that is, if  $M = (m_{k,l})$ , then  $M^T = (m_{l,k})$ .)