Let $f : [0,1] \to \mathbb{R}$ be a continuous function. Say that $f$ "crosses the axis" at $x$ if $f(x) = 0$ but in any neighbourhood of $x$ there are $y, z$ with $f(y) < 0$ and $f(z) > 0$.

a) Give an example of a continuous function that "crosses the axis" infinitely often.

b) Can a continuous function "cross the axis" uncountably often?

Justify your answer.