Let $f : \mathbb{R} \to (0, \infty)$ be an increasing differentiable function for which $\lim_{x\to\infty} f(x) = \infty$ and f' is bounded.
Let $F(x) = \int_0^x f$. Define the sequence (a_n) inductively by

$$a_0 = 1, \qquad a_{n+1} = a_n + \frac{1}{f(a_n)},$$

and the sequence (b_n) simply by $b_n = F^{-1}(n)$. Prove that $\lim_{n\to\infty}(a_n - b_n) = 0$.