Let $n \geq 2$ be an integer. Find all real numbers a such that there exist real numbers $x_1, \ldots, x_n$ satisfying

$$x_1(1 - x_2) = x_2(1 - x_3) = \ldots$$
$$= x_{n-1}(1 - x_n) = x_n(1 - x_1) = a.$$