

Let $A = (a_{ij})_{i,j=1}^n$ be a symmetric $n \times n$ matrix with real entries, and let $\lambda_1, \lambda_2, \dots, \lambda_n$ denote its eigenvalues. Show that

$$\sum_{1 \leq i < j \leq n} a_{ii}a_{jj} \geq \sum_{1 \leq i < j \leq n} \lambda_i \lambda_j,$$

and determine all matrices for which equality holds.