

In a town every two residents who are not friends have a friend in common, and no one is a friend of everyone else. Let us number the residents from 1 to  $n$  and let  $a_i$  be the number of friends of the  $i$ -th resident. Suppose that  $\sum_{i=1}^n a_i^2 = n^2 - n$ . Let  $k$  be the smallest number of residents (at least three) who can be seated at a round table in such a way that any two neighbors are friends. Determine all possible values of  $k$ .