

Let X be a set of $\binom{2k-4}{k-2} + 1$ real numbers, $k \geq 2$. Prove that there exists a monotone sequence $\{x_i\}_{i=1}^k \subseteq X$ such that

$$|x_{i+1} - x_1| \geq 2|x_i - x_1|$$

for all $i = 2, \dots, k-1$.