Suppose that a, b, c are real numbers in the interval $[-1, 1]$ such that

$$1 + 2abc \geq a^2 + b^2 + c^2.$$

Prove that

$$1 + 2(abc)^n \geq a^{2n} + b^{2n} + c^{2n}$$

for all positive integers n.