

For an integer $n \geq 3$ consider the sets

$$S_n = \{(x_1, x_2, \dots, x_n) : \forall i \ x_i \in \{0, 1, 2\}\}$$

$$A_n = \{(x_1, x_2, \dots, x_n) \in S_n : \forall i \leq n-2 \\ |\{x_i, x_{i+1}, x_{i+2}\}| \neq 1\}$$

and

$$B_n = \{(x_1, x_2, \dots, x_n) \in S_n : \forall i \leq n-1 \\ (x_i = x_{i+1} \implies x_i \neq 0)\}.$$

Prove that $|A_{n+1}| = 3 \cdot |B_n|$.

($|A|$ denotes the number of elements of the set A .)