

For any positive integer m , denote by $P(m)$ the product of positive divisors of m (e.g. $P(6) = 36$). For every positive integer n define the sequence

$$a_1(n) = n, \quad a_{k+1}(n) = P(a_k(n)) \quad (k = 1, 2, \dots, 2016).$$

Determine whether for every set $S \subseteq \{1, 2, \dots, 2017\}$, there exists a positive integer n such that the following condition is satisfied:

For every k with $1 \leq k \leq 2017$, the number $a_k(n)$ is a perfect square if and only if $k \in S$.