

Determine all pairs  $(a, b)$  of real numbers for which there exists a unique symmetric  $2 \times 2$  matrix  $M$  with real entries satisfying  $\text{trace}(M) = a$  and  $\det(M) = b$ .