Suppose that the differentiable functions $a, b, f, g : \mathbb{R} \to \mathbb{R}$ satisfy

$$f(x) \geq 0, f'(x) \geq 0, g(x) > 0, g'(x) > 0 \text{ for all } x \in \mathbb{R},$$

$$\lim_{x\to\infty} a(x) = A > 0, \qquad \lim_{x\to\infty} b(x) = B > 0,$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty,$$

and

$$\frac{f'(x)}{g'(x)} + a(x)\frac{f(x)}{g(x)} = b(x).$$

Prove that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{B}{A+1}.$$