

Let $n > 1$ be an odd positive integer and $A = (a_{ij})_{i,j=1\dots n}$ be the $n \times n$ matrix with

$$a_{ij} = \begin{cases} 2 & \text{if } i = j \\ 1 & \text{if } i - j \equiv \pm 2 \pmod{n} \\ 0 & \text{otherwise.} \end{cases}$$

Find $\det A$.