Let n be a positive integer, and let $p(x)$ be a polynomial of degree n with integer coefficients. Prove that

$$\max_{0 \le x \le 1} \left| p(x) \right| > \frac{1}{e^n}.$$