We say that a square-free positive integer n is *almost prime* if

$$n \mid x^{d_1} + x^{d_2} + ... + x^{d_k} - kx$$

for all integers x, where $1 = d_1 < d_2 < \cdots < d_k = n$ are all the positive divisors of n. Suppose that r is a Fermat prime (i.e., it is a prime of the form $2^{2^m} + 1$ for an integer $m \geq 0$), p is a prime divisor of an almost prime integer n, and $p \equiv 1 \pmod{r}$. Show that, with the above notation, $d_i \equiv 1 \pmod{r}$ for all $1 \leq i \leq k$.
(An integer n is called *square-free* if it is not divisible by d^2 for any integer $d > 1$.)