Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Suppose $f(0) = 0$. Prove that there exists $\xi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that

$$f''(\xi) = f(\xi)(1 + 2\tan^2 \xi).$$