Suppose $f : \mathbb{R} \to \mathbb{R}$ is a two times differentiable function satisfying $f(0) = 1$, $f'(0) = 0$, and for all $x \in [0, \infty)$,

$$f''(x) - 5f'(x) + 6f(x) \geq 0.$$

Prove that for all $x \in [0, \infty)$,

$$f(x) \geq 3e^{2x} - 2e^{3x}.$$