Let α be a real number, $1 < \alpha < 2$.

a) Show that α has a unique representation as an infinite product

$$\alpha = \left(1 + \frac{1}{n_1}\right)\left(1 + \frac{1}{n_2}\right)\cdots$$

where each n_i is a positive integer satisfying

$$n_i^2 \leq n_{i+1}.$$

b) Show that α is rational if and only if its infinite product has the following property: For some m and all $k \geq m$,

$$n_{k+1} = n_k^2.$$