Let A be an $n \times n$ matrix with complex entries and suppose that $n > 1$. Prove that

$$A\overline{A} = I_n \iff \exists S \in \mathrm{GL}_n(\mathbb{C}) \text{ such that } A = S\overline{S}^{-1}.$$

(If $A = [a_{ij}]$ then $\overline{A} = [\overline{a_{ij}}]$, where $\overline{a_{ij}}$ is the complex conjugate of a_{ij}; $\mathrm{GL}_n(\mathbb{C})$ denotes the set of all $n \times n$ invertible matrices with complex entries, and I_n is the identity matrix.)