Let f be a $C^3(\mathbb{R})$ non-negative function, $f(0) = f'(0) = 0$, $0 < f''(0)$. Let

$$g(x) = \left(\frac{\sqrt{f(x)}}{f'(x)} \right)'$$

for $x \neq 0$ and $g(0) = 0$. Show that g is bounded in some neighbourhood of 0. Does the theorem hold for $f \in C^2(\mathbb{R})$?