Let A be a $n \times n$ complex matrix whose eigenvalues have absolute value at most 1. Prove that

$$\|A^n\| \leq \frac{n}{\ln 2}\|A\|^{n-1}.$$

(Here $\|B\| = \sup_{\|x\| \leq 1} \|Bx\|$ for every $n \times n$ matrix B and $\|x\| = \sqrt{\sum_{i=1}^{n} |x_i|^2}$ for every complex vector $x \in \mathbb{C}^n$.)