

Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a twice differentiable function such that

$$2f'(x) + xf''(x) \geq 1 \quad \text{for } x \in (-1, 1).$$

Prove that

$$\int_{-1}^1 xf(x) \, dx \geq \frac{1}{3}.$$