Define the sequence $a_0, a_1, \ldots$ of numbers by the following recurrence:

$$a_0 = 1, \quad a_1 = 2,$$

$$(n+3)a_{n+2} = (6n+9)a_{n+1} - na_n \quad \text{for } n \geq 0.$$

Prove that all terms of this sequence are integers.