

Let  $k$  and  $n$  be positive integers. A sequence  $(A_1, \dots, A_k)$  of  $n \times n$  real matrices is *preferred* by *Ivan the Confessor* if  $A_i^2 \neq 0$  for  $1 \leq i \leq k$ , but  $A_i A_j = 0$  for  $1 \leq i, j \leq k$  with  $i \neq j$ . Show that  $k \leq n$  in all preferred sequences, and give an example of a preferred sequence with  $k = n$  for each  $n$ .