Compute the determinant of the $n \times n$ matrix $A = [a_{ij}],$

$$a_{ij} = \begin{cases} (-1)^{|i-j|}, & \text{if } i \neq j \\ 2, & \text{if } i = j. \end{cases}$$