

Let $A = (a_{k,l})_{k,l=1,\dots,n}$ be an $n \times n$ complex matrix such that for each $m \in \{1, \dots, n\}$ and $1 \leq j_1 < \dots < j_m \leq n$ the determinant of the matrix $(a_{j_k, j_l})_{k,l=1,\dots,m}$ is zero. Prove that $A^n = 0$ and that there exists a permutation $\sigma \in S_n$ such that the matrix

$$(a_{\sigma(k), \sigma(l)})_{k,l=1,\dots,n}$$

has all of its nonzero elements above the diagonal.