Let $A_1, A_2, \ldots, A_n$ be finite, nonempty sets. Define the function

$$f(t) = \sum_{k=1}^{n} \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n} (-1)^{k-1} t^{|A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_k}|}.$$

Prove that f is nondecreasing on $[0, 1]$.
($|A|$ denotes the number of elements in A.)