

Let A be a subset of $\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z}$ containing at most $\frac{1}{100} \ln n$ elements. Define the r th Fourier coefficient of A for $r \in \mathbb{Z}_n$ by

$$f(r) = \sum_{s \in A} \exp\left(\frac{2\pi i}{n} sr\right).$$

Prove that there exists an $r \neq 0$, such that $|f(r)| \geq \frac{|A|}{2}$.