Let a be a rational number and let n be a positive integer. Prove that the polynomial $X^{2^n}(X+a)^{2^n}+1$ is irreducible in the ring $\mathbb{Q}[X]$ of polynomials with rational coefficients.