Let $a_0 = \sqrt{2}$, $b_0 = 2$, $a_{n+1} = \sqrt{2 - \sqrt{4 - a_n^2}}$, $b_{n+1} = \frac{2b_n}{2+\sqrt{4+b_n^2}}$.

a) Prove that the sequences (a_n), (b_n) are decreasing and converge to 0.

b) Prove that the sequence $(2^n a_n)$ is increasing, the sequence $(2^n b_n)$ is decreasing and that these two sequences converge to the same limit.

c) Prove that there is a positive constant C such that for all n the following inequality holds: $0 < b_n - a_n < \frac{C}{8^n}$.