Let n be a positive integer. Find all $n \times n$ matrices A with only real eigenvalues satisfying

$$A + A^k = A^T$$

for some integer $k \geq n$.
(A^T denotes the transpose of A.)