

Let $d \geq 2$ be an integer. Prove that there exists a constant $C(d)$ such that the following holds: For any convex polytope $K \subset \mathbb{R}^d$, which is symmetric about the origin, and any $\varepsilon \in (0, 1)$, there exists a convex polytope $L \subset \mathbb{R}^d$ with at most $C(d)\varepsilon^{1-d}$ vertices such that

$$(1 - \varepsilon)K \subseteq L \subseteq K.$$

(For a real α , a set $T \subset \mathbb{R}^d$ with nonempty interior is a convex polytope with at most α vertices, if T is a convex hull of a set $X \subset \mathbb{R}^d$ of at most α points, i.e.,

$$T = \left\{ \sum_{x \in X} t_x x \mid t_x \geq 0, \sum_{x \in X} t_x = 1 \right\}.$$

For a real λ , put $\lambda K = \{\lambda x \mid x \in K\}$. A set $T \subset \mathbb{R}^d$ is symmetric about the origin if $(-1)T = T$.)