Let G be a group and $n \geq 2$ be an integer. Let H_1 and H_2 be two subgroups of G that satisfy

$$[G : H_1] = [G : H_2] = n$$

and

$$[G : (H_1 \cap H_2)] = n(n-1).$$

Prove that H_1 and H_2 are conjugate in G.
(Here $[G : H]$ denotes the *index* of the subgroup H, i.e. the number of distinct left cosets xH of H in G. The subgroups H_1 and H_2 are *conjugate* if there exists an element $g \in G$ such that $g^{-1}H_1g = H_2$.)