Determine all pairs $(a, b) \in \mathbb{C} \times \mathbb{C}$ satisfying

$$|a| = |b| = 1 \quad \text{and} \quad a + b + a\bar{b} \in \mathbb{R}.$$