

- a) Prove that for every $\epsilon > 0$ there is a positive integer n and real numbers $\lambda_1, \dots, \lambda_n$ such that

$$\max_{x \in [-1, 1]} \left| x - \sum_{k=1}^n \lambda_k x^{2k+1} \right| < \epsilon.$$

- b) Prove that for every odd continuous function f on $[-1, 1]$ and for every $\epsilon > 0$ there is a positive integer n and real numbers μ_1, \dots, μ_n such that

$$\max_{x \in [-1, 1]} \left| f(x) - \sum_{k=1}^n \mu_k x^{2k+1} \right| < \epsilon.$$

Recall that f is odd means that $f(x) = -f(-x)$ for all $x \in [-1, 1]$.