Let f be continuous and nowhere monotone on $[0, 1]$. Show that the set of points on which f attains local minima is dense in $[0, 1]$.

(A function is nowhere monotone if there exists no interval where the function is monotone. A set is dense if each non-empty open interval contains at least one element of the set.)