

For a positive integer x , denote its n^{th} decimal digit by $d_n(x)$, i.e. $d_n(x) \in \{0, 1, \dots, 9\}$ and $x = \sum_{n=1}^{\infty} d_n(x) 10^{n-1}$. Suppose that for some sequence $(a_n)_{n=1}^{\infty}$, there are only finitely many zeros in the sequence $(d_n(a_n))_{n=1}^{\infty}$. Prove that there are infinitely many positive integers that do not occur in the sequence $(a_n)_{n=1}^{\infty}$.