For each positive integer k, find the smallest number n_k for which there exist real $n_k \times n_k$ matrices $A_1, A_2, \ldots, A_k$ such that all of the following conditions hold:

(1) $A_1^2 = A_2^2 = \cdots = A_k^2 = 0$,

(2) $A_i A_j = A_j A_i$ for all $1 \leq i, j \leq k$, and

(3) $A_1 A_2 \ldots A_k \neq 0$.