Let n be a positive integer. Show that there are positive real numbers $a_0, a_1, \ldots, a_n$ such that for each choice of signs the polynomial

$$\pm a_n x^n \pm a_{n-1} x^{n-1} \pm \cdots \pm a_1 x \pm a_0$$

has n distinct real roots.