

Let p and q be relatively prime positive integers.
Prove that

$$\sum_{k=0}^{pq-1} (-1)^{\lfloor \frac{k}{p} \rfloor + \lfloor \frac{k}{q} \rfloor} = \begin{cases} 0 & \text{if } pq \text{ is even,} \\ 1 & \text{if } pq \text{ is odd.} \end{cases}$$

(Here $\lfloor x \rfloor$ denotes the integer part of x .)