A matrix $A = (a_{ij})$ is called *nice*, if it has the following properties:

(i) the set of all entries of A is $\{1, 2, \ldots, 2t\}$ for some integer t;

(ii) the entries are non-decreasing in every row and in every column: $a_{i,j} \leq a_{i,j+1}$ and $a_{i,j} \leq a_{i+1,j}$;

(iii) equal entries can appear only in the same row or the same column: if $a_{i,j} = a_{k,\ell}$, then either $i = k$ or $j = \ell$;

(iv) for each $s = 1, 2, \ldots, 2t-1$, there exist $i \neq k$ and $j \neq \ell$ such that $a_{i,j} = s$ and $a_{k,\ell} = s+1$.

Prove that for any positive integers m and n, the number of nice $m \times n$ matrices is even.
For example, the only two nice 2×3 matrices are $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 & 3 \\ 2 & 4 & 4 \end{pmatrix}$.