Let $f : [0, +\infty) \to \mathbb{R}$ be a continuous function such that $\lim\limits_{x \to +\infty} f(x) = L$ exists (it may be finite or infinite). Prove that

$$\lim_{n \to \infty} \int_0^1 f(nx)\, \mathrm{d}x = L.$$