

Let $a_1 = 1$, $a_n = \frac{1}{n} \sum_{k=1}^{n-1} a_k a_{n-k}$ for $n \geq 2$. Show that

$$(i) \limsup_{n \rightarrow \infty} |a_n|^{1/n} < 2^{-1/2};$$

$$(ii) \limsup_{n \rightarrow \infty} |a_n|^{1/n} \geq 2/3.$$