

Let n be a positive integer and let V be a $(2n - 1)$ -dimensional vector space over the two-element field. Prove that for arbitrary vectors $v_1, \dots, v_{4n-1} \in V$, there exists a sequence $1 \leq i_1 < \dots < i_{2n} \leq 4n - 1$ of indices such that $v_{i_1} + \dots + v_{i_{2n}} = 0$.