Fix positive integers n and k such that $2 \leq k \leq n$ and a set M consisting of n fruits. A *permutation* is a sequence $x = (x_1, x_2, \ldots, x_n)$ such that $\{x_1, \ldots, x_n\} = M$. Ivan *prefers* some (at least one) of these permutations. He realized that for every preferred permutation x, there exist k indices $i_1 < i_2 < \ldots < i_k$ with the following property: for every $1 \leq j < k$, if he swaps x_{i_j} and $x_{i_{j+1}}$, he obtains another preferred permutation.

Prove that he prefers at least $k!$ permutations.