Let n be a positive integer. Suppose that A and B are invertible $n \times n$ matrices with complex entries such that $A + B = I$ (where I is the identity matrix) and

$$(A^2 + B^2)(A^4 + B^4) = A^5 + B^5.$$

Find all possible values of $\det(AB)$ for the given n.