Let $f : (0,1) \to [0,\infty)$ be a function that is zero except at the distinct points $a_1, a_2, \ldots$. Let $b_n = f(a_n)$.

(a) Prove that if $\sum_{n=1}^{\infty} b_n < \infty$, then f is differentiable at at least one point $x \in (0,1)$.

(b) Prove that for any sequence of non-negative real numbers $(b_n)_{n=1}^{\infty}$, with $\sum_{n=1}^{\infty} b_n = \infty$, there exists a sequence $(a_n)_{n=1}^{\infty}$ such that the function f defined as above is nowhere differentiable.