Let $p > 1$. Show that there exists a constant $K_p > 0$ such that for every $x, y \in \mathbb{R}$ satisfying $|x|^p + |y|^p = 2$, we have

$$(x - y)^2 \leq K_p \left(4 - (x + y)^2 \right).$$