Denote by V the real vector space of all real polynomials in one variable, and let $P : V \to \mathbb{R}$ be a linear map. Suppose that for all $f, g \in V$ with $P(fg) = 0$ we have $P(f) = 0$ or $P(g) = 0$. Prove that there exist real numbers x_0, c such that $P(f) = cf(x_0)$ for all $f \in V$.