Find all positive integers n for which there exists a family $\mathcal{F}$ of three-element subsets of $S = \{1, 2, \ldots, n\}$ satisfying the following two conditions:

(i) for any two different elements $a, b \in S$, there exists exactly one $A \in \mathcal{F}$ containing both a, b;

(ii) if a, b, c, x, y, z are elements of S such that if $\{a, b, x\}, \{a, c, y\}, \{b, c, z\} \in \mathcal{F}$, then $\{x, y, z\} \in \mathcal{F}$.