

For each $n \geq 1$ let

$$a_n = \sum_{k=0}^{\infty} \frac{k^n}{k!}, \quad b_n = \sum_{k=0}^{\infty} (-1)^k \frac{k^n}{k!}.$$

Show that $a_n \cdot b_n$ is an integer.