Prove that if p and q are rational numbers and $r = p + q\sqrt{7}$, then there exists a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \pm \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ with integer entries and with $ad - bc = 1$ such that

$$\frac{ar + b}{cr + d} = r.$$