

Let $f: \mathbb{R} \rightarrow (0, \infty)$ be a differentiable function, and suppose that there exists a constant $L > 0$ such that

$$|f'(x) - f'(y)| \leq L|x - y|$$

for all x, y . Prove that

$$(f'(x))^2 < 2Lf(x)$$

holds for all x .