Find all polynomials $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ $(a_n \neq 0)$ satisfying the following two conditions:

(i) $(a_0, a_1, \ldots, a_n)$ is a permutation of the numbers $(0, 1, \ldots, n)$

and

(ii) all roots of $P(x)$ are rational numbers.