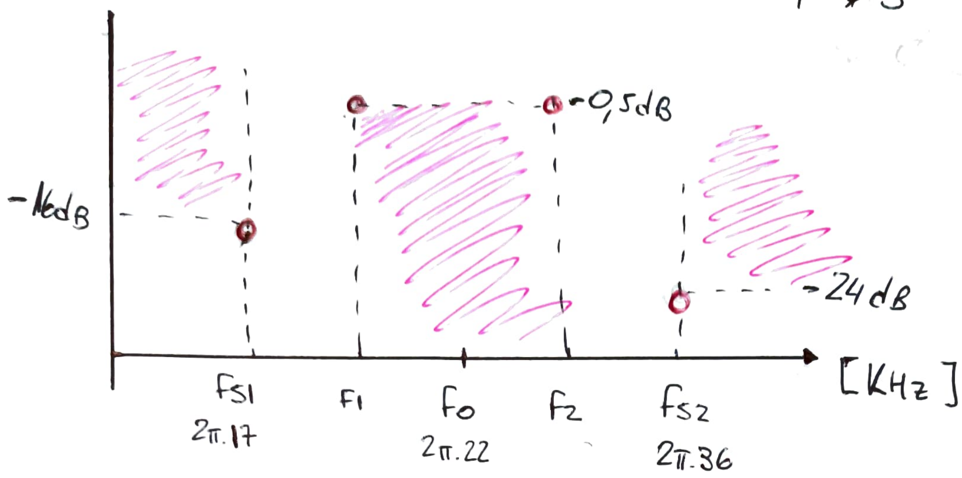
Tarea Semana 3

$\omega_0 = 2\pi \cdot 22\,KHz$

$q = 5$



$$q = \frac{\omega_0}{B\omega}$$

$$B\omega = \omega_{p2} - \omega_{p1}$$

$$\omega_0 = \sqrt{\omega_{p1} \cdot \omega_{p2}}$$

$$\Rightarrow q \cdot (\omega_{p2} - \omega_{p1}) = \omega_0$$

; Como estoy normalizando $\Omega_0 = 1$

$$q(\omega_{p2} - \omega_{p1}) = 1$$

$$q \cdot \omega_{p2} - q\,\omega_{p1} = 1$$

$1 = \sqrt{\omega_{p1} \cdot \omega_{p2}}$

$\omega_{p1} = 1/\omega_{p2}$

$$q \cdot \omega_{p2} - q \frac{1}{\omega_{p2}} = 1$$

$$q \cdot \omega_{p2}^2 - q = \omega_{p2}$$

$$\omega_{p2}^2 \cdot q - \omega_{p2} - q = 0$$

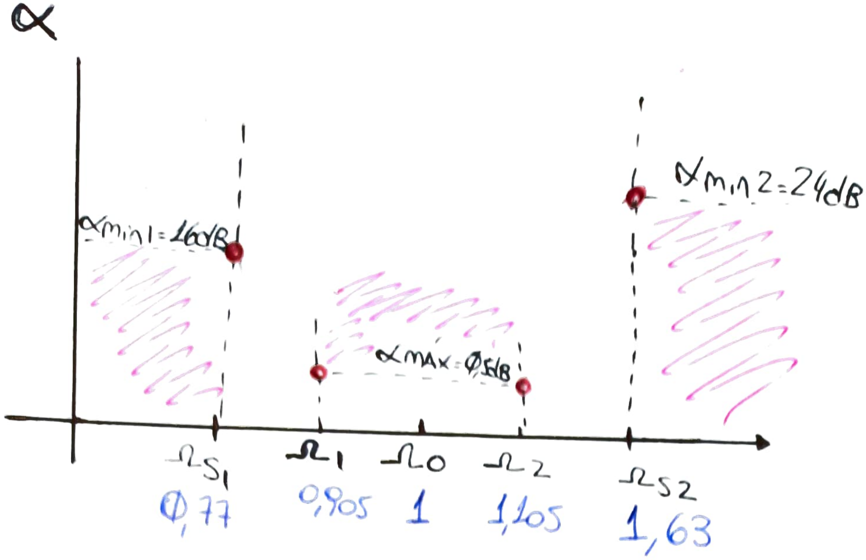
$$\omega_{p2}^2 \cdot 5 - \omega_{p2} - 5 = 0$$

$\omega_{p2} \to 1{,}1049 \cong 1{,}105 \therefore \omega_{p1} \cong 0{,}905$

$\quad \to -0{,}9 \; \otimes$

$$\Rightarrow \boxed{\begin{array}{l} \omega_{p1} = 0{,}905 \\ \omega_{p2} = 1{,}105 \end{array}}$$

Ya estoy en condiciones de hacer mi plantilla



$\boxed{\text{Plantilla FBp normalizada}}$

tengo 2 Rq a cumplir
Como es un FBp geométrico tendré que cumplir solo el más exigente

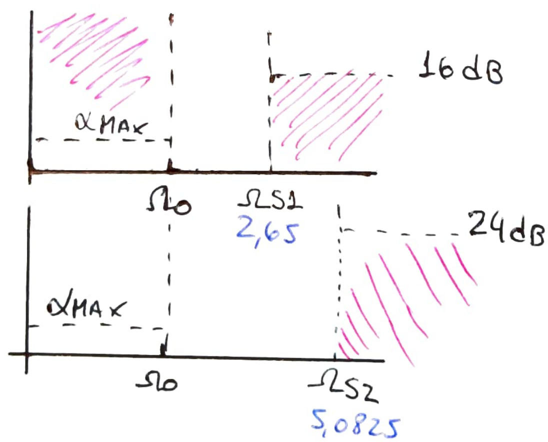
$$\Omega_{s_{i\,LP}} = q \frac{(\Omega_{s_i}^2 - 1)}{\Omega_{s_i}}$$

$\Omega_{s1} = +2,65$

$\Omega_{s2} = 5,0825$

$$\xi^2 = 10^{\alpha_{MAX}/10} - 1$$

$$\boxed{\xi^2 = 0,122}$$



$$10 \log\left(1 + \xi^2 \cdot \cosh^2\left[n \cdot \cosh^{-1}(\Omega_{s_i})\right]\right) \geq \alpha_{min\,1}$$

$n_1 = 2 = 13,37$

$n_1 = 3 = 27,32\,dB$

$n_2 = 2 = 33,12\,dB$

=> Debo diseñar un Cheby orden 3

Mi problema ahora es hallar un Cheby de $n=3$

$\xi^2 = \emptyset,122$ $\qquad C_n = 2\omega \, C_{n-1} - C_{n-2}$

$n = 3$ $\quad ; \quad \therefore C_3 = 4\omega^3 - 3\omega$

$$\Rightarrow |T(j\omega)|^2 = \frac{1}{1 + C_3^2 \cdot \xi^2}$$

$$|T(j\omega)|^2 = \frac{1}{1 + \xi^2 (4\omega^3 - 3\omega)^2}$$

$$= \frac{1}{1 + \xi^2 (16\omega^6 - 24\omega^4 + 9\omega^2)}$$

$$|T(j\omega)|^2 = \frac{1}{\xi^2 \cdot 16 \cdot \omega^6 - \xi^2 \cdot 24\omega^4 + \xi^2 \cdot 9 \cdot \omega^2 + 1}$$

$$|T(j\omega)|^2 \Big|_{\omega = \frac{\$}{j}} = \frac{1}{-\xi^2 \cdot 16 \cdot \$^6 - \xi^2 \cdot 24 \cdot \$^4 - \xi^2 \cdot 9 \cdot \$^2 + 1}$$

$$|T(\$)|^2 = T(\$) \cdot T(-\$)$$

$$\frac{1}{\$^3 \cdot a_3 + \$^2 \cdot a_2 + \$ \cdot a_1 + a_0} \cdot \frac{1}{-\$ a_3 + \$^2 \cdot a_2 - \$ \cdot a_1 + a_0} = \frac{1}{\underbrace{-\xi^2 16 \cdot \$^6}\underbrace{-\xi^2 24 \cdot \$^4}\underbrace{-\xi^2 9 \cdot \$^2}\underbrace{+1}}$$

- $-\xi^2 \cdot 16 = \$^3 \cdot \$^3 \Rightarrow -a_3^2 \Rightarrow \cancel{-}\xi^2 16 = \cancel{-}a_3^2 \therefore \boxed{a_3 = 4\xi = 1{,}397}$
- $1 = a_0^2 \Rightarrow \boxed{a_0 = 1}$
- $-\xi^2 \cdot 24 = \$^2 \cdot \$^2 + 2 \cdot \$^3 \cdot \$ \Rightarrow -\xi^2 \cdot 24 = a_2^2 - 2 \cdot a_3 \cdot a_1$

  $-\xi^2 \cdot 24 = a_2^2 - 2 \cdot 1{,}397 \cdot a_1$
- $-\xi^2 \cdot 9 = \$ \cdot \$ + 2\$^2 \cdot \$^0 \Rightarrow -\xi^2 \cdot 9 = -a_1^2 + 2 \cdot a_2 \cdot \underset{1}{a_0}$

  $-\xi^2 \cdot 9 = -a_1^2 + 2 \cdot a_2$

$$\Rightarrow \quad -\xi^2 \cdot 24 = a_2^2 - 2{,}794 \cdot a_1 \Rightarrow a_2 = \sqrt{-\xi^2 24 + 2{,}794 \cdot a_1}$$

$$-\xi^2 \cdot 9 = -a_1^2 + 2 \cdot a_2$$

$$-\xi^2 \cdot 9 = -a_1^2 + 2\sqrt{2{,}794 \cdot a_1 - \xi^2 24}$$

Uso un solucionador online....

$$-2{,}928 = a_2^2 - 2{,}794 \cdot a_1$$

$$-1{,}098 = 2a_2 - a_1^2$$

$$\frac{a_2^2 + 2{,}928}{2{,}794} = a_1$$

$$\Rightarrow \quad -1{,}098 = 2a_2 - \left(\frac{a_2^2 + 2{,}928}{2{,}794}\right)^2$$

$$-1{,}098 = 2a_2 - \frac{(a_2^4 + 5{,}856\,a_2^2 + 8{,}57)}{7{,}8064}$$

$$\Rightarrow \quad -8{,}57 - 15{,}61 \cdot a_2 = -a_2^4 \overset{(-)}{+} 5{,}856\,a_2^2 \overset{(-)}{+} 8{,}57$$

$$-a_2^4 \overset{(-)}{+} 5{,}856 \cdot a_2^2 + 15{,}61 \cdot a_2 + \overset{0}{17{,}14} = 0$$ → Acá S. no Solucionable

- $a_2 = \mathbb{C}$ conj
- $a_2 = \mathbb{C}$ conj
- $a_2 = -1{,}644$
- $a_2 = 3{,}441 \quad \therefore \quad a_1 = 5{,}2857$

$$-a_2^4 - 5{,}856 \cdot a_2^2 + 15{,}61 \cdot a_2 = 0$$

$$a_2(-a_2^3 - 5{,}856 \cdot a_2 + 15{,}61)$$

$a_2 = 0$

$a_2 = \mathbb{C}$ conj $= -0{,}87 \pm j2{,}85$

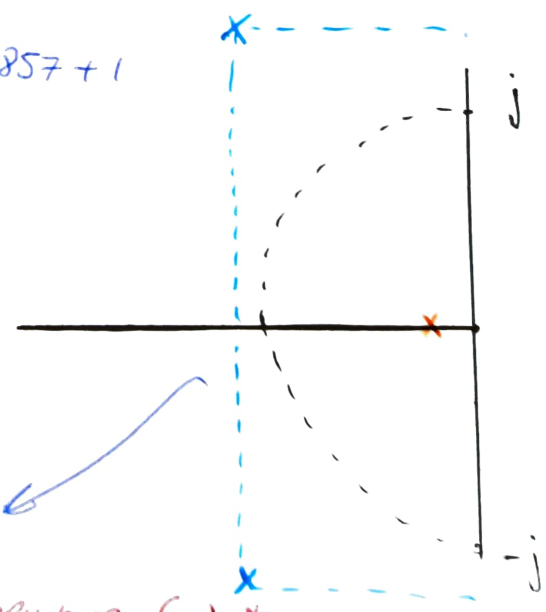
$a_2 = 1{,}75 \quad \therefore \quad a_1 = 2{,}144$

$$\Rightarrow \quad T(s) = \frac{1}{s^3 \cdot 1{,}397 + s^2 \cdot 3{,}441 + s \cdot 5{,}2857 + 1}$$

Polos

$\mathbb{R}$: $s = -0{,}2171$

$\mathbb{C}$ conj $s = -1{,}12 \pm j\,1{,}42$



No se

Parece a un elipse ~~[illegible]~~ me olvide un (−)

Viendo que no era una elipse encontré varios (-) que me comí

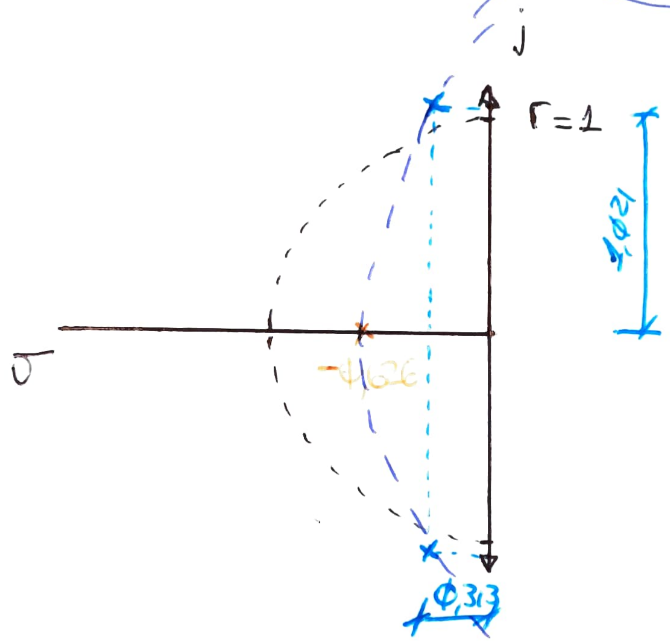
$$T(s) = \frac{1}{s^3 \cdot 1{,}397 + s^2 \cdot 1{,}75 + s \cdot 2{,}144 + 1}$$

$\times \quad s - (-0{,}626)$

$\times \quad s - (-0{,}313 \pm j\, 1{,}021)$

esto sí es una elipse

ES una elipse



$$\Rightarrow \boxed{T_{LP}(s) = \frac{0{,}626}{s + 0{,}626} \cdot \frac{1{,}14}{s^2 + s \cdot 2 \cdot 0{,}313 + 1{,}14}}$$

→ Filtro Low-Pass normalizado

Lo chequeo con python ... Cumple ???

$$T_{BP}(\$) = T_{LP}(\$)\Big|_{\$ = q\,\frac{\$^2+\omega_0^2}{\$\cdot\omega_0} = q\cdot\frac{\$^2+1}{\$}}$$

$$T_{BP}(\$) = \frac{0,626}{q\cdot\frac{\$^2+1}{\$}+0,626} \qquad \frac{1,14}{q^2\frac{(\$^2+1)^2}{\$^2}+q\frac{(\$^2+1)}{\$}\cdot 2\cdot 0,313+1,14}$$

$$= \frac{\$\cdot 0,626}{q\cdot\$^2+q+\$\cdot 0,626} \qquad \frac{1,14}{\;}$$

$$\frac{\$\cdot 0,626}{q\left(\$^2+\$\cdot\frac{0,626}{q}+1\right)}$$

$$\frac{\$\cdot 0,626/q}{\$^2+\$\cdot\frac{0,626}{q}+1} = \frac{\$\cdot 0,626/5}{\$^2+\$\cdot 0,626/5+1} = \boxed{H\,\frac{\$\cdot\sigma/q}{\$^2+\$\,\frac{\sigma}{q}+1}}$$

yo partí de

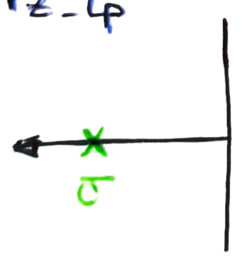
$$\frac{\sigma}{\$+\sigma}$$

$H=1 \Rightarrow F_{BP}$ 0 dB

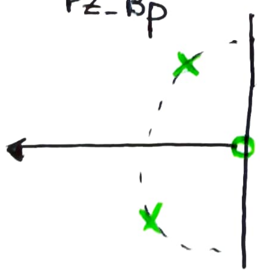
$Q_1=\frac{q}{\sigma}$ → ya estoy normalizado en $\omega_0=1$

$$\frac{\sigma}{q}=\frac{0,626}{5}=0,1252 \Rightarrow Q_1=7,987$$

Pz_Lp



Pz_Bp



$$\frac{1{,}14 \cdot \$^2}{q^2 \$^4 + q^2 \cdot 2 \$^2 + q^2 + (q \$^3 + q \$) \cdot 2 \cdot 0{,}313 + 1{,}14 \$^2}$$

$$\frac{\$^2 \, 1{,}14}{q^2 \$^4 + q \$^3 \cdot 0{,}626 + (2q^2 + 1{,}14)\$^2 + q \cdot 0{,}626 \$ + q^2}$$

$$\frac{\$^2 \cdot 1{,}14}{25 \$^4 + 3{,}13 \$^3 + 51{,}14 \$^2 + 3{,}13 \$ + 25}$$

Me ayudo de roots

↳ Polos c conj: $-0{,}034 \pm j\, 1{,}106 \Rightarrow \omega_{01}^2 = 1{,}226$

$-0{,}028 \pm j\, 0{,}902 \Rightarrow \omega_{02}^2 = 0{,}815$

$$\Rightarrow 1{,}14 \frac{\$}{\$^2 + \$ \cdot 2 \cdot 0{,}034 + 1{,}226} \cdot \frac{\$}{\$^2 + \$ \cdot 2 \cdot 0{,}028 + 0{,}815}$$

$$H \quad \frac{\$ \cdot \omega_{01}/Q_1}{\$^2 + \$ \cdot \frac{\omega_{01}}{Q_1} + \omega_{01}^2} \qquad \frac{\$ \cdot \omega_{02}/Q_2}{\$^2 + \$ \cdot \frac{\omega_{02}}{Q_2} + \omega_{02}^2}$$

Son iguales

$$0{,}068 = \frac{\omega_{01}}{Q_1} \qquad 0{,}056 = \frac{\omega_{02}}{Q_2}$$

$\Rightarrow Q_1 = 16{,}28 \quad \cong \quad Q_2 = 16{,}22 \rightarrow$ Hay error de decimales

⇒ lo Reescribo como F_{BP}

$$\frac{s \cdot \omega_0/Q}{s^2 + s \cdot \frac{\omega_0}{Q} + \omega_0^2}$$

$$\textcolor{red}{H} \quad \frac{s \cdot \frac{1,107}{16,28}}{s^2 + s \, \frac{1,107}{16,28} + 1,226} \qquad \frac{s \cdot \frac{0,902}{16,28}}{s^2 + s \, \frac{0,902}{16,28} + 0,815}$$

$\bigcirc \cdot \bigcirc = 0,00376$ ⇒ Necesito una ganancia $H = \frac{1,14}{0,00376}$ ⇒ $H = 302,59$

$$\boxed{T_{Bp}(\$) = \frac{H\ \$ \cdot \sigma/Q}{\$^2 + \$ \cdot \sigma/q + 1} \cdot \frac{H\ \$ \cdot \omega_{01}/Q}{\$^2 + \$ \cdot \omega_{01}/Q + \omega_{01}^2} \cdot \frac{H\ \$ \cdot \omega_{02}/Q}{\$^2 + \$ \cdot \omega_{02}/q + \omega_{02}^2}}$$

Aclaraciones:

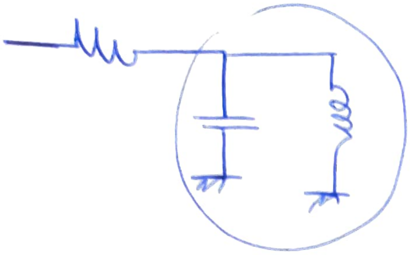
- El 1° Bp no tiene ω_0^2 en el numerador porque está normalizado
- Aparecen ganancias H a pesar de ser un filtro pasivo porque ~~a~~ al tener distintas bandas de paso al matchear las respuestas no todo da Ø dB.

$$T_{Bp}(\$) = \underbrace{\frac{1\ \$ \cdot 0{,}626/5}{\$^2 + \$ \cdot \frac{0{,}626}{5} + 1}}_{\text{Filtro I}} \cdot \underbrace{\frac{17{,}39\ \$ \cdot \frac{1{,}107}{16{,}28}}{\$^2 + \$ \cdot \frac{1{,}107}{16{,}28} + 1{,}226}}_{\text{Filtro II}} \cdot \underbrace{\frac{17{,}39\ \$ \cdot \frac{0{,}902}{16{,}28}}{\$^2 + \$ \cdot \frac{0{,}902}{16{,}28} + 0{,}815}}_{\text{Filtro III}}$$

HG

FILTRO BP

$$\cancel{Y}_p = sC + \frac{1}{sL} = \frac{s^2LC+1}{sL}$$



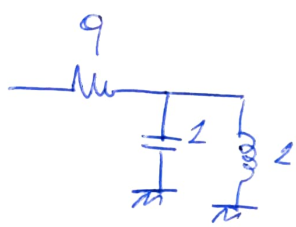
$$T(s) = \frac{Y_1}{Y_1+Y_p} = \frac{G}{G + \frac{s^2LC+1}{sL}}$$

$$T(s) = \frac{sLG}{sLG + s^2LC+1} = \frac{sLG}{s^2LC + sLG+1} = \frac{\cancel{L}G}{\cancel{L}C}\,\frac{s}{s^2 + s\frac{\cancel{L}G}{\cancel{L}C} + \frac{1}{LC}}$$

$$T(s) = \frac{1}{RC}\,\frac{s}{s^2 + s\,1/RC + 1/LC} \Rightarrow \omega_0^2 = 1/LC \qquad \frac{\omega_0}{Q} = 1/RC$$

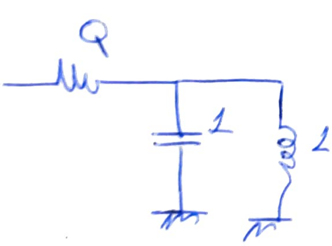
Si normalizo $\omega_0 = 1$

$$T(s) = \frac{s\;1/R}{s^2 + s\;1/R + 1} \Rightarrow$$



$\Rightarrow$

FILTRO (I)



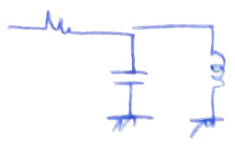
$$\Rightarrow R = \frac{5}{0{,}626} \Rightarrow$$

$R_n = 7{,}98$

$L_n = 1$

$C_n = 1$

Filtro (II)



$\omega_0^2 = 1/LC = 1,226$

$\frac{\omega_0}{Q} = 0,0679 = \frac{1}{RC}$

$\Rightarrow C = 1 \therefore 1,226 = \frac{1}{L} \Rightarrow L = 0,815$

$0,0679 = \frac{1}{R} \Rightarrow R = 14,7$

$$\boxed{\begin{array}{l} C_n = 1 \\ L_n = 0,815 \\ R_n = 14,7 \end{array}}$$

Filtro (III)

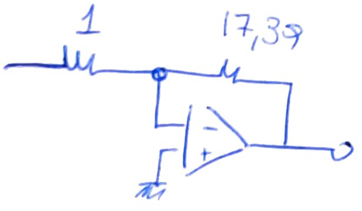
$C = 1 \Rightarrow 0,815 = \frac{1}{L} \Rightarrow L = 1,226$

$\frac{0,902}{16,28} = \frac{1}{R} \Rightarrow R = 18,048$

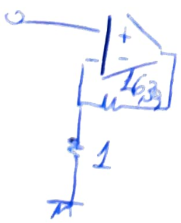
$$\boxed{\begin{array}{l} C_n = 1 \\ L_n = 1,226 \\ R_n = 18,048 \end{array}}$$

Me falta el tema de la ganancia...

Acoplo con 2 OpAmp inv que ganan 17,39 veces c/u

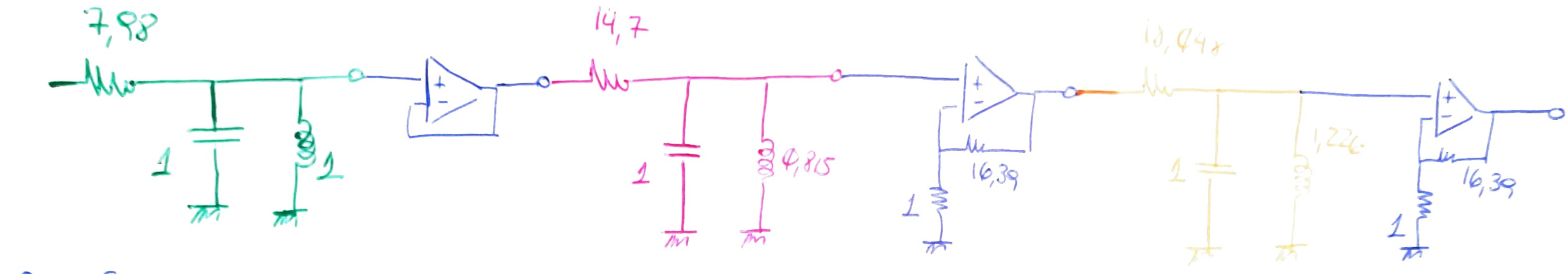


... el problema es que me carga el filtro anterior esa R=2



$A_v = 1 + \frac{16,39}{1}$

esto no me carga. Obvio que se pueden juntar las ganancias



$\Omega_w = 2\pi \cdot 22KHz$; $\Omega_z = 1K\Omega$

$R = 7{,}98 \cdot \Omega_z = 7{,}98K$

$L = \frac{1 \cdot \Omega_z}{\Omega_w} = 7{,}23\ mF$

$C = \frac{1}{\Omega_z \cdot \Omega_w} = 7{,}23\ nF$

$R = 14{,}7 \cdot \Omega_z = 14{,}7K$

$L = \frac{0{,}815 \cdot \Omega_z}{\Omega_w} = 5{,}89\ mF$

$C = 7{,}23\ nF$

$R_F = 16{,}39 \cdot \Omega_z = 16{,}39K$

$R_g = 1 \cdot \Omega_z = 1K$

$R = 18{,}048 \cdot \Omega_z = 18{,}048K$

$L = \frac{1{,}226 \cdot \Omega_z}{\Omega_w} = 8{,}86\ mF$

$C = 7{,}23\ nF$

Lo llevo al spice para ver si cumple ... Está ganando 28 dB .... 28 dB = 20 log (AV veces)

28 dB ≡ 25 veces más ...

Tengo algún problema con las ganancias .... Reviso los cálculos

#13

Después de 1 hora veo que usé Roots sin término mónico =>

Faltó • 25 ∴ ganó 25dB +

$$T_{BP}(\$) = \frac{\$ \cdot 0{,}626/5}{\$^2 + \$ \cdot \frac{0{,}626}{5} + 2} \cdot H^2 \cdot \frac{\$ \cdot \frac{1{,}107}{16{,}28}}{\$^2 + \$ \frac{1{,}107}{16{,}28} + 1{,}226} \cdot \frac{\$ \cdot \frac{0{,}902}{16{,}28}}{\$^2 + \$ \frac{0{,}902}{16{,}28} + 0{,}815} \rightsquigarrow$$

Antes igualaba a 1,14

Ahora igualo a $\frac{1{,}14}{25}$

$$\cancel{\frac{H^2}{25} \cdot \frac{1{,}107}{16{,}28} \cdot \frac{0{,}902}{16{,}28} = 1{,}14 \Rightarrow H^2 =}$$

$$\cancel{\frac{H^2 \cdot \frac{1{,}107}{16{,}28} \cdot \frac{0{,}902}{16{,}28}}{25} = 1{,}14 \Rightarrow H^2 \cdot 0{,}00376 = 1{,}14}$$

$$H^2 \cdot \frac{1{,}107}{16{,}28} \cdot \frac{0{,}902}{16{,}28} = \frac{1{,}14}{25} \Rightarrow H^2 = 12{,}1 \Rightarrow H = 3{,}47$$

Sólo cambio $R_F = 2{,}47$

Ahora sí tengo 0 dB

α MÁX

#14

$$\frac{3{,}47 \cdot 1{,}107 \cdot \$ \, \frac{1{,}107}{16{,}28}}{\$^2 \, 1{,}107^2 + \$ \cdot \frac{1{,}107 \cdot 1{,}107}{16{,}28} + 1{,}107^2} \cdot \frac{3{,}47 \cdot 0{,}902 \cdot \$ \cdot \frac{0{,}902}{16{,}28}}{\$^2 \, 0{,}902^2 + \$ \cdot \frac{0{,}902 \cdot 0{,}902}{16{,}28} + 0{,}902^2}$$

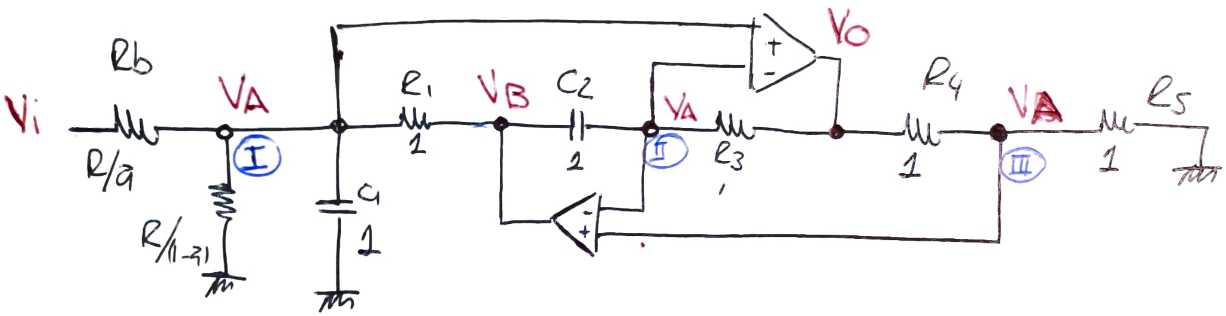
$$\cancel{\frac{1{,}107^2}{1{,}107^2}} \; \frac{3{,}47 \quad \$ \; 1/16{,}28}{\$^2 + \$ \; 1/16{,}28 + 1} \qquad \cancel{\frac{0{,}902^2}{0{,}902^2}} \; \frac{3{,}47 \cdot \$ \; 1/16{,}28}{\$^2 + \$ \; 1/16{,}28 + 1}$$

1- NORMALIZAR TODO

2- SACAR $\frac{V_O}{V_i}$ TOTAL y elegir $H=1$ y meter toda la ganancia junta

Elaboró Gustavo Salis

#15



$+\ 1/R/2$

I $\quad V_A\left(\frac{1}{R(1-a)} + \$C_1 + \frac{1}{R_1}\right) = \frac{V_i}{R/a} + \frac{V_B}{R_1}$

II $\quad V_A\left(\$C_2 + \frac{1}{R_3}\right) = \frac{V_o}{R_3} + V_B \cdot \$C_2 \Rightarrow V_o\left[\frac{R_5}{R_5+R_4}\left(\$C_2 + \frac{1}{R_3}\right) - \frac{1}{R_3}\right] = V_B \cdot \$C_2 \Rightarrow V_o\left[\frac{1}{2}(\$+1) - 1\right] = V_B \cdot \$$

III $\quad V_A\left(\frac{1}{R_4} + \frac{1}{R_5}\right) = \frac{V_o}{R_4} \Rightarrow V_A \frac{R_5+R_4}{\cancel{R_4} \cdot R_5} \cdot \cancel{R_4} = V_o \Rightarrow V_A = V_o \cdot \frac{R_5}{R_5+R_4} \Rightarrow \boxed{V_A = V_o \cdot \frac{1}{2}}$

$$V_o\left(\frac{\$}{2} - \frac{1}{2}\right) = V_B \cdot \$$$

$$\boxed{V_B = \frac{V_o}{2} \frac{(\$-1)}{\$}}$$

$$\Rightarrow \cancel{V_o \frac{1}{2}\left(\frac{1}{R(1-a)} + \$ + 1\right) = \frac{V_i}{R/a}}$$

R_b

$$V_o \cdot \frac{1}{2}\left(\frac{1}{R_b} + \frac{1}{R_c} + \$ + 1\right) = \frac{V_i}{R_b} + V_o \, \frac{1}{2} \, \frac{(\$-1)}{\$}$$

$$V_o \, \frac{1}{2}\left[\frac{1}{R_b} + \frac{1}{R_c} + \$ + 1 - \frac{(\$-1)}{\$}\right] = \frac{V_i}{R_b}$$

$$V_o \, \frac{1}{2}\left[\frac{1}{R_b} + \frac{1}{R_c} + \$ + \cancel{1} - \cancel{\frac{\$}{\$}} + \frac{1}{\$}\right] = \frac{V_i}{R_b}$$

$$\frac{V_o}{2}\left(\frac{1}{R_b} + \frac{1}{R_c} + \frac{\$^2+1}{\$}\right) = \frac{V_i}{R_b} \Rightarrow \frac{V_o}{V_i} = \frac{2}{R_b} \, \frac{1}{\left(\frac{1}{R_b} + \frac{1}{R_c} + \frac{\$^2+1}{\$}\right)}$$

~~No me sirve. Necesito ...~~

$$\frac{V_o}{V_i} = \frac{2a}{R} \, \frac{\$}{\$^2 + \$ \, {}^1/_R + 1}$$

$$\frac{2a}{R} = \frac{3{,}47}{9} \Rightarrow a = 0$$

$$\frac{1}{R} = \frac{1}{9}$$

$$\frac{V_o}{V_i} = \frac{2}{R_b} \frac{\$ \cdot R_b \cdot R_c}{\$(R_c + R_b) + \$^2 \cdot R_b \cdot R_c + R_b \cdot R_c} = \frac{2}{\frac{R}{a}} \frac{\$ \cdot \frac{R}{a} \cdot \frac{R}{(1-a)}}{\$^2 \cdot \frac{R}{a} \cdot \frac{R}{1-a} + \$\left(\frac{R}{a} + \frac{R}{(1-a)}\right) + \frac{R}{a} \cdot \frac{R}{1-a}}$$

$$= \frac{2}{R/a} \frac{\$ \quad R^2/a(1-a)}{\$^2 \, R^2/a(1-a) + \$ \, \frac{R - \cancel{Ra} + \cancel{Ra}}{a(1-a)} + \frac{R^2}{a(1-a)}} = \frac{2}{R/a} \frac{R^2/\cancel{a(1-a)}}{R^2/\cancel{a(1-a)}} \frac{\$}{\$^2 + \$ \cdot 1/R + 1}$$

$$\frac{V_o}{V_i} = \frac{2}{R/a} \cdot \frac{\$}{\$^2 + \$ \cdot 1/R + 1} \Rightarrow \frac{2a}{R} \cdot \frac{\$}{\$^2 + \$ \, 1/R + 1} \Rightarrow \frac{1}{q} = \frac{1}{R} \Rightarrow \boxed{R = q}$$

$$2$$

$$\boxed{2a = H} \Rightarrow$$ Adopto usar $a = 0{,}5$