

## Exercise sheet 3

Deadline: Tuesday, 6.11.2018 12:00 AM

### Exercise 1 (6 points)

Show that  $\log_2 n = \mathcal{O}(n)$  holds. Directly use the definition of  $\mathcal{O}$  by determining  $n_0$  and  $C$  such that  $\log_2 n \leq C \cdot n$  holds for all  $n \geq n_0$ .

Show that  $\log_2 n = \Omega(n)$  does not hold. Directly use the definition of  $\Omega$  by showing that for each given  $C > 0$  and  $n_0$  there exists a  $n \geq n_0$  which violates the definition of  $\Omega$  (i.e. that  $\log_2 n \leq C \cdot n$ ). Consider also that  $C$  can be smaller than 1.

### Exercise 2 (7 points)

Argue that the propositions in Exercise 1 do not only hold for  $\log_2$ , but in the general case  $\log_b$  for any single given  $b > 1$  that does not depend on  $n$ . Remark: That is why in runtime analyses you often find  $\log$  written without the base. Why is it important that  $b > 1$ ? Describe what happens if  $b = 1$  or  $b < 1$ . Why is it important that  $b$  does not depend on  $n$ ? Give an example where  $b$  does depend on  $n$  and one of the propositions above does not hold anymore (you can choose which one you want to use).

### Exercise 3 (7 points)

Order the following functions  $f_1, f_2, f_3, f_4, f_5$  according to their runtime complexity such that  $f_i = \mathcal{O}(f_{i+1})$  holds for  $i = 1, 2, 3, 4$ . Also determine for which  $i$   $f_i = \Theta(f_{i+1})$  holds and for which not. Justify your decisions, particularly for the  $i$  cases where  $f_i = \Theta(f_{i+1})$  does not hold. You can use the limit definition of  $\mathcal{O}$  and  $\Theta$  for all your justifications.

$$\begin{aligned} & n^2 \\ & n \log_{10} n \\ & n^2 \log_2(n^2) \\ & \sqrt{n} \\ & n \log_2(n^2) \end{aligned}$$

## Commit

Commit your solutions in PDF format within a new subdirectory **uebungsblatt\_03** into your SVN repository. Also commit a text file *erfahrungen.txt*. Therein describe your experience with the exercise sheet in a few sentences: was it manageable for you? how much time did it take you? Did you have problems with specific parts?