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## Exercise sheet 11

**Exercise 1** (20 points)

In the lecture it was shown by means of a potential function that the costs for  $n$  *insert* / *remove* operations for a  $(2,4)$  tree is  $O(n)$ . Prove in a similar fashion and with a suitable potential function that the costs for  $n$  arbitrary *insert* / *remove* operations for a  $(4,9)$  tree are still  $O(n)$ .

Pay attention to provide a complete proof. Be especially careful that you consider every event that may occur. Similar cases can be proven together, different cases should be considered separately. For example, when you add a child there is no difference whether the node has 6 or 7 children before (in both cases a split is not necessary, and the potential function does not change, if chosen appropriately). In case of a previous node with 9 children you should consider a different case, as now a split has to happen and maybe the parent node changes as well as the potential function.

A complete proof does not imply much text. A proof may be very short, and a shorter proof with the same information content is always better.

Allocate enough time to write down your proof in a clean way. This is the main work of this task, in addition to the understanding of the proof from the lecture.