Entwurf, Analyse und Umsetzung von Algorithmen

Graphs, Depth-/Breadth-first Search, Graph-Connectivity

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Bioinformatics Group / Department of Computer Science

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Structure

Graphs
- Introduction
- Implementation
- Application example

Graphs - Overview:

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- Representation of graphs in the computer
- Breadth-first search (BFS)
- Depth-first search (DFS)
- Connected components of a graph

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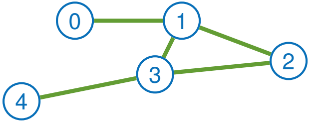

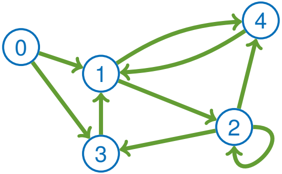

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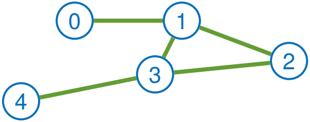

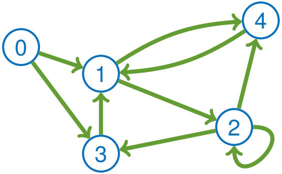

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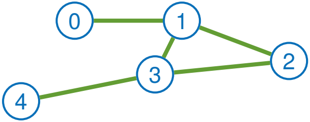

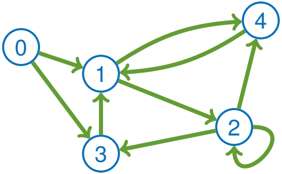

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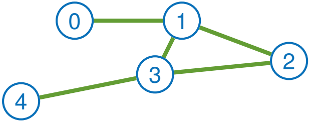

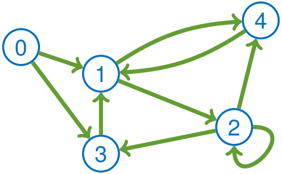

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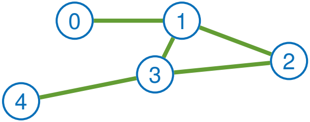

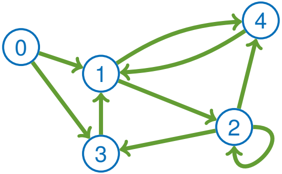

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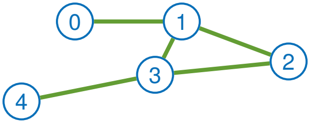

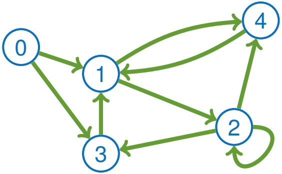

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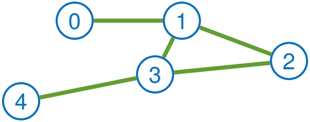

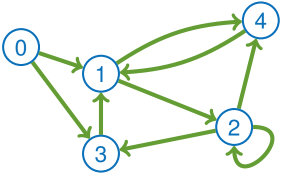

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- Self-loops are also possible: $e = (u, u)$ or $e = \{u, u\}$

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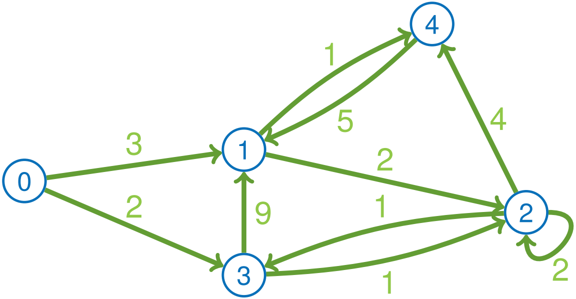

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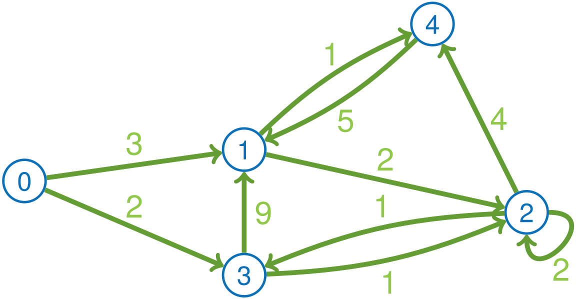

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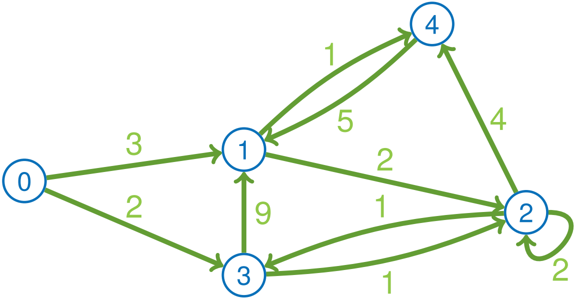

- Each edge is marked with a real number named weight
- The weight is also named length or cost of the edge depending on the application

Example: Road network

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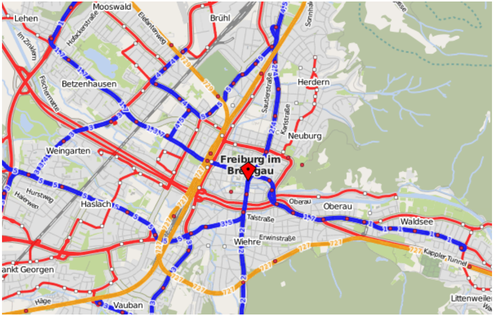

Figure: Map of Freiburg © OpenStreetMap

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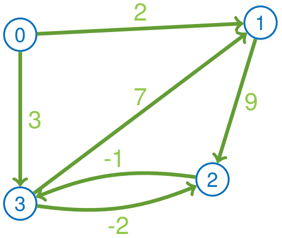

Figure: Weighted graph with $|V| = 4$, $|E| = 6$

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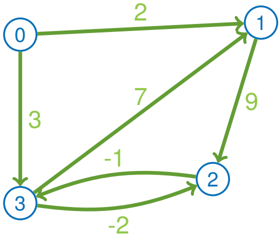

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end-vertice

start-vertice	0	1	2	3
0		2		3
1			9	
2				-1
3		7	-2	

Figure: Adjacency matrix

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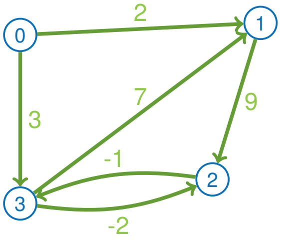

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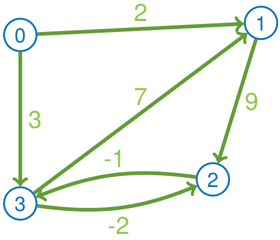

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Figure: Adjacency list

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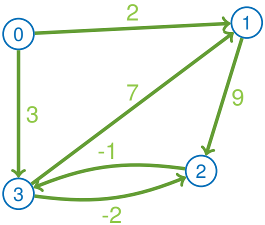

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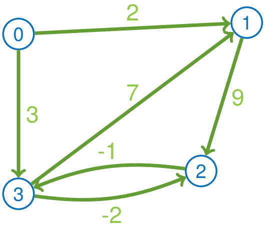

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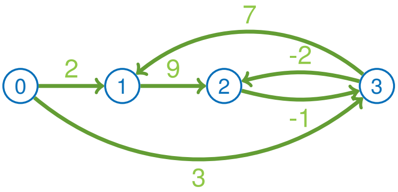

Figure: Same graph ordered by number - outer planar graph

```python
class Graph:
    def __init__(self):
        self.vertices = []
        self.edges = []

    def addVertice(self, vert):
        self.vertices.append(vert)

    def addEdge(self, fromVert, toVert, cost):
        self.edges.append( \
            (fromVert, toVert, cost))

    ...
```

Degree of a vertex: Directed graph: $G = (V, E)$

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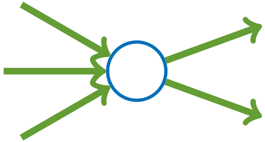

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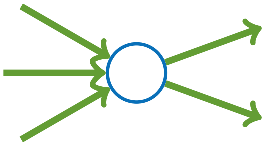

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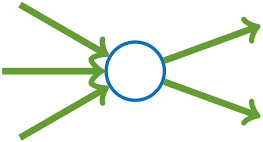

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- Outdegree of a vertex u is the number of edge tail ends adjacent to the vertex

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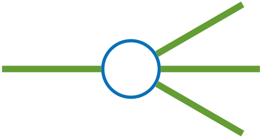

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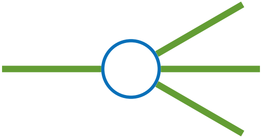

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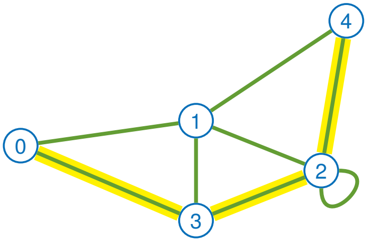
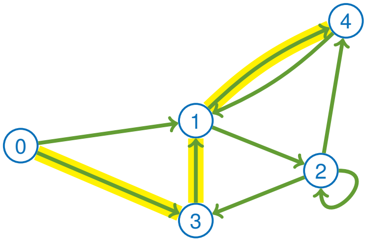


Figure: Undirected path of length 3
$P = (0, 3, 2, 4)$



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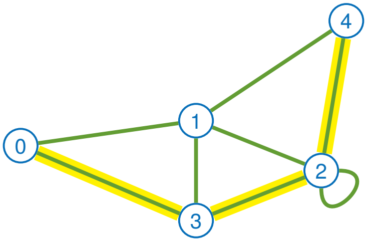

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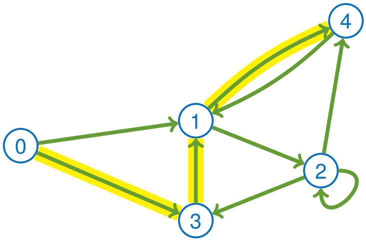

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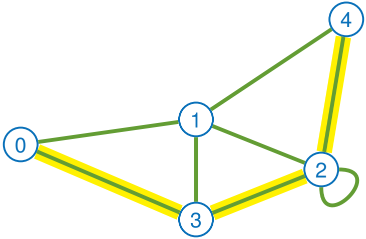

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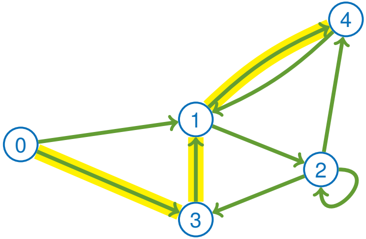

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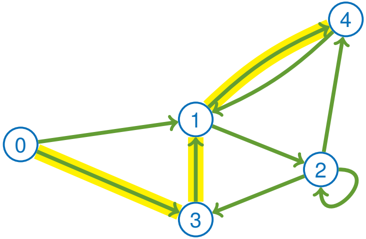

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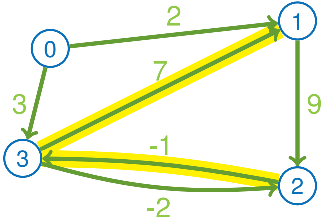

Figure: Weighted path with cost 6
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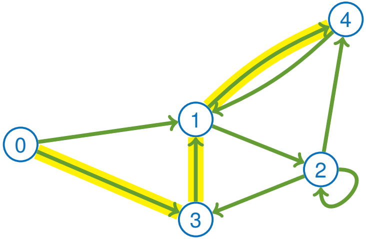

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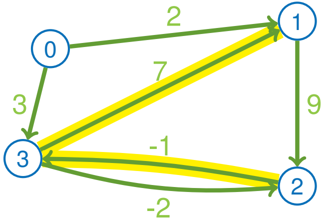

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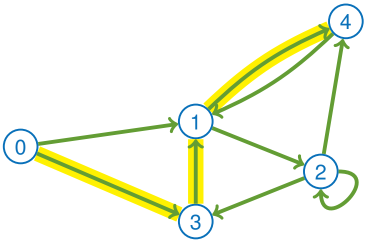

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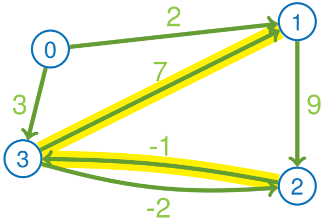

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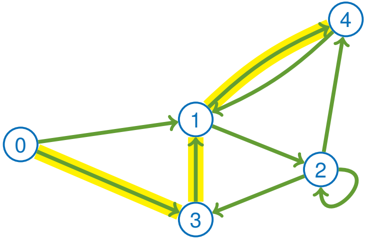

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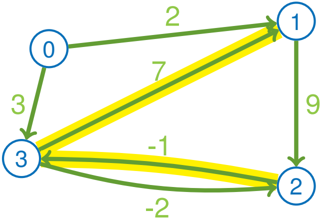

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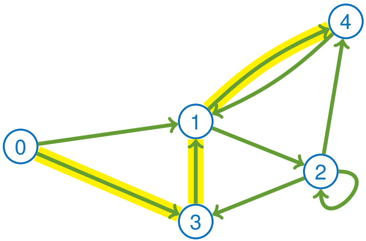

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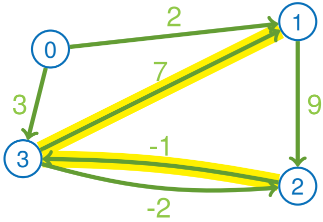

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$P = (2, 3, 1)$

- The length of a path is: (also costs of a path)
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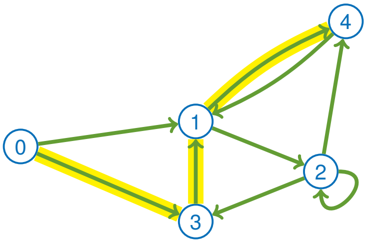

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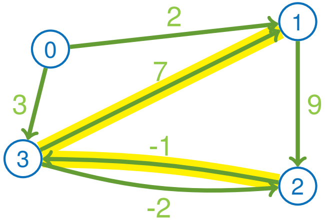

Figure: Weighted path with cost 6
$P = (2, 3, 1)$

- The length of a path is: (also costs of a path)
  - Without weights: number of edges taken
  - With weights: sum of weigths of edges taken

Shortest path in a graph: $G = (V, E)$

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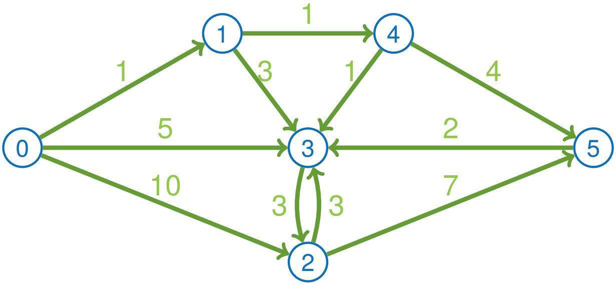

Figure: Shortest path from 0 to 2 with cost / distance $d(0, 2) = ?$

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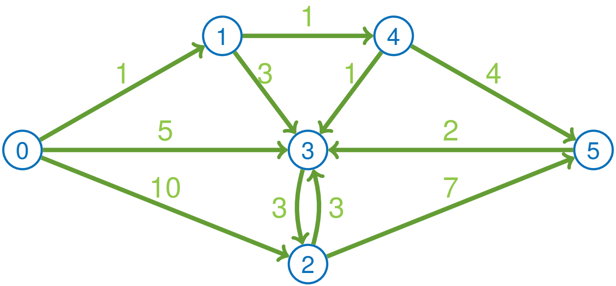

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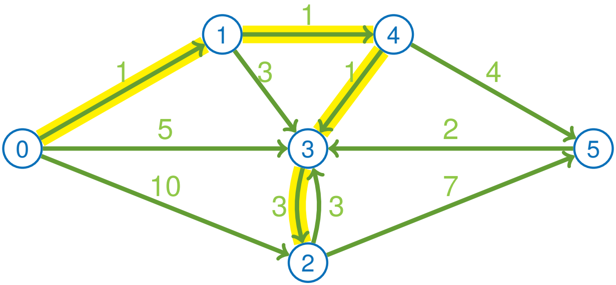

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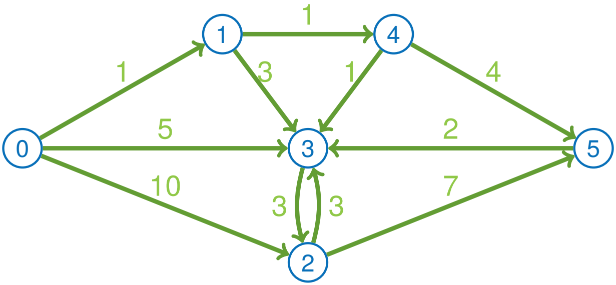

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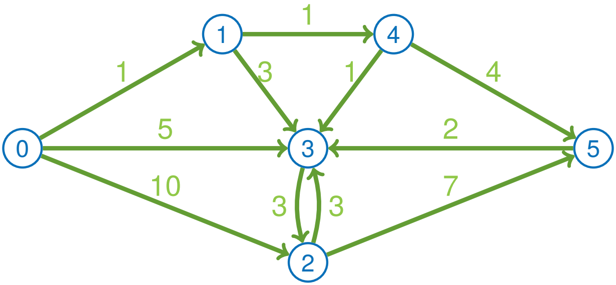

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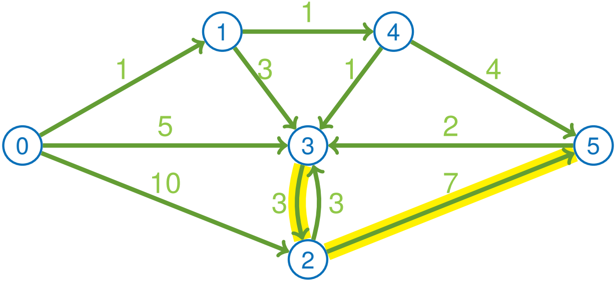

Figure: Diameter of graph is $d = 10$, $P = (3, 2, 5)$

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Connected components: $G = (V, E)$

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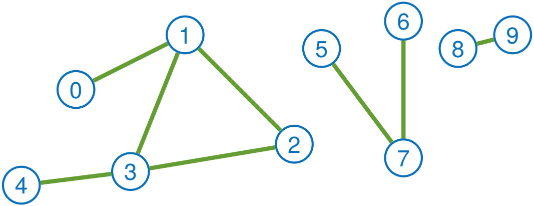

Figure: Three connected components

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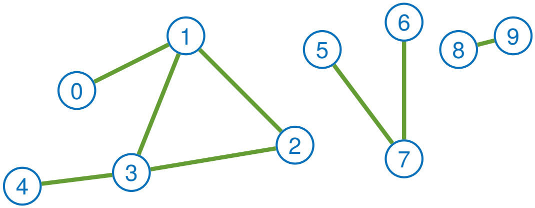

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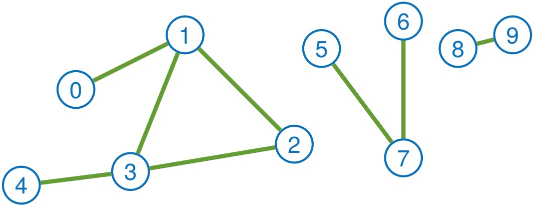

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6. All connected nodes are now marked and in the same connected component as the start vertex *s*

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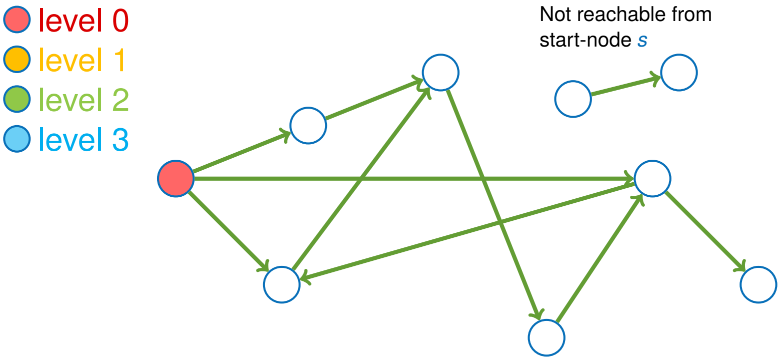

Figure: spanning tree of a breadth-first search

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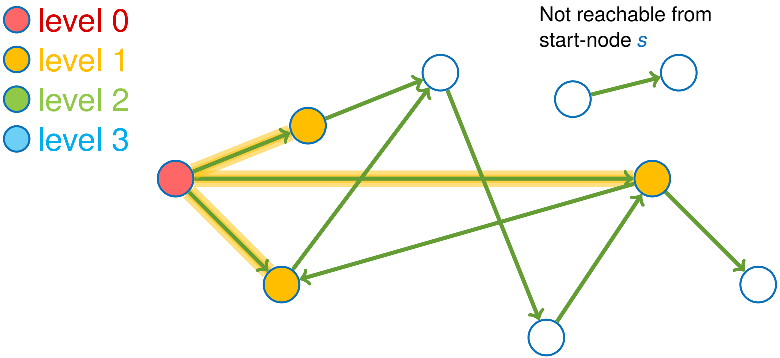

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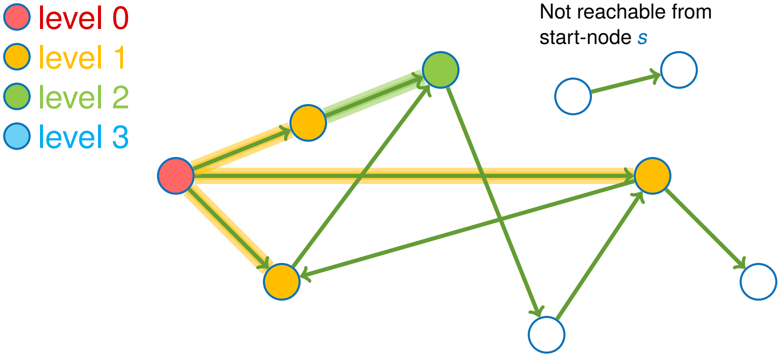

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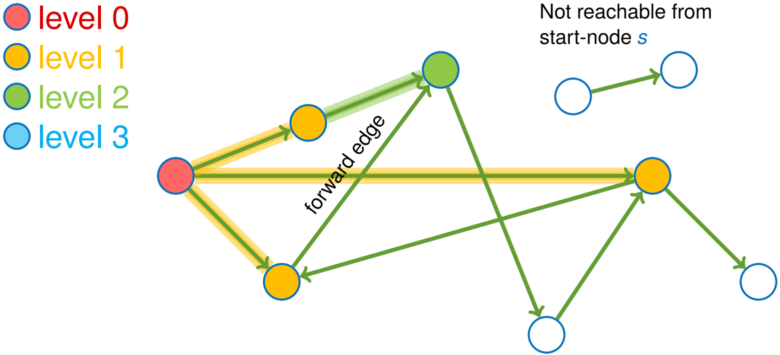

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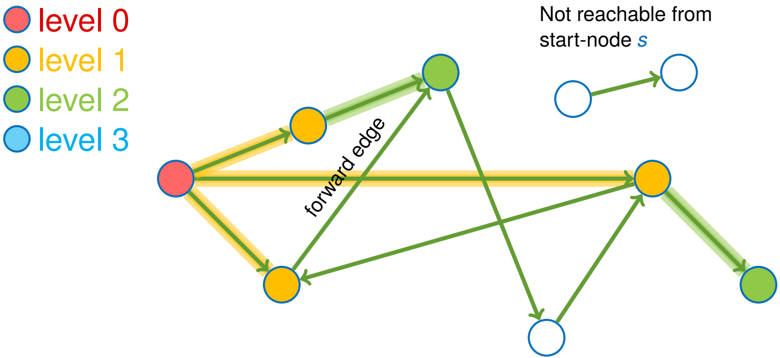

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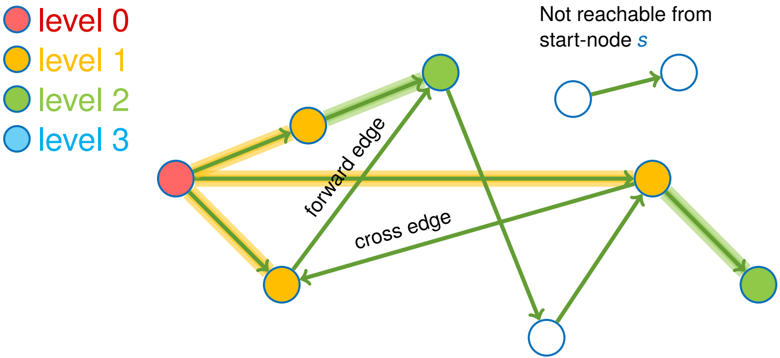

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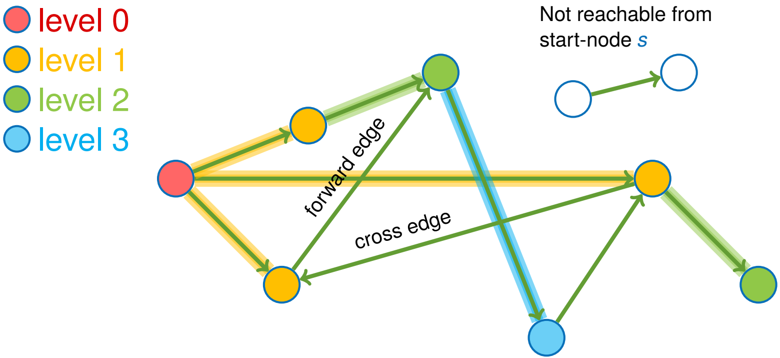

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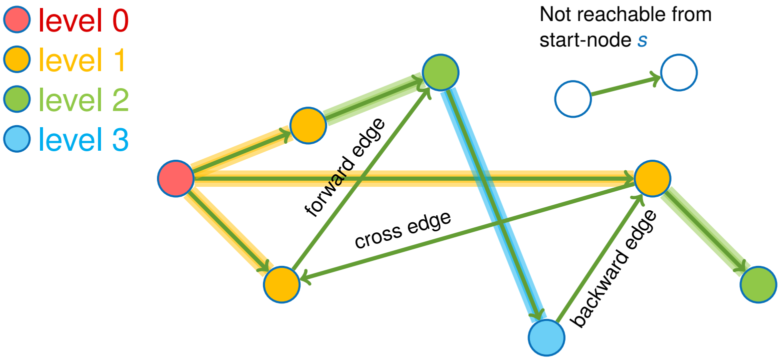

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(reduce the recursion level by one)

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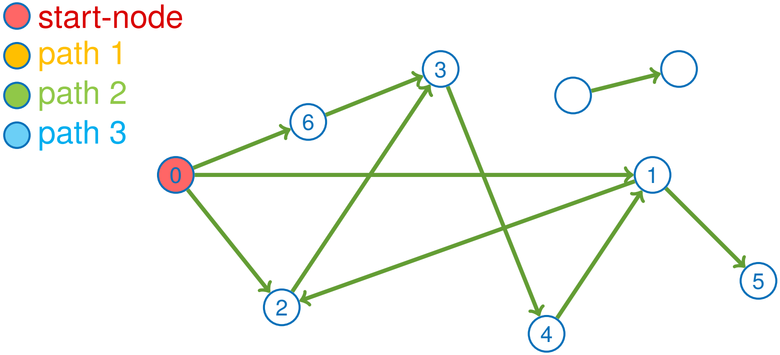

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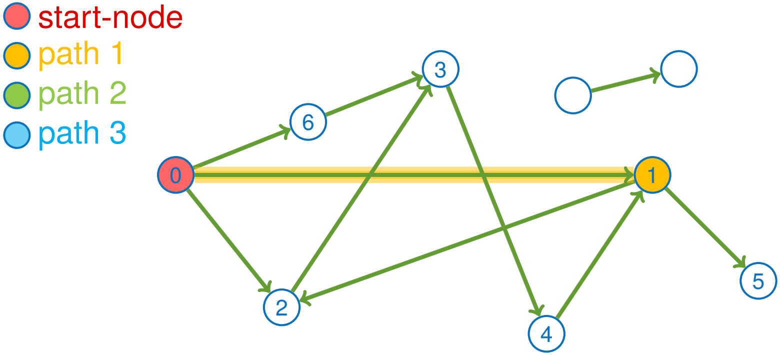

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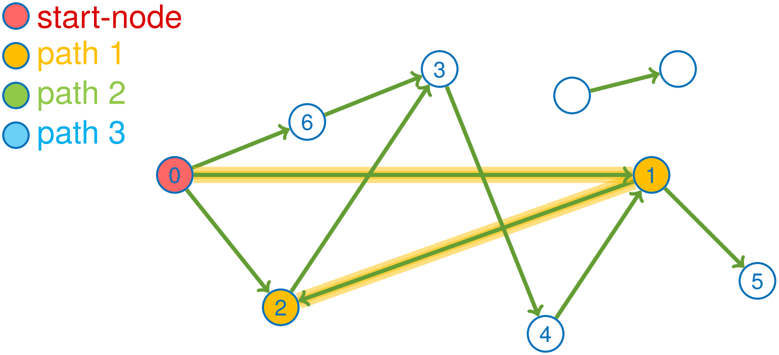

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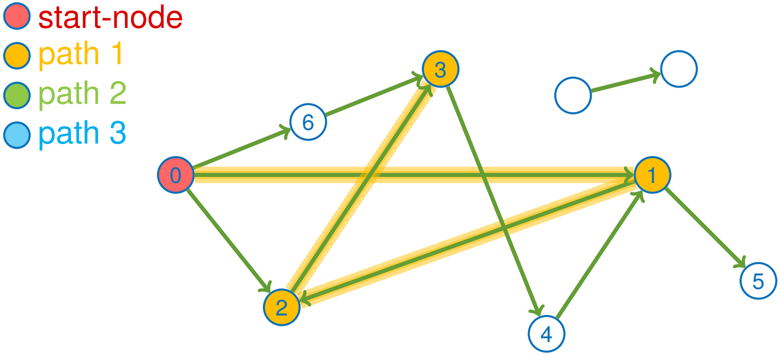

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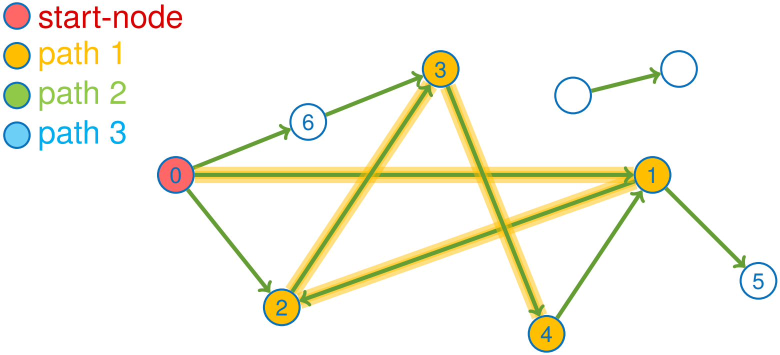

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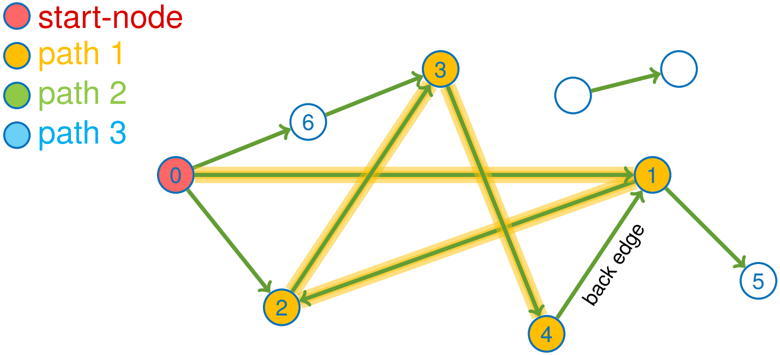

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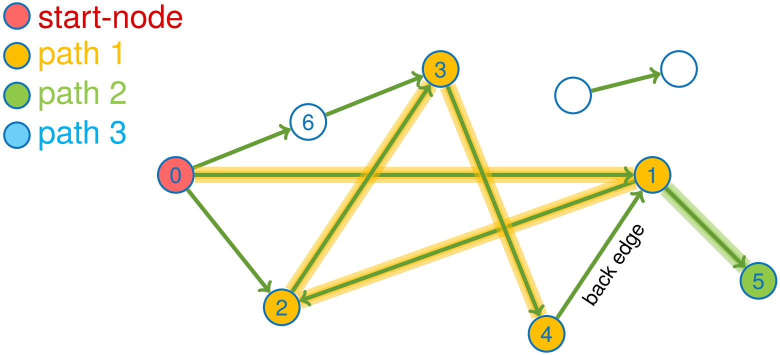

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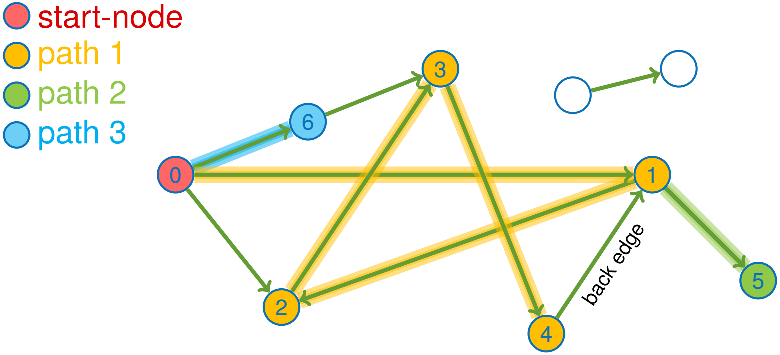

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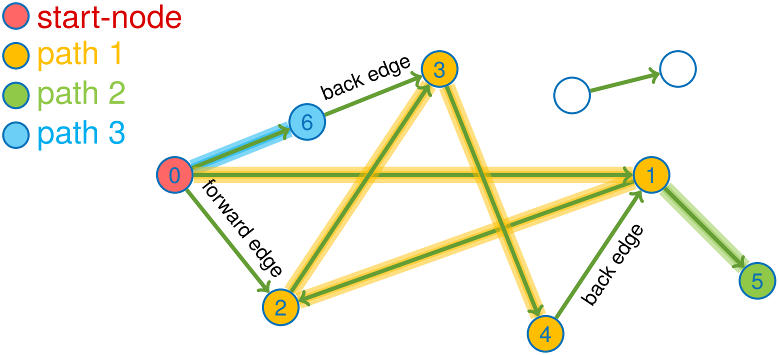

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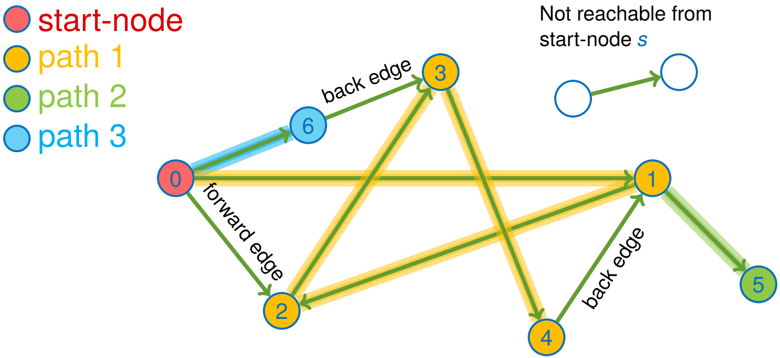

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Structure

Application example

Image processing

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Image processing

- Connected component labeling

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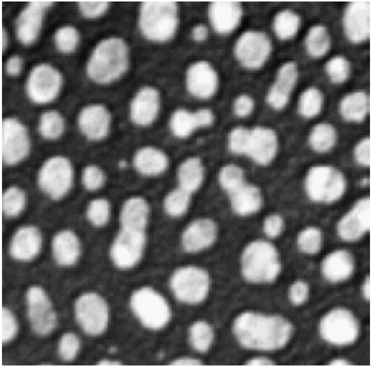

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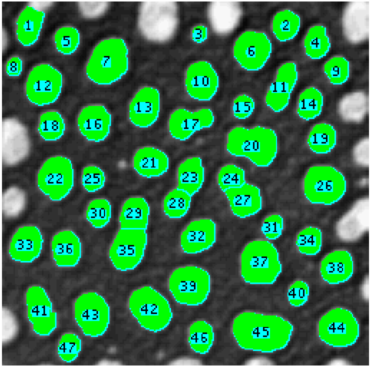
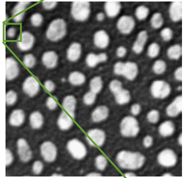


What is object, what is background?



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
35	104	80	56	40	16	16	8	16	16	24	32	32	32	32	32	32	32	32	24	24	16
36	80	64	48	32	16	16	16	24	32	40	40	40	40	40	40	40	32	32	24	24	24
37	56	48	32	24	8	16	16	32	40	48	48	48	40	40	40	40	32	32	24	24	24
38	40	32	24	24	16	32	48	64	72	80	80	72	56	56	48	48	40	40	32	32	32
39	16	16	16	24	24	48	72	88	104	112	112	96	72	64	56	48	40	40	40	40	40
40	16	16	24	40	56	88	120	128	136	144	144	120	96	88	72	56	48	48	40	40	40
41	8	16	24	56	80	120	160	168	168	168	168	144	120	104	80	64	48	48	40	40	32
42	16	32	40	80	112	144	176	176	176	176	168	152	128	112	88	64	48	40	32	32	24
43	24	40	56	96	136	160	184	184	176	176	168	152	136	112	88	64	40	32	24	24	16
44	40	56	80	112	152	168	184	184	176	176	168	152	136	112	80	64	40	32	16	16	16
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46	48	72	96	136	168	176	192	192	184	184	176	160	136	104	72	56	32	24	16	24	32
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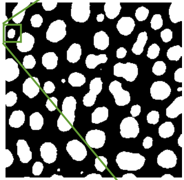
53	40	48	48	56	64	72	72	80	80	80	72	64	48	40	24	32	32	72	104	144	184
54	48	48	48	48	48	56	56	56	64	56	56	48	40	32	24	40	48	88	128	160	200

Convert to black and white using threshold:

```
value = 255 if value > 100 else 0
```



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
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36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
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49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Interpret image as graph:

Interpret image as graph:

- Each white pixel is a node

Application example
Image processing

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)

Application example
Image processing

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)
- Edges are not saved externally, algorithm works directly on array

Application example

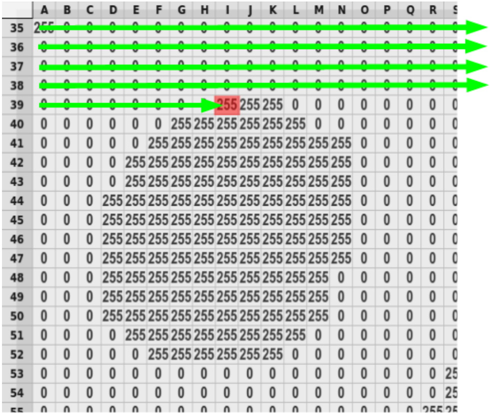
Image processing

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)
- Edges are not saved externally, algorithm works directly on array
- Breadth- / depth-first search find all connected components (particles)

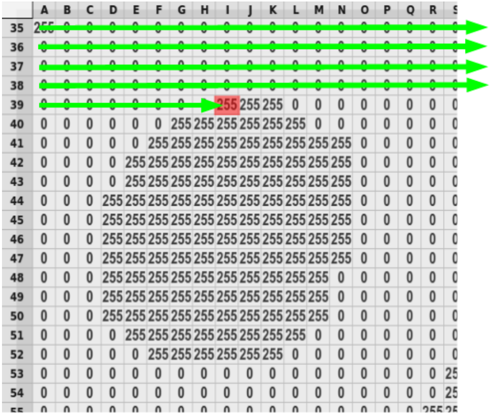
Find connected components:

Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

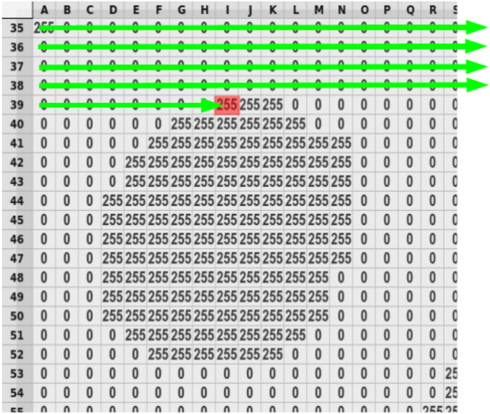
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity

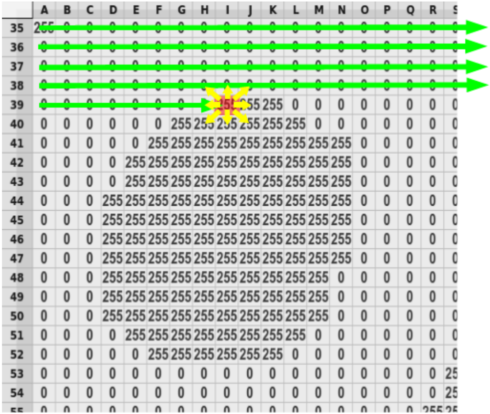
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1

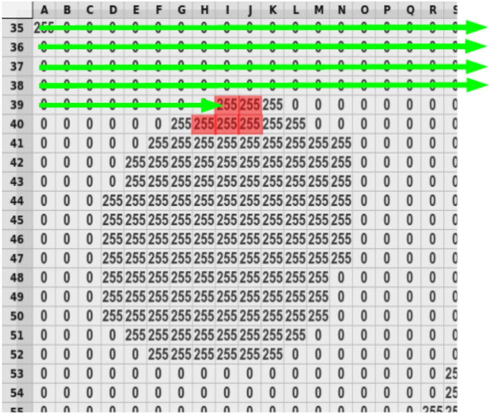
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels

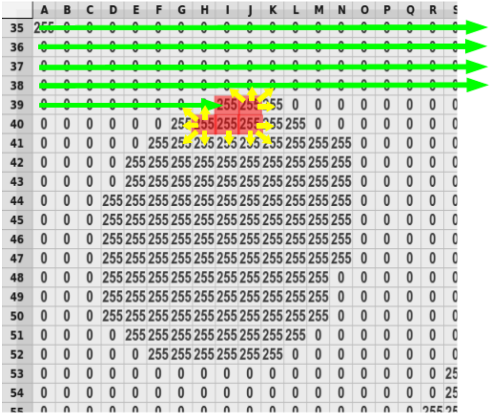
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

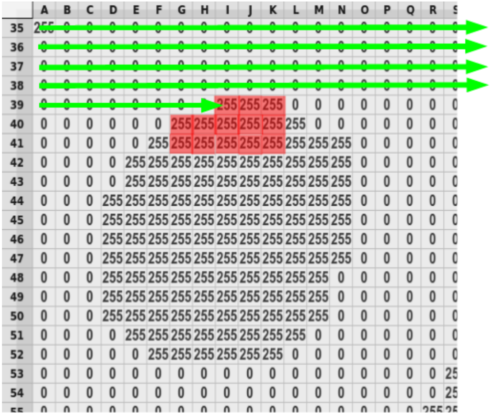
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

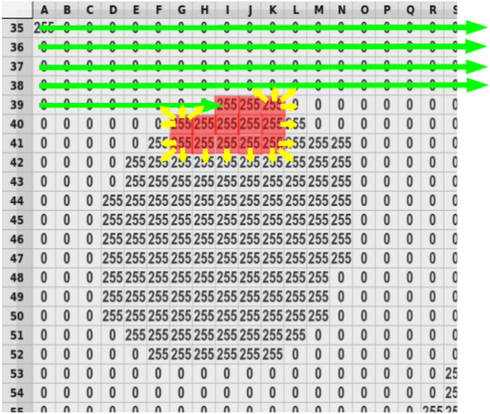
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

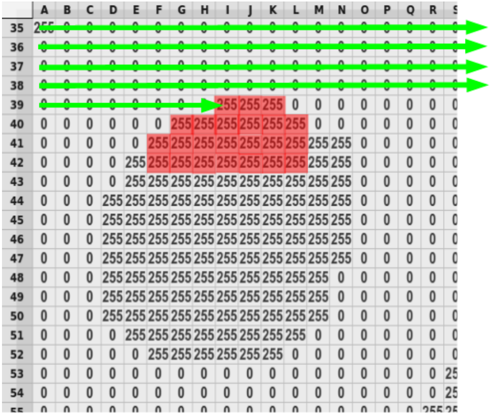
- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:



- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

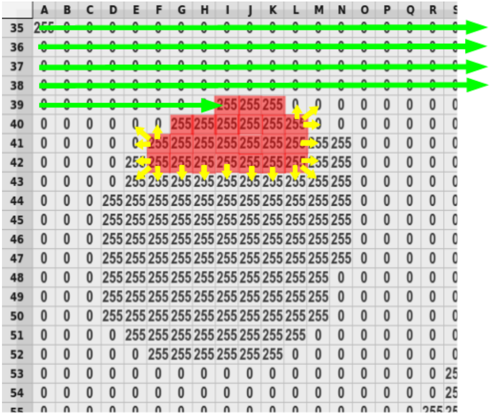
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

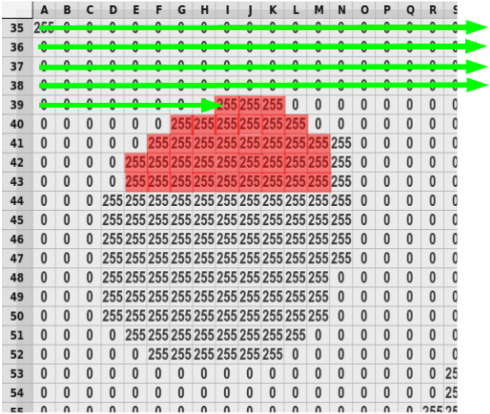
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

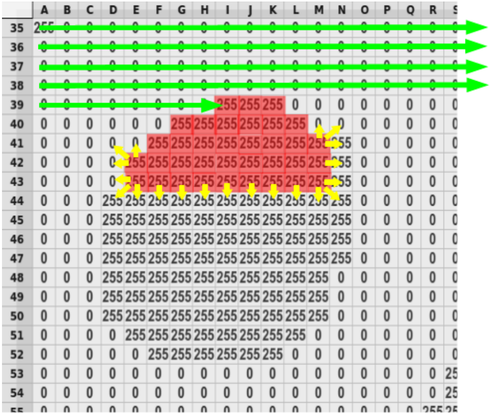
Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

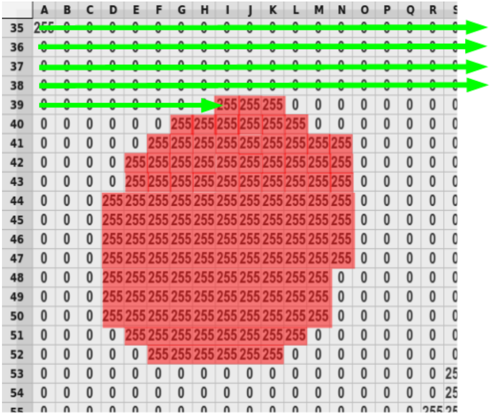
- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:



- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:



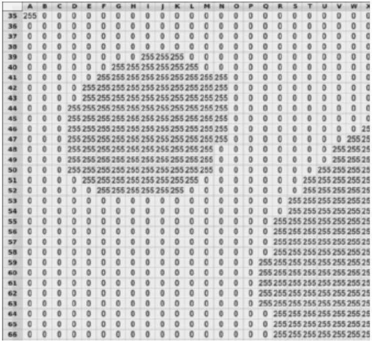
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Find connected components:

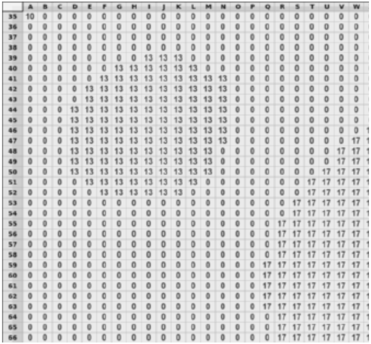
- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 2
- ...

Result of connected component labeling:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	25
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	25
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	25
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	25
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	255	255	25
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	255	255	255	25
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	255	255	25
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25

$\Rightarrow$



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	
35	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
39	0	0	0	0	0	0	0	0	13	13	13	0	0	0	0	0	0	0	0	0	0	0	0	
40	0	0	0	0	0	0	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0	0	
41	0	0	0	0	0	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	
42	0	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	
43	0	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	
44	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	
45	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	
46	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	1
47	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	1
48	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17	1
49	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17	1
50	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17	17	1
51	0	0	0	0	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17	17	17	1
52	0	0	0	0	0	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17	17	17	1
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	1
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	1
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17	1
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17	1
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17	1
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17	1
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17	1
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	1

Figure: Result: particle indices instead of intensities

Further Literature

- **General**

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.
Introduction to Algorithms.
MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.
Algorithms and data structures, 2008.
https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf.

Further Literature

- **Graph Search**

[Wika] Breadth-first search
https://en.wikipedia.org/wiki/Breadth-first_search

[Wikb] Depth-first search
https://en.wikipedia.org/wiki/Depth-first_search

- **Graph Connectivity**

[Wik] Connectivity (graph theory)
https://en.wikipedia.org/wiki/Connectivity_(graph_theory)