Bioinformatics Group
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Entwurf, Analyse und Umsetzung von Algorithmen

Exercise sheet 9

Exercise 1 (5 points)
a) The accession of data structures is often directed by the following recursion:

$$T(n) = \begin{cases} a & \text{for } n = 1 \\ c + T(n/2) & \text{else} \end{cases} \tag{1}$$

Proof that $T(n) = \mathcal{O}(\log n)$.

b) The following recursion is given:

$$T(n) = \begin{cases} a & \text{for } n = 1 \\ 2T(n/2) + n^3 & \text{else} \end{cases} \tag{2}$$

Provide an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem.

Exercise 2 (10 points)
A recursive algorithm has the cost of:

$$T(n) = \begin{cases} 1 & \text{for } n = 1 \\ 4T(n/2) + n^2 & \text{else} \end{cases} \tag{3}$$

Provide an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Write a program that proves experimentally that this algorithm satisfies the calculated time complexity.

Exercise 3 (5 points)
Given is the following recurrence relation:

$$T(n) = \begin{cases} a & \text{for } n \leq 2 \\ T(\sqrt{n}) + a & \text{else} \end{cases} \tag{4}$$

Provide an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Hint: Find a suitable substitution for $\sqrt{n}$, such that the Master Theorem can be used.