

Algorithms and Datastructures

Graphs, Depth-/Breadth-first Search, Graph-Connectivity

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Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, January 2017

Structure

Feedback

- Exercises

- Lecture

Graphs

- Introduction

- Implementation

- Application example

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Feedback from the exercises

The upcoming exercise sheet 12 and 13 will be merged together
(finding largest connected component + Dijkstra)

Some people were asking for more solution sheets for the exercises

We are working on it.

Feedback from the lecture

Code in the lecture will be a litte bit different from exercise sheet.

One person asked for additional explanations regarding proofs.

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Graphs

Introduction

Graphs - Overview:

Graphs

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- ▶ Besides arrays, lists and trees the most common datastructure (Trees are a special type of graph)

Graphs

Introduction

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- ▶ Representation of graphs in the computer

Graphs

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Graphs

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Graphs - Overview:

- ▶ Besides arrays, lists and trees the most common datastructure (Trees are a special type of graph)
- ▶ Representation of graphs in the computer
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- ▶ Depth first search (DFS)

Graphs

Introduction

Graphs - Overview:

- ▶ Besides arrays, lists and trees the most common datastructure (Trees are a special type of graph)
- ▶ Representation of graphs in the computer
- ▶ Breadth first search (BFS)
- ▶ Depth first search (DFS)
- ▶ Connected components of a graph

Graphs

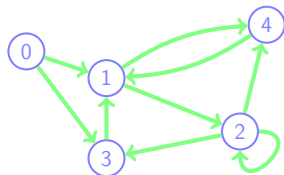
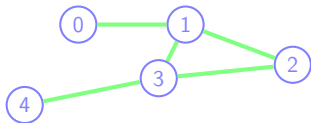
Introduction

Terminology:

Graphs

Introduction

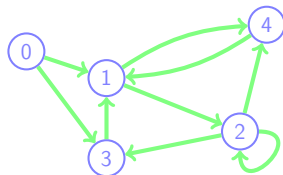
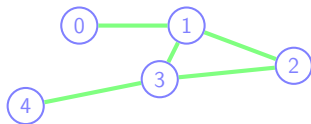
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Graphs

Introduction

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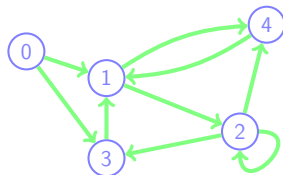
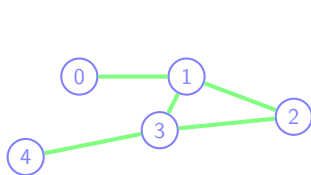


- Each Graph $G = (V, E)$ consists of:

Graphs

Introduction

Terminology:

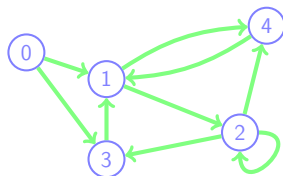
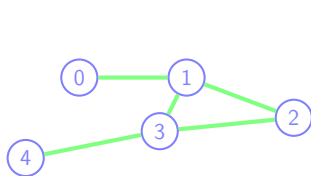


- ▶ Each Graph $G = (V, E)$ consists of:
 - ▶ A set of vertices (nodes) $V = \{v_1, v_2, \dots\}$

Graphs

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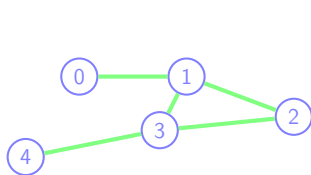


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Graphs

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Graphs

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 - ▶ Undirected edge: $e = \{u, v\}$ (set)

Graphs

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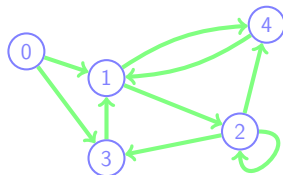
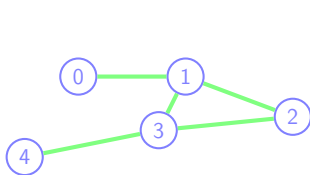


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Graphs

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Terminology:



- ▶ Each Graph $G = (V, E)$ consists of:
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 - ▶ Undirected edge: $e = \{u, v\}$ (set)
 - ▶ Directed edge: $e = (u, v)$ (tuple)
- ▶ Self-loops are also possible: $e = (u, u)$ or $e = \{u, u\}$

Graphs

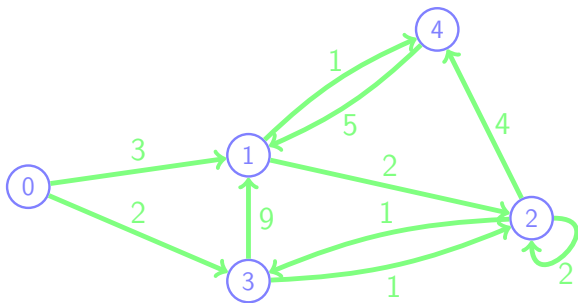
Introduction

Weighted graph:

Graphs

Introduction

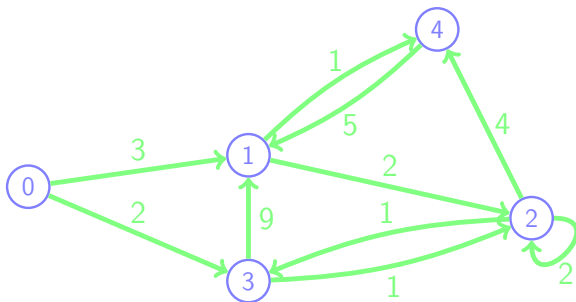
Weighted graph:



Graphs

Introduction

Weighted graph:

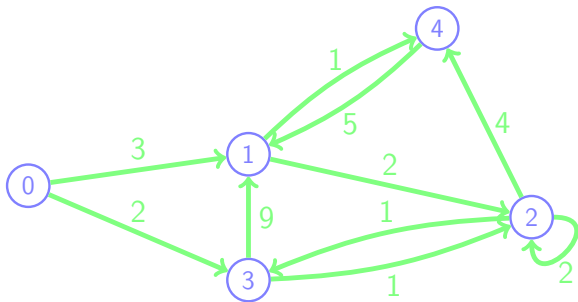


- Each edge is marked with a real number named **weight**

Graphs

Introduction

Weighted graph:



- ▶ Each edge is marked with a real number named **weight**
- ▶ The **weight** is also named **length** or **cost** of the edge depending on the application

Graphs

Introduction

Example: Road network

Graphs

Introduction

Example: Road network

- ▶ Intersections: **vertices**

Graphs

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Example: Road network

- ▶ Intersections: **vertices**
- ▶ Roads: **edges**

Graphs

Introduction

Example: Road network

- ▶ Intersections: vertices
- ▶ Roads: edges
- ▶ Travel time:
costs of the edges

Graphs

Introduction

Example: Road network

- Intersections: **vertices**
- Roads: **edges**
- Travel time:
costs of the edges

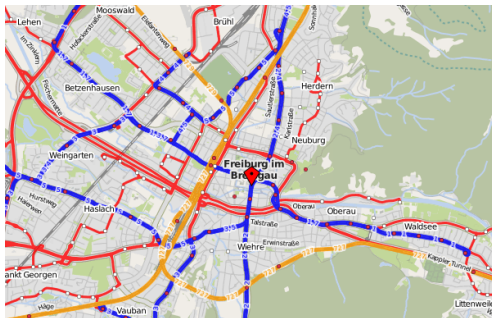


Figure: Map of Freiburg © OpenStreetMap

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Implementation

How to represent this graph computationally?

Graphs

Implementation

How to represent this graph computationally?

- ▶ Two classic variants

Graphs

Implementation

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1. **Adjacency matrix** with space consumption $\Theta(|V|^2)$

Graphs

Implementation

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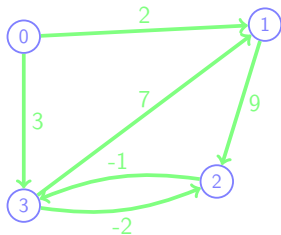


Figure: Weighted graph with
 $|V| = 4$, $|E| = 6$

Graphs

Implementation

How to represent this graph computationally?

- ▶ Two classic variants

1. **Adjacency matrix** with space consumption $\Theta(|V|^2)$

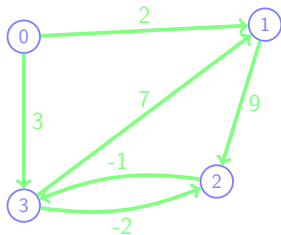


Figure: Weighted graph with
 $|V| = 4$, $|E| = 6$

		end-vertex			
		0	1	2	3
start-vertex	0		2		3
	1			9	
	2				-1
	3		7	-2	

Figure: Adjacency matrix

Graphs

Implementation

How to represent this graph computationally?

Graphs

Implementation

How to represent this graph computationally?

- ▶ Two classic variants
 2. Adjacency list / fields with space consumption $\Theta(|V| + |E|)$

Graphs

Implementation

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 - ▶ Each list item stores the target vertice and the cost of the edge

Graphs

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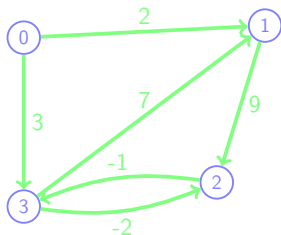


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Graphs

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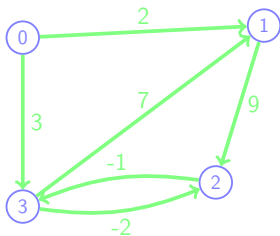


Figure: Weighted graph with
 $|V| = 4$, $|E| = 6$

start-vertex	0	1, 2	3, 3
	1	2, 9	
	2	3, -1	
	3	1, 7	2, -2

Figure: Adjacency list

Graphs

Implementation

Graph: Arrangement

Graphs

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- ▶ Graph is fully defined through the [adjacency matrix / list](#)

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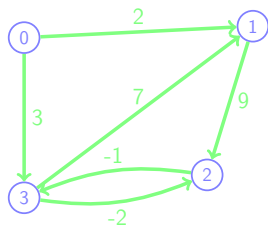


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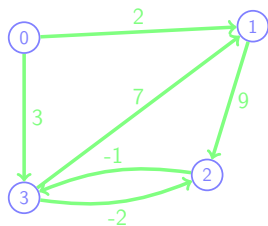


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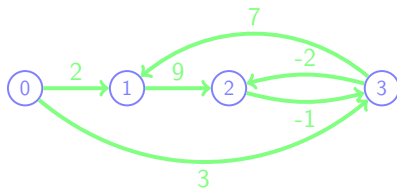


Figure: Same graph ordered by number -
outer planar graph

Graphs

Implementation - Python

```
class Graph:
    def __init__(self):
        self.vertices = []
        self.edges = []

    def addVertice(self, vert):
        self.vertices.append(vert)

    def addEdge(self, fromVert, toVert):
        self.edges.append((fromVert, toVert))

    ...
```


Graphs

Implementation - Python

...

```
def toString(self):  
    return '{'  
        + ', '.join( \  
            [str(len(self.vertices)), \  
              str(len(self.edges))] \  
        + ["(%s, %s)" % tup \  
           for tup in self.edges]) \  
    + '}'
```

Graphs

Degrees (Valency)

Degree of a vertex: Directed graph: $G = (V, E)$

Graphs

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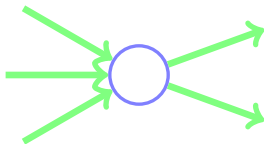


Figure: Vertex with in- / outdegree of 3 / 2

Graphs

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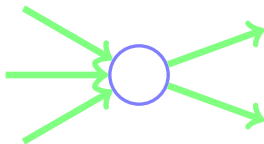


Figure: Vertex with in- / outdegree of 3 / 2

- **Indegree** of a vertex u is the number of **edge heads** adjacent to the vertex

$$\deg^+(u) = |\{(v, u) : (v, u) \in E\}|$$

Graphs

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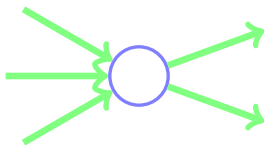


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- **Indegree** of a vertex u is the number of **edge heads** adjacent to the vertex

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Graphs

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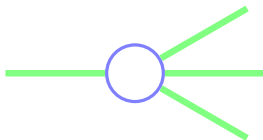


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Graphs

Degrees (Valency)

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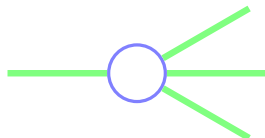


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- **Degree** of a vertex u is the number of **vertices** adjacent to the vertex

$$\deg(u) = |\{\{v, u\} : \{v, u\} \in E\}|$$

Graphs

Paths

Paths in a graph: $G = (V, E)$

Graphs

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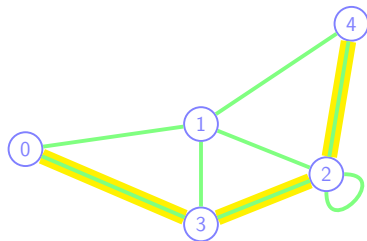


Figure: Undirected path of length 3
 $P = (0, 3, 2, 4)$

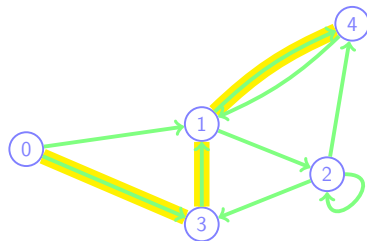


Figure: Directed path of length 3
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Graphs

Paths

Paths in a graph: $G = (V, E)$

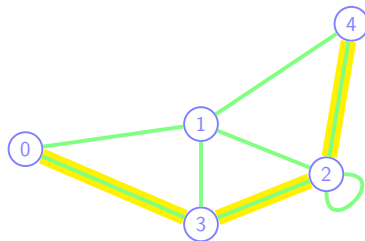


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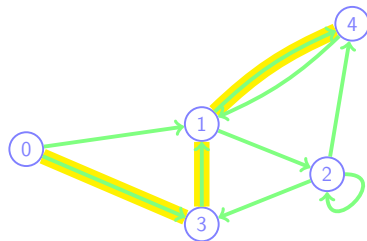


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- ▶ A path of G is a sequence of edges $u_1, u_2, \dots, u_i \in V$ with

Graphs

Paths

Paths in a graph: $G = (V, E)$

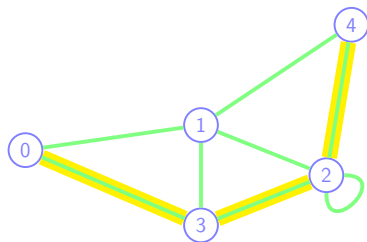


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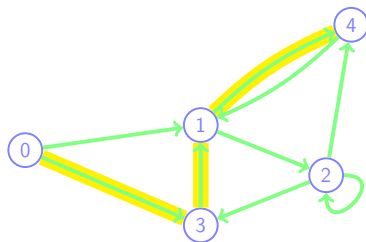


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- ▶ A path of G is a sequence of edges $u_1, u_2, \dots, u_i \in V$ with
 - ▶ Undirected graph: $\{u_1, u_2\}, \{u_2, u_3\}, \dots, \{u_{i-1}, u_i\} \in E$
 - ▶ Directed graph: $(u_1, u_2), (u_2, u_3), \dots, (u_{i-1}, u_i) \in E$

Graphs

Paths

Paths in a graph: $G = (V, E)$

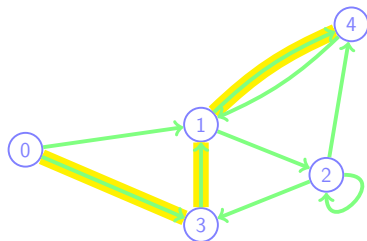


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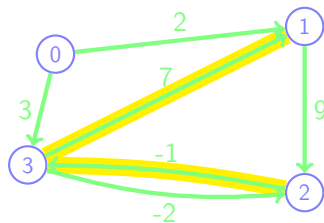


Figure: Weighted path with cost 6
 $P = (2, 3, 1)$

Graphs

Paths

Paths in a graph: $G = (V, E)$

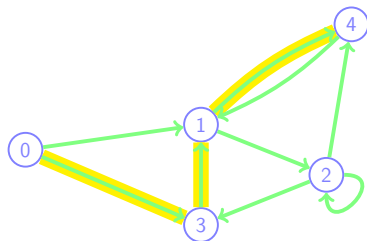


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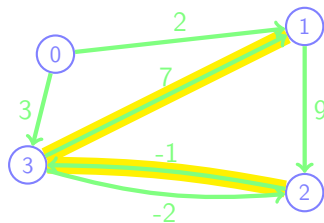


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Graphs

Paths

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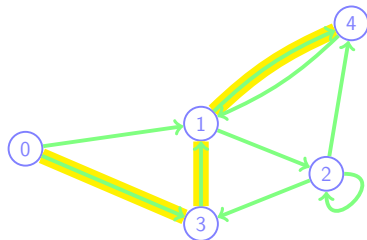


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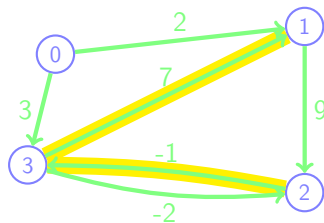


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Graphs

Paths

Paths in a graph: $G = (V, E)$



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Figure: Weighted path with cost 6
 $P = (2, 3, 1)$

- The length of a path is: (also costs of a path)

Graphs

Paths

Paths in a graph: $G = (V, E)$



Figure: Directed path of length 3
 $P = (0, 3, 1, 4)$



Figure: Weighted path with cost 6
 $P = (2, 3, 1)$

- ▶ The length of a path is: (also costs of a path)
 - ▶ Without weights: number of edges taken

Graphs

Paths

Paths in a graph: $G = (V, E)$



Figure: Directed path of length 3
 $P = (0, 3, 1, 4)$



Figure: Weighted path with cost 6
 $P = (2, 3, 1)$

- ▶ The length of a path is: (also costs of a path)
 - ▶ Without weights: number of edges taken
 - ▶ With weights: sum of weights of edges taken

Graphs

Paths

Shortest path in a graph: $G = (V, E)$

Graphs

Paths

Shortest path in a graph: $G = (V, E)$

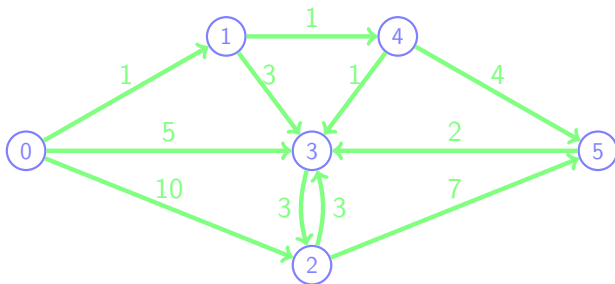


Figure: Shortest path from 0 to 2 with cost / distance $d(0, 2) = ?$

Graphs

Paths

Shortest path in a graph: $G = (V, E)$

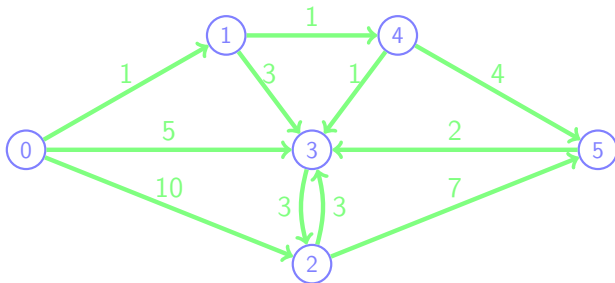


Figure: Shortest path from 0 to 2 with cost / distance $d(0, 2) = ?$

- ▶ The **shortest path** between two vertices u, v is the path $P = (u, \dots, v)$ with the shortest length $d(u, v)$ or lowest costs

Graphs

Paths

Shortest path in a graph: $G = (V, E)$

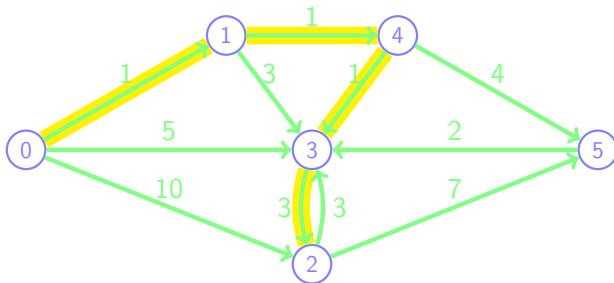


Figure: Shortest path from 0 to 2 with cost / distance $d(0,2) = 6$
 $P = (0, 1, 4, 3, 2)$

- The shortest path between two vertices u, v is the path $P = (u, \dots, v)$ with the shortest length $d(u, v)$ or lowest costs

Graphs

Paths

Diameter of a graph: $G = (V, E)$

Graphs

Paths

Diameter of a graph: $G = (V, E)$

$$d = \max_{u,v \in V} d(u, v)$$

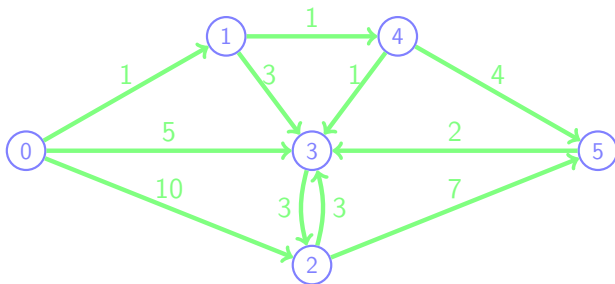


Figure: Diameter of graph is $d = ?$

Graphs

Paths

Diameter of a graph: $G = (V, E)$

$$d = \max_{u,v \in V} d(u, v)$$

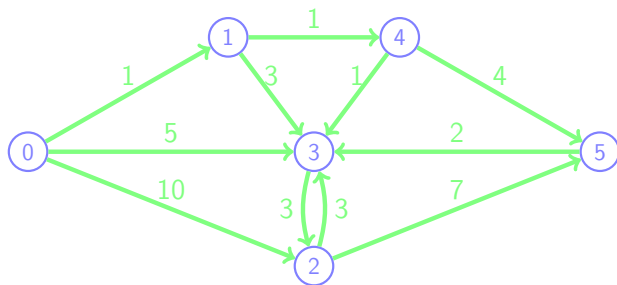


Figure: Diameter of graph is $d = ?$

- The **diameter** of a graph is the length / the costs of the longest shortest path

Graphs

Paths

Diameter of a graph: $G = (V, E)$

$$d = \max_{u,v \in V} d(u, v)$$

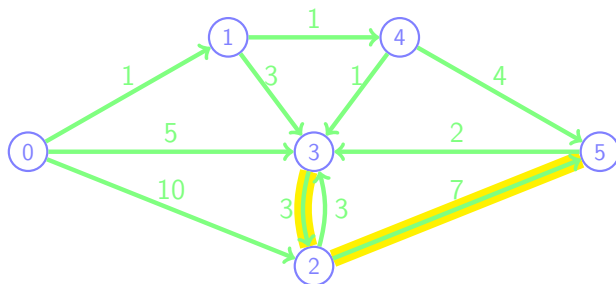


Figure: Diameter of graph is $d = 10$, $P = (3, 2, 5)$

- The **diameter** of a graph is the length / the costs of the longest shortest path

Graphs

Connected Components

Connected components: $G = (V, E)$

Graphs

Connected Components

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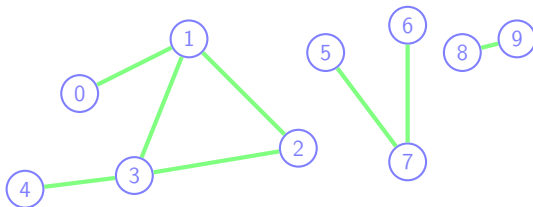


Figure: Three connected components

- Undirected graph:

Graphs

Connected Components

Connected components: $G = (V, E)$

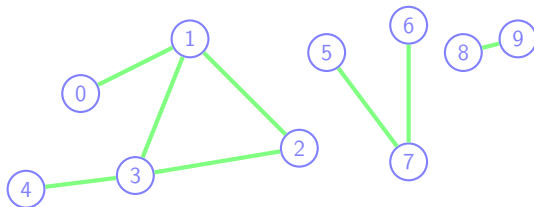


Figure: Three connected components

- Undirected graph:
 - All connected components are a partition of V

$$V = V_1 \cup \dots \cup V_k$$

Graphs

Connected Components

Connected components: $G = (V, E)$



Figure: Three connected components

- ▶ Undirected graph:
 - ▶ All connected components are a partition of V

$$V = V_1 \cup \dots \cup V_k$$

- ▶ Two vertices u, v are in the same connected component if a path between u and v exists

Graphs

Connected Components

Connected components: $G = (V, E)$

Graphs

Connected Components

Connected components: $G = (V, E)$

- ▶ Directed graph:

Graphs

Connected Components

Connected components: $G = (V, E)$

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 - ▶ Named **strongly connected components**

Graphs

Connected Components

Connected components: $G = (V, E)$

- ▶ Directed graph:
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 - ▶ Direction of edge has to be regarded

Graphs

Connected Components

Connected components: $G = (V, E)$

- ▶ Directed graph:
 - ▶ Named **strongly connected components**
 - ▶ Direction of edge has to be regarded
 - ▶ Not part of this lecture

Graphs

Connected Components - Graph Exploration

Graph Exploration: (Informal definition)

Graphs

Connected Components - Graph Exploration

Graph Exploration: (Informal definition)

- ▶ Let $G = (V, E)$ be a graph and $s \in V$ a start vertex

Graphs

Connected Components - Graph Exploration

Graph Exploration: (Informal definition)

- ▶ Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
- ▶ We visit each reachable vertex connected to s

Graphs

Connected Components - Graph Exploration

Graph Exploration: (Informal definition)

- ▶ Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
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Graphs

Connected Components - Graph Exploration

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- ▶ **Depth-first search:** in sequence of the largest distance to s

Graphs

Connected Components - Graph Exploration

Graph Exploration: (Informal definition)

- ▶ Let $G = (V, E)$ be a graph and $s \in V$ a start vertex
- ▶ We visit each reachable vertex connected to s
- ▶ **Breadth-first search:** in sequence of the smallest distance to s
- ▶ **Depth-first search:** in sequence of the largest distance to s
- ▶ Not a problem on its own but is often used as subroutine of other algorithms

Graphs

Connected Components - Breadth-First Search

Idea:

Graphs

Connected Components - Breadth-First Search

Idea:

1. We start with all vertices unmarked and mark visited vertices

Graphs

Connected Components - Breadth-First Search

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1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)

Graphs

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Idea:

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2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)

Graphs

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1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)
4. Mark all unmarked vertices connected to a level 1-vertex (level 2)

Graphs

Connected Components - Breadth-First Search

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)
4. Mark all unmarked vertices connected to a level 1-vertex (level 2)
5. Iteratively mark reachable vertices for all levels

Graphs

Connected Components - Breadth-First Search

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s (level 0)
3. Mark all unmarked connected vertices (level 1)
4. Mark all unmarked vertices connected to a level 1-vertex (level 2)
5. Iteratively mark reachable vertices for all levels
6. All connected nodes are now marked and in the same connected component as the start vertex s

Graphs

Connected Components - Breadth-First Search

Graphs

Connected Components - Breadth-First Search

- ▶ The marked vertices create a “spanning tree” containing all reachable nodes

Graphs

Connected Components - Breadth-First Search

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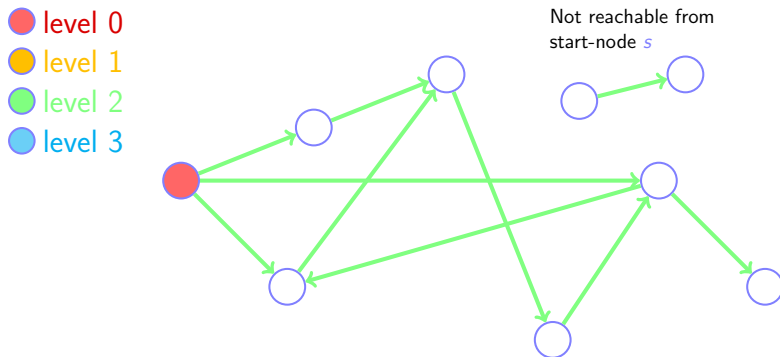


Figure: spanning tree of a breadth-first search

Graphs

Connected Components - Breadth-First Search

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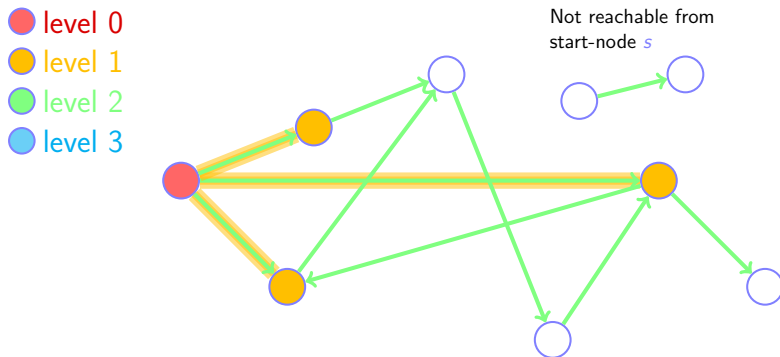


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Graphs

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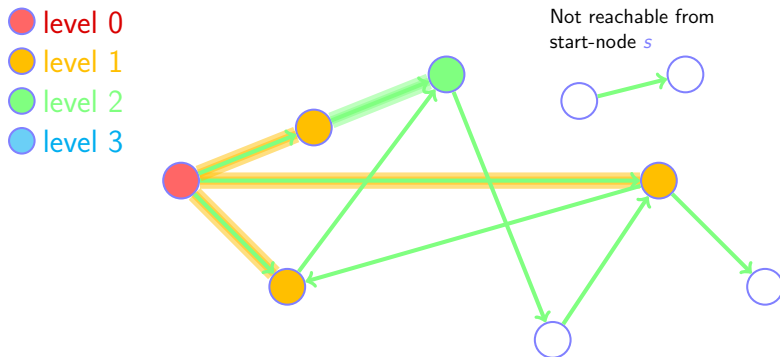


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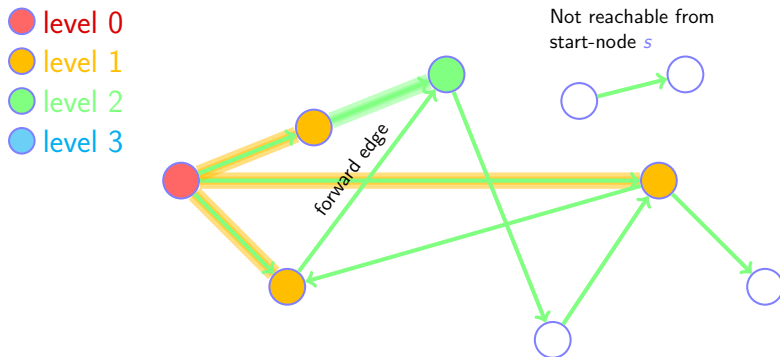


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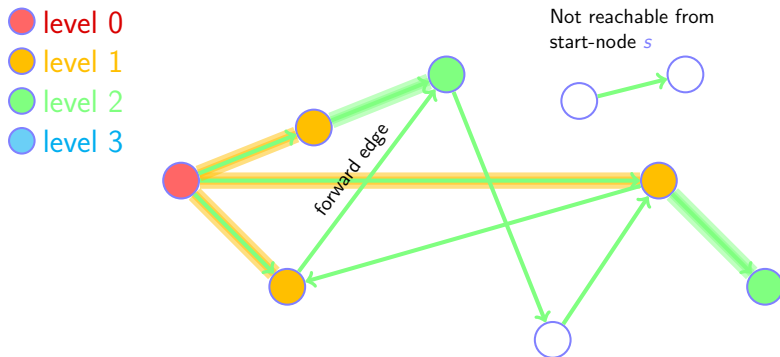


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Graphs

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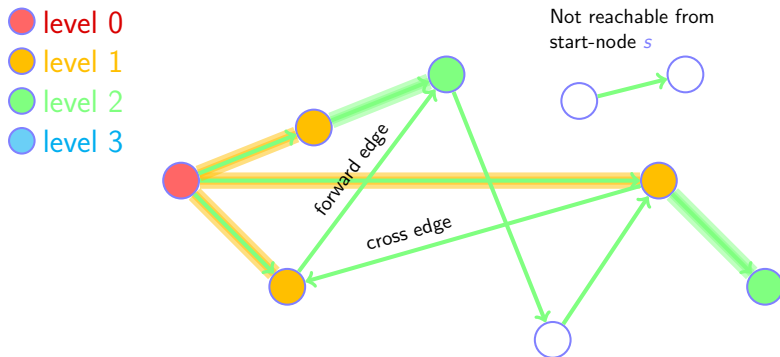


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Graphs

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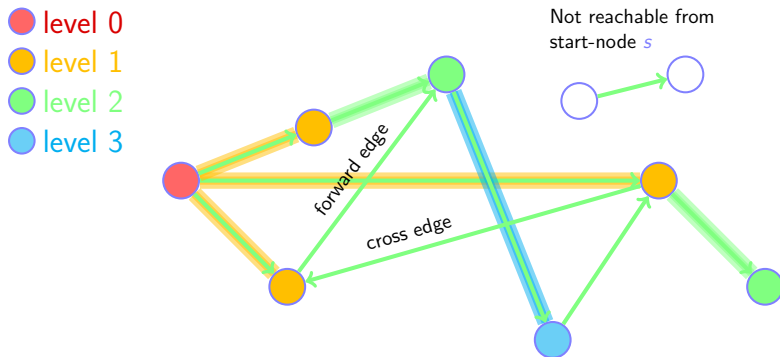


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Graphs

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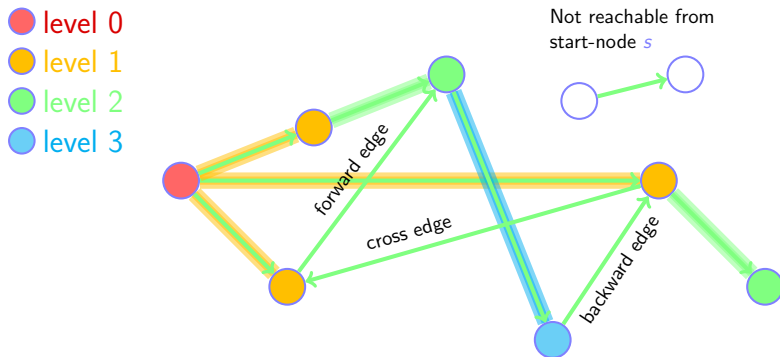


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Graphs

Connected Components - Depth-First Search

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Graphs

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Graphs

Connected Components - Depth-First Search

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3. Pick an unmarked connected vertex and start a recursive depth-first search with the vertex as start vertex (continue on step 2)

Graphs

Connected Components - Depth-First Search

Idea:

1. We start with all vertices unmarked and mark visited vertices
2. Mark the start vertex s
3. Pick an unmarked connected vertex and start a recursive depth-first search with the vertex as start vertex (continue on step 2)
4. If no unmarked connected vertex exists go one vertex back (reduce the recursion level by one)

Graphs

Connected Components - Depth-First Search

Depth-first search:

Graphs

Connected Components - Depth-First Search

Depth-first search:

- ▶ Search starts with **long paths** (searching with depth)

Graphs

Connected Components - Depth-First Search

Depth-first search:

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- ▶ Marks like **breadth-first search** all connected vertices

Graphs

Connected Components - Depth-First Search

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Graphs

Connected Components - Depth-First Search

Depth-first search:

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Graphs

Connected Components - Depth-First Search

Depth-first search:

- ▶ Search starts with **long paths** (searching with depth)
- ▶ Marks like **breadth-first search** all connected vertices
- ▶ If the graph is acyclic we get a **topological sorting**
 - ▶ Each newly visited vertex gets marked by an increasing number
 - ▶ The numbers increase with path from the start vertex

Graphs

Connected Components - Depth-First Search

Graphs

Connected Components - Depth-First Search

- ▶ The marked vertices create a different spanning tree containing all reachable nodes

Graphs

Connected Components - Depth-First Search

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Figure: spanning tree of a depth-first search

Graphs

Connected Components - Depth-First Search

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● start-node

● path 1

● path 2

● path 3

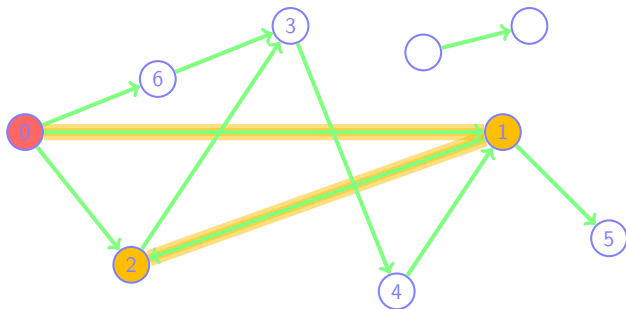


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Graphs

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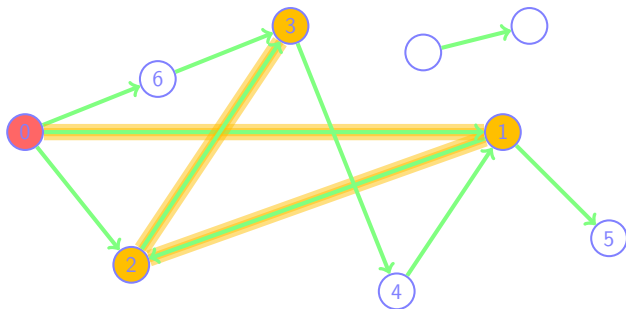


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Graphs

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Graphs

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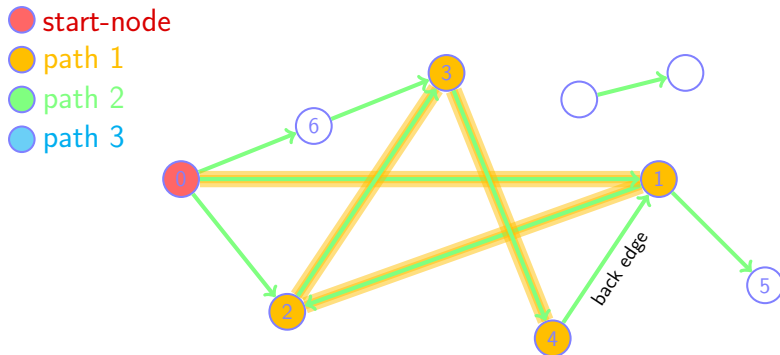


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Graphs

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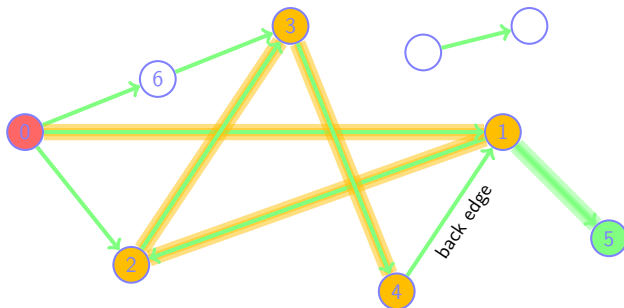


Figure: spanning tree of a depth-first search

Graphs

Connected Components - Depth-First Search

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Graphs

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Figure: spanning tree of a depth-first search

Graphs

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Figure: spanning tree of a depth-first search

Graphs

Why is this called Breadth - and Depth First Search?

Graphs

Connected Components - Breadth-/Depth-First Search

Runtime complexity:

Graphs

Connected Components - Breadth-/Depth-First Search

Runtime complexity:

- ▶ Constant costs for each visited vertex and edge

Graphs

Connected Components - Breadth-/Depth-First Search

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- ▶ We get a runtime complexity of $\Theta(|V'| + |E'|)$

Graphs

Connected Components - Breadth-/Depth-First Search

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Graphs

Connected Components - Breadth-/Depth-First Search

Runtime complexity:

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Graphs

Connected Components - Breadth-/Depth-First Search

Runtime complexity:

- ▶ Constant costs for each visited vertex and edge
- ▶ We get a runtime complexity of $\Theta(|V'| + |E'|)$
- ▶ Let V' and E' be the reachable vertices and edges
- ▶ All vertices of V' are in the same connected component as our start vertex s
- ▶ This can only be improved by a constant factor

Structure

Feedback

Exercises

Lecture

Graphs

Introduction

Implementation

Application example

Application example

Image processing

Application example

Image processing

- ▶ Connected component labeling

Application example

Image processing

- ▶ Connected component labeling
- ▶ Counting of objects in an image

Application example

Image processing

- ▶ Connected component labeling
- ▶ Counting of objects in an image



Application example

Image processing

What's object, what's background?



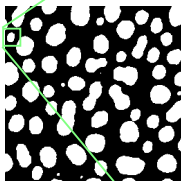
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
35	104	80	56	40	16	16	8	16	16	24	32	32	32	32	32	32	32	24	24	16	
36	80	64	48	32	16	16	16	24	32	40	40	40	40	40	40	40	32	32	24	24	24
37	56	48	32	24	8	16	16	32	40	48	48	48	40	40	40	40	32	32	24	24	24
38	40	32	24	24	16	32	48	64	72	80	80	72	56	56	48	48	40	40	32	32	32
39	16	16	16	24	24	48	72	88	104	112	112	96	72	64	56	48	40	40	40	40	40
40	16	16	24	40	56	88	120	128	136	144	144	120	96	88	72	56	48	48	40	40	40
41	8	16	24	56	80	120	160	168	168	168	168	144	120	104	80	64	48	48	40	40	32
42	16	32	40	80	112	144	176	176	176	168	152	128	112	88	64	48	40	32	32	24	
43	24	40	56	96	136	160	184	184	176	176	168	152	136	112	88	64	40	32	24	24	16
44	40	56	80	112	152	168	184	184	176	176	168	152	136	112	80	64	40	32	16	16	16
45	48	72	96	128	160	176	184	184	176	176	168	152	136	104	72	56	32	24	8	16	16
46	48	72	96	136	168	176	192	192	184	184	176	160	136	104	72	56	32	24	16	24	32
47	48	72	96	136	168	184	192	192	192	192	184	160	136	104	72	48	24	24	16	32	48
48	48	72	96	128	168	184	200	200	200	192	184	160	128	96	64	48	24	32	32	56	72
49	48	72	88	128	160	184	200	200	200	192	184	152	120	88	56	40	24	32	40	72	96
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51	48	64	72	96	112	128	144	152	152	144	136	112	88	64	40	40	32	48	64	112	152
52	48	56	64	80	88	104	112	112	120	112	104	88	72	56	32	32	32	64	88	128	168
53	40	48	48	56	64	72	72	80	80	80	72	64	48	40	24	32	32	72	104	144	184
54	48	48	48	48	48	56	56	56	64	56	56	48	40	32	24	40	48	88	128	160	200

Application example

Image processing

Convert to black white using threshold:

value = 255 **if** value > 100 **else** 0



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
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47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
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56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Application example

Image processing

Interpret image as graph:

Application example

Image processing

Interpret image as graph:

- ▶ Each white pixel is a node

Application example

Image processing

Interpret image as graph:

- ▶ Each white pixel is a node
- ▶ Edges between adjacent pixels (normally 4 or 8 neighbors)

Application example

Image processing

Interpret image as graph:

- ▶ Each white pixel is a node
- ▶ Edges between adjacent pixels (normally 4 or 8 neighbors)
- ▶ Edges are not saved externally, algorithm works directly on array

Application example

Image processing

Interpret image as graph:

- ▶ Each white pixel is a node
- ▶ Edges between adjacent pixels (normally 4 or 8 neighbors)
- ▶ Edges are not saved externally, algorithm works directly on array
- ▶ Breadth- / depth-first search find all connected components (particles)

Application example

Image processing

Find connected components:

Application example

Image processing

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
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54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

Application example

Image processing

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- Search pixel-by-pixel for non-zero intensity

Application example

Image processing

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1

Application example

Image processing

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255
40	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255
41	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255
42	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
51	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255
52	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	255	255
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels

Application example

Image processing

Find connected components:

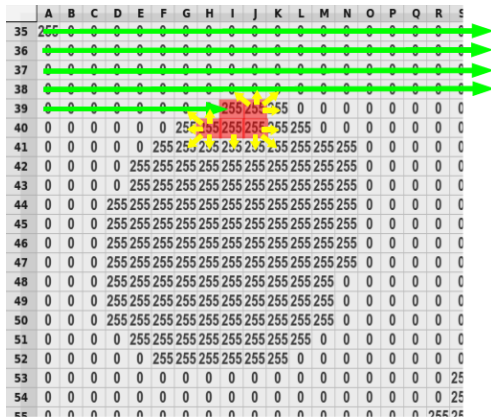
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

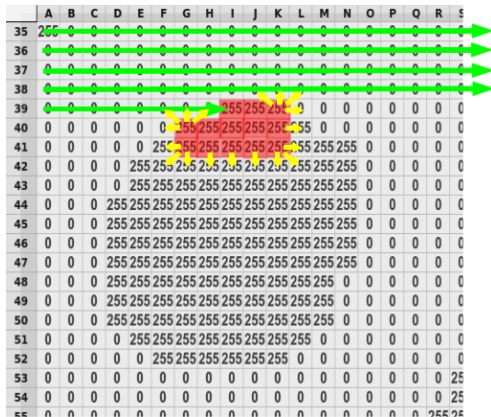
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	255	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

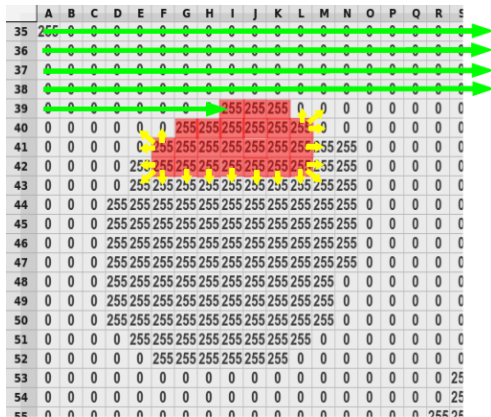
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

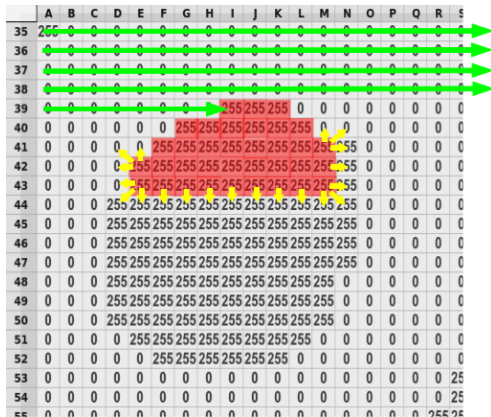
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
52	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 1
- ▶ Check neighbors of all new labeled pixels
- ▶ Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	25	255	255	255	255	255	255	255	255	255
40	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
41	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
42	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
43	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
44	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
45	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
46	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
47	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
48	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
49	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
50	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
51	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
52	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- ▶ Search pixel-by-pixel for non-zero intensity
- ▶ Label found pixel as component 2
- ▶ ...

Application example

Image processing

Result of connected component labeling:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0
45	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0
46	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	25
47	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	25
48	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	255	255	25
49	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	255	255	25
50	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	255	255	255	25
51	0	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	255	255	255	255	25
52	0	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	255	255	255	255	25
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
35	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	13	13	13	13	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0
43	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0
44	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
45	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
46	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
47	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17
48	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17
49	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17
50	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17
51	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17
52	0	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17

Figure: Result: particle indices instead of intensities

Further Literature

► General

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Further Literature

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