

# Algorithms and Datastructures

Runtime analysis Minsort / Heapsort, Induction

Prof. Dr. Rolf Backofen

Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, October 2018

# Structure

## Algorithms and Datastructures

- Structure

- Links

- Organisation

  - Daphne

  - Forum

  - Checkstyle

  - Unit Tests

  - Version management

  - Jenkins

## Sorting

- Minsort

- Heapsort

# Algorithms and Datastructures

## Topics of this Lecture

### Topics of the Lecture:

- ▶ Algorithms and Data Structures
- ▶ **Algorithm**  $\hat{=}$  Solving of complex computational problems
- ▶ **Datastructure**  $\hat{=}$  Representation of data on computer

# Algorithms and Datastructures

## Topics of this Lecture

### Topics of the Lecture:

- ▶ Algorithms and Data Structures  
Efficient data handling and processing  
... for problems that occur in practical **any** larger program / project
- ▶ **Algorithm**  $\hat{=}$  Solving of complex computational problems
- ▶ **Datastructure**  $\hat{=}$  Representation of data on computer

## Example 1: Sorting



Figure: Sorting with *Minsort*

## Example 2: Navigation

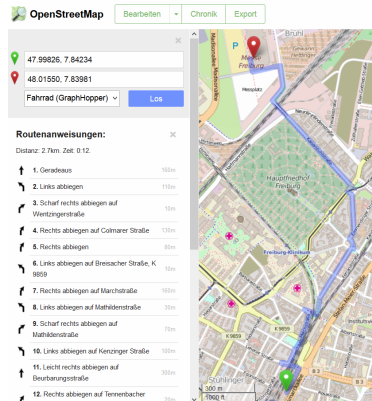


Figure: Navigationplan  
© OpenStreetMap

## Example 2: Navigation

- **Datastructures:** How to represent the map as data?

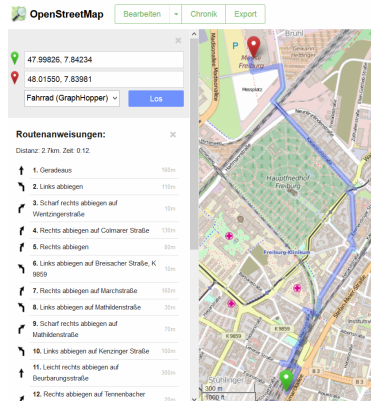


Figure: Navigationsplan  
© OpenStreetMap

## Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?



Figure: Navigationplan  
© OpenStreetMap



# Structure

## Algorithms and Datastructures

### Structure

Links

### Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort

# Content of the Lecture 1 / 2

## **General:**

- ▶ Most of you had a lecture on basic programming ...  
performance was not an issue

# Content of the Lecture 1 / 2

## General:

- ▶ Most of you had a lecture on basic programming ...  
performance was not an issue
- ▶ Here it is going to be:

# Content of the Lecture 1 / 2

## General:

- ▶ Most of you had a lecture on basic programming ...  
performance was not an issue
- ▶ Here it is going to be:
  1. How fast is our program?

# Content of the Lecture 1 / 2

## General:

- ▶ Most of you had a lecture on basic programming ...  
performance was not an issue
- ▶ Here it is going to be:
  1. How fast is our program?
  2. How can we make it faster?

# Content of the Lecture 1 / 2

## General:

- ▶ Most of you had a lecture on basic programming ...  
performance was not an issue
- ▶ Here it is going to be:
  1. How fast is our program?
  2. How can we make it faster?
  3. How can we proof that it will always be that fast?

# Content of the Lecture 1 / 2

## General:

- ▶ Most of you had a lecture on basic programming ...  
performance was not an issue
- ▶ Here it is going to be:
  1. How fast is our program?
  2. How can we make it faster?
  3. How can we proof that it will always be that fast?
- ▶ **Important** issues:
  - ▶ Most of the time: application runtime
  - ▶ Sometimes also: resource / space consumption

## Content of the Lecture 2 / 2

**Algorithms:**



# Content of the Lecture 2 / 2

## **Algorithms:**

- ▶ Sorting
- ▶ Dynamic Arrays
- ▶ Associative Arrays
- ▶ Hashing
- ▶ Edit distance
- ▶ Priority Queue
- ▶ Linked Lists
- ▶ Pathfinding / Dijkstra Algorithm
- ▶ Search Trees

# Content of the Lecture 2 / 2

## **Algorithms:**

- ▶ Sorting
- ▶ Dynamic Arrays
- ▶ Associative Arrays
- ▶ Hashing
- ▶ Edit distance
- ▶ Priority Queue
- ▶ Linked Lists
- ▶ Pathfinding / Dijkstra Algorithm
- ▶ Search Trees

## **Mathematics:**

# Content of the Lecture 2 / 2

## Algorithms:

- ▶ Sorting
- ▶ Dynamic Arrays
- ▶ Associative Arrays
- ▶ Hashing
- ▶ Edit distance
- ▶ Priority Queue
- ▶ Linked Lists
- ▶ Pathfinding / Dijkstra Algorithm
- ▶ Search Trees

## Mathematics:

- ▶ Runtime analysis
- ▶ Proof of correctness
- ▶  $\mathcal{O}$ -Notation

## After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](#)

## After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](#)

- Hopefully your parents will still invite you

# Structure

## Algorithms and Datastructures

Structure

**Links**

Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort

# Links



## Homepage:

- ▶ Exercise sheets
- ▶ Lectures
- ▶ Materials

Link to [Homepage](#)

# Structure

## Algorithms and Datastructures

Structure

Links

### Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort



## Lecture:

- ▶ Tuesday, 12:00 - 14:00, HS 00 006, Build. 082
- ▶ Recordings of the lecture will be uploaded to the webpage

## Exercises:

- ▶ One exercise sheet per week
- ▶ Submission / Correction / Assistance online
- ▶ Tutorial: (if needed)  
Wednesday, 13:00-14:00 - HS 00 006, Build. 082

## Exam:

- ▶ Planned: Sa. 23th March 2019, 10:00-12:00, Build. 101, Lec. theater 026 & 036

# Organisation 2 / 5

## **Exercises:**

- ▶ 80 % practical, 20 % theoretical

## Exercises:

- ▶ 80 % practical, 20 % theoretical
- ▶ We expect **everyone** to solve **every** exercise sheet

# Organisation 2 / 5

## Exercises:

- ▶ 80 % practical, 20 % theoretical
- ▶ We expect **everyone** to solve **every** exercise sheet

## Exam:

# Organisation 2 / 5

## Exercises:

- ▶ 80 % practical, 20 % theoretical
- ▶ We expect **everyone** to solve **every** exercise sheet

## Exam:

- ▶ 50 % of all points from the exercise sheets are needed

## Exercises:

- ▶ 80 % practical, 20 % theoretical
- ▶ We expect **everyone** to solve **every** exercise sheet

## Exam:

- ▶ 50 % of all points from the exercise sheets are needed
- ▶ Content of exam: whole lecture **and all exercises**

# Organisation - Exercises 3 / 5

## Exercises:

- ▶ Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller

# Organisation - Exercises 3 / 5

## Exercises:

- ▶ Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller
- ▶ Coordinators: Michael Uhl, Florian Eggenhofer and Björn Grüning



# Organisation - Exercises 3 / 5

## Exercises:

- ▶ Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller
- ▶ Coordinators: Michael Uhl, Florian Eggenhofer and Björn Grüning
- ▶ Deadline: ESE: 1 week, IEMS: none

# Organisation - Exercises 3 / 5

## **Exercises:**

- ▶ Post questions into the forum (link later)

# Organisation - Exercises 3 / 5

## **Exercises:**

- ▶ Post questions into the forum (link later)
- ▶ Submission via “commit” through svn and Daphne

# Organisation - Exercises 3 / 5

## Exercises:

- ▶ Post questions into the forum (link later)
- ▶ Submission via “commit” through svn and Daphne
- ▶ Feedback one week after deadline through “update” (svn)

# Organisation - Exercises 3 / 5

## Exercises:

- ▶ Post questions into the forum (link later)
- ▶ Submission via “commit” through svn and Daphne
- ▶ Feedback one week after deadline through “update” (svn)
- ▶ Unit test / checkstyle via Jenkins

# Organisation - Exercises 4 / 5

## Exercises - Points:

- ▶ Practical:
  - ▶ 60 % functionality
  - ▶ 20 % tests
  - ▶ 20 % documentation, Checkstyle, etc.
  - ▶ Program is not running  $\Rightarrow$  0 points

# Organisation - Exercises 4 / 5

## Exercises - Points:

- ▶ Practical:
  - ▶ 60 % functionality
  - ▶ 20 % tests
  - ▶ 20 % documentation, Checkstyle, etc.
  - ▶ Program is not running  $\Rightarrow$  0 points
- ▶ Theoretical (mathematical proof):
  - ▶ 40 % general idea / approach
  - ▶ 60 % clean / complete

## Effort:

- ▶ 4 ECTS (ESE), 6 ECTS (IEMS)
- ▶ 120 / 180 working hours per semester
- ▶ 14 Lectures each 6 h / 8 h + exam
- ▶ 4 h / 6 h per exercise sheet (one per week)



# Structure

## Algorithms and Datastructures

Structure

Links

### Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort

# Daphne

## Daphne:

- ▶ Provides the following information:
  - ▶ Name / contact information of your tutor
  - ▶ Download of / info needed for exercise sheets
  - ▶ Collected points of all exercise sheets
  - ▶ Links to:
    1. Coding standards
    2. Build system
    3. The other systems
- ▶ Link: [Daphne](#)

## Forum:

- ▶ Please don't hesitate to ask if something is unclear
- ▶ Ask in the forum and not separate. Others might also be interested in the answer
- ▶ The [tutors](#) or the [coordinators](#) will reply as soon as possible
- ▶ Link: [Forum](#)

# Checkstyle

flake8

## Checkstyle / Linting (flake8):

- ▶ Installation: **python3** -m pip install flake8
- ▶ Check file: **python3** -m flake8 path/to/files/\*.py
- ▶ Link: [flake8](#)

# Unit Tests

## **Why unit tests?**

1. A non-trivial method without a unit test is probably wrong

# Unit Tests

## **Why unit tests?**

1. A non-trivial method without a unit test is probably wrong
2. Simplifies debugging

# Unit Tests

## **Why unit tests?**

1. A non-trivial method without a unit test is probably wrong
2. Simplifies debugging
3. We and you can automatically check correctness of code

# Unit Tests

## **Why unit tests?**

1. A non-trivial method without a unit test is probably wrong
2. Simplifies debugging
3. We and you can automatically check correctness of code

## **What is a good unit test?**



# Unit Tests

## **Why unit tests?**

1. A non-trivial method without a unit test is probably wrong
2. Simplifies debugging
3. We and you can automatically check correctness of code

## **What is a good unit test?**

- ▶ Unit test checks desired output for a given input

# Unit Tests

## Why unit tests?

1. A non-trivial method without a unit test is probably wrong
2. Simplifies debugging
3. We and you can automatically check correctness of code

## What is a good unit test?

- ▶ Unit test checks desired output for a given input
- ▶ At least one **typical** input

# Unit Tests

## Why unit tests?

1. A non-trivial method without a unit test is probably wrong
2. Simplifies debugging
3. We and you can automatically check correctness of code

## What is a good unit test?

- ▶ Unit test checks desired output for a given input
  - ▶ At least one **typical** input
  - ▶ At least one **critical** case
- E.g. double occurrence of a value in sorting

# Unit Tests

## doctest

### Testing (doctest):

```
def subtract_one(n):  
    """Subtracts 1 from n
```

- ▶ Tests are contained in docstrings

```
>>> subtract_one(5)  
4
```

```
>>> subtract_one(3)  
2  
"""
```

```
return n-1
```

```
if __name__ == "__main__":  
    print("2 - 1 = %d" % subtract_one(2))
```

# Unit Tests

## doctest

### Testing (doctest):

```
def subtract_one(n):  
    """Subtracts 1 from n
```

```
>>> subtract_one(5)  
4
```

```
>>> subtract_one(3)  
2  
"""
```

```
    return n-1
```

```
if __name__ == "__main__":  
    print("2 - 1 = %d" % subtract_one(2))
```

- ▶ Tests are contained in docstrings
- ▶ Module doctest runs them

# Unit Tests

## doctest

### Testing (doctest):

```
def subtract_one(n):  
    """Subtracts 1 from n
```

```
>>> subtract_one(5)  
4
```

```
>>> subtract_one(3)  
2  
"""
```

```
return n-1
```

```
if __name__ == "__main__":  
    print("2 - 1 = %d" % subtract_one(2))
```

- ▶ Tests are contained in docstrings
- ▶ Module doctest runs them
- ▶ Run check with:  
**python3 -m doctest**  
*path/to/files/\*.py -v*

# Version management

## Subversion

### Version management (subversion):

- ▶ Keeps a history of code changes
- ▶ Initialize / update directory: **svn** checkout <URL>
- ▶ Add files / folders: **svn** add <file> --all
- ▶ Create snapshot: **svn** commit -m "<Your Message>"  
Data is uploaded to Jenkins automatically
- ▶ Link: [Subversion](#)

# Jenkins

## Jenkins:

- ▶ Provides our build system
- ▶ You can check if your uploaded code runs
  - ▶ Especially whether all **unit test** pass
  - ▶ And if **checkstyle** (flake8) is statisfied
- ▶ Will be shown in the first exercise
- ▶ Link: [Jenkins](#)



# Structure

## Algorithms and Datastructures

Structure

Links

Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort

# Sorting 1 / 2

## Problem:

- ▶ Input:  $n$  elements  $x_1, \dots, x_n$
- ▶ Transitive operator “**i**” which returns **true** if the left value is smaller than the right one
  - ▶ Transitivity:  $x < y, y < z \rightarrow x < z$
- ▶ Output:  $x_1, \dots, x_n$  sorted with operator

# Sorting 1 / 2

## Problem:

- ▶ Input:  $n$  elements  $x_1, \dots, x_n$
- ▶ Transitive operator “ $i$ ” which returns **true** if the left value is smaller than the right one
  - ▶ Transitivity:  $x < y, y < z \rightarrow x < z$
- ▶ Output:  $x_1, \dots, x_n$  sorted with operator

## Example

Input: 14, 4, 32, 19, 8, 44, 65

Output:

### Why do we need sorting?

- ▶ Nearly **every** program needs a sorting algorithm
- ▶ **Examples:**
  - ▶ Index of a search engine
  - ▶ Listing filesystem in explorer / finder
  - ▶ (Music) library
  - ▶ Highscore list

# Structure

## Algorithms and Datastructures

Structure

Links

Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*



# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...

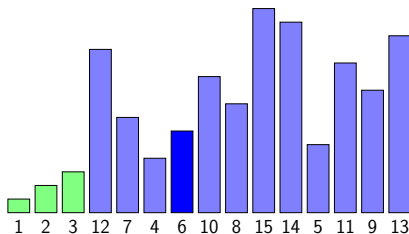


Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: Minsort

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...

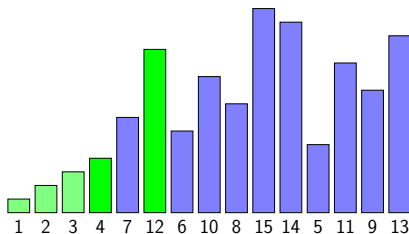


Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*



# Minsort - Algorithm

## Minsort in Python:

```
def minsort(lst):  
    for i in range(0, len(lst)-1):  
        minimum = i  
  
        for j in range(i+1, len(lst)):  
            if lst[j] < lst[minimum]:  
                minimum = j  
  
        if minimum != i:  
            lst[i], lst[minimum] = \  
                lst[minimum], lst[i]  
  
    return lst
```

# MinSort - Runtime

**How long does our program run?**

- ▶ We test it for different input sizes

# MinSort - Runtime

How long does our program run?

Table: Runtime for *Minsort*

- We test it for different input sizes

$n$	Runtime / ms
$2 \times 10^3$	5.24
$4 \times 10^3$	16.92
$6 \times 10^3$	39.11
$8 \times 10^3$	67.80
$10 \times 10^3$	105.50
$12 \times 10^3$	150.38
$14 \times 10^3$	204.00
$16 \times 10^3$	265.98
$18 \times 10^3$	334.94

# MinSort - Runtime

## How long does our program run?

Table: Runtime for *Minsort*

- ▶ We test it for different input sizes
- ▶ **Observation:**  
It is going to be “disproportionately” slower the more numbers are being sorted

$n$	Runtime / ms
$2 \times 10^3$	5.24
$4 \times 10^3$	16.92
$6 \times 10^3$	39.11
$8 \times 10^3$	67.80
$10 \times 10^3$	105.50
$12 \times 10^3$	150.38
$14 \times 10^3$	204.00
$16 \times 10^3$	265.98
$18 \times 10^3$	334.94

# MinSort - Runtime

## How long does our program run?

- ▶ We test it for different input sizes
- ▶ **Observation:**  
It is going to be “disproportionately” slower the more numbers are being sorted



Figure: Runtime of *Minsort*

# MinSort - Runtime

## Runtime analysis:

- ▶ *Minsort* runtime depicted in a diagram
  - ▶ That is what you should do in the first exercise sheet



Figure: Runtime of *Minsort*

# MinSort - Runtime

## Runtime analysis:

- ▶ *Minsort* runtime depicted in a diagram
  - ▶ That is what you should do in the first exercise sheet
- ▶ **We observe:**
  - ▶ The runtime **grows faster than linear**
  - ▶ With double the input size we need four times the time

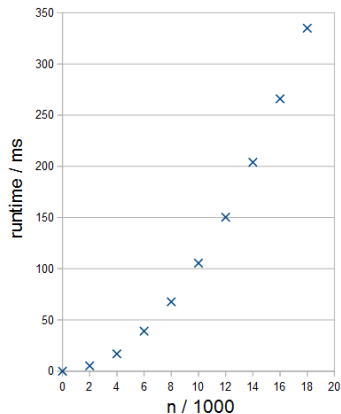


Figure: Runtime of *Minsort*

# MinSort - Runtime

## Runtime analysis:

- ▶ *Minsort* runtime depicted in a diagram
  - ▶ That is what you should do in the first exercise sheet
- ▶ **We observe:**
  - ▶ The runtime **grows faster than linear**
  - ▶ With double the input size we need four times the time

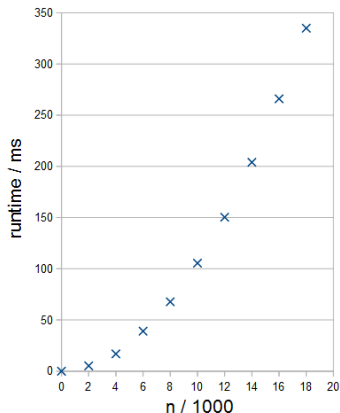


Figure: Runtime of *Minsort*

- ▶ Next lecture we will analyze deeper with other methods



# Structure

## Algorithms and Datastructures

Structure

Links

Organisation

Daphne

Forum

Checkstyle

Unit Tests

Version management

Jenkins

## Sorting

Minsort

Heapsort

# Heapsort - Algorithm 1 / 10

## **Heapsort:**

- ▶ The principle stays the same
- ▶ Better structure for finding the smallest element quicker

# Heapsort - Algorithm 1 / 10

## Heapsort:

- ▶ The principle stays the same
- ▶ Better structure for finding the smallest element quicker

## Binary heap:

- ▶ Preferably a complete binary tree
- ▶ **Heap property:** Each child is **smaller** (larger) than the parent element

## Heapsort - Algorithm 2 / 10

### Min heap:

- **Heap property:** Each child is **smaller** (larger) than the parent element



Figure: Valid min heap



Figure: Invalid min heap

# Heapsort - Algorithm 2 / 10

## Min heap:

- ▶ **Heap property:** Each child is **smaller** (larger) than the parent element
- ▶ A valid heap fulfills the property at each node



Figure: Valid min heap



Figure: Invalid min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array



Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$

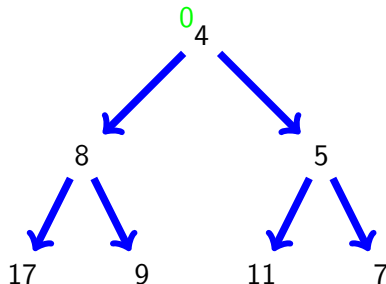


Table: Elements can be stored in array

0						
4						

Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

0	1					
4	8					

Figure: Min heap



# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

0	1	2				
4	8	5				

Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

0	1	2	3			
4	8	5	17			

Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

0	1	2	3	4		
4	8	5	17	9		

Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

0	1	2	3	4	5	
4	8	5	17	9	11	

Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

0	1	2	3	4	5	6
4	8	5	17	9	11	7

Figure: Min heap

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

- ▶ Remove the smallest element (root node)

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node



# Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ **Sift** the new root node down until the **heap property** is satisfied

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ **Sift** the new root node down until the **heap property** is satisfied

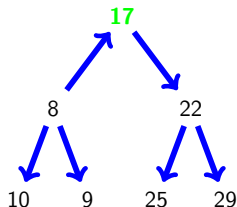


Figure: Repairing a min heap

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ **Sift** the new root node down until the **heap property** is satisfied

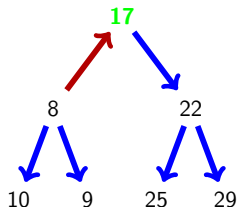


Figure: Repairing a min heap

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ **Sift** the new root node down until the **heap property** is satisfied

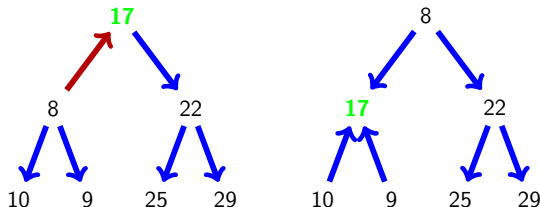


Figure: Repairing a min heap

## Heapsort - Algorithm 4 / 10

### Repairing after taking the smallest element: `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ Sift the new root node down until the **heap property** is satisfied

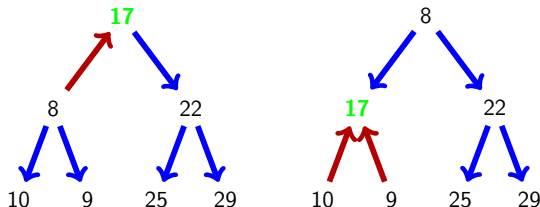


Figure: Repairing a min heap

## Heapsort - Algorithm 4 / 10

### Repairing after taking the smallest element: `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ Sift the new root node down until the **heap property** is satisfied

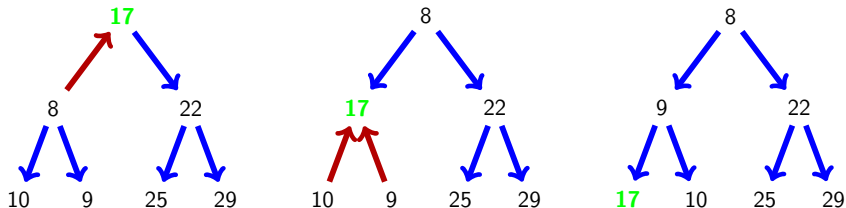


Figure: Repairing a min heap

# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described

# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described
- ▶ Output: 4

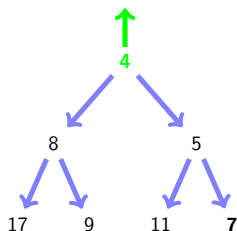


Figure: One iteration of Heapsort



# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described
- ▶ Output: 4

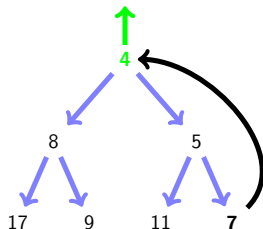


Figure: One iteration of Heapsort

# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described
- ▶ Output: 4



Figure: One iteration of Heapsort

# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described
- ▶ Output: 4

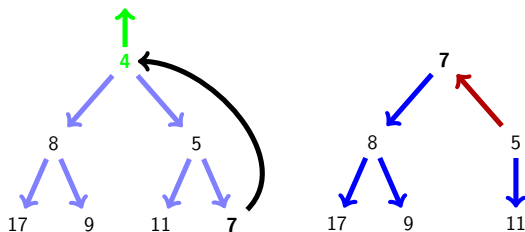


Figure: One iteration of Heapsort

# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described
- ▶ Output: 4, 5, ...



Figure: One iteration of Heapsort

# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
  - ▶ Take the smallest element
  - ▶ Move the last node to the root
  - ▶ Repair the heap as described
- ▶ Output: 4, 5, ...



Figure: One iteration of Heapsort

# Heapsort - Algorithm 6 / 10

## **Creating a heap:**

- ▶ This operation is called **heapify**

# Heapsort - Algorithm 6 / 10

## Creating a heap:

- ▶ This operation is called **heapify**
- ▶ The  $n$  elements are already stored in an array

# Heapsort - Algorithm 6 / 10

## Creating a heap:

- ▶ This operation is called **heapify**
- ▶ The  $n$  elements are already stored in an array
- ▶ Interpret the array as binary heap where the **heap property** is not yet satisfied



# Heapsort - Algorithm 6 / 10

## Creating a heap:

- ▶ This operation is called **heapify**
- ▶ The  $n$  elements are already stored in an array
- ▶ Interpret the array as binary heap where the **heap property** is not yet satisfied
- ▶ We repair the heap from bottom up (in layers) with **sift**

# Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4



Figure: Heapify lower layer

# Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4



Figure: Heapify lower layer

## Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

## Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

## Heapsort - Algorithm 9 / 10



Figure: Resulting heap

# Heapsort - Algorithm 10 / 10

## Finding the minimum is intuitive:

- ▶ **Minsort:** Iterate through all non-sorted elements
- ▶ **Heapsort:** Finding the minimum is trivial (concept)  
*Just take the root of the heap*

# Heapsort - Algorithm 10 / 10

## Finding the minimum is intuitive:

- ▶ **Minsort:** Iterate through all non-sorted elements
- ▶ **Heapsort:** Finding the minimum is trivial (concept)

*Just take the root of the heap*

## Removing the minimum in Heapsort:

- ▶ Repair the heap and restore the **heap property**
  - ▶ We don't have to repair the whole heap
- ▶ More of this in the next lecture



# Further Literature

## ► Course literature

- [CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.

*Introduction to Algorithms.*

MIT Press, Cambridge, Mass, 2001.

- [MS08] Kurt Mehlhorn and Peter Sanders.

*Algorithms and Data Structures.*

Springer, Berlin, 2008.

<https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf>.

# Further Literature

## ► **Sorting**

[Wika] [Wikipedia - Heapsort](https://en.wikipedia.org/wiki/Heapsort)

`https://en.wikipedia.org/wiki/Heapsort`

[Wikb] [Wikipedia - Selectionsort](https://de.wikipedia.org/wiki/Selectionsort)

`https://de.wikipedia.org/wiki/Selectionsort`

# Further Literature

- ▶ **Subversion**

[Apa] [Apache Subversion](https://subversion.apache.org/)

`https://subversion.apache.org/`