Algorithms and Datastructures

Runtime analysis Minsort / Heapsort, Induction

Prof. Dr. Rolf Backofen

Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, October 2018

Structure

Topics of the Lecture:

- Algorithms and Data Structures
- **Algorithm** $\hat{=}$ Solving of complex computional problems
- **Datastructure** $\hat{=}$ Representation of data on computer

Topics of the Lecture:

- Algorithms and Data Structures
  Efficient data handling and processing
  ... for problems that occur in practical **any** larger program / project
- **Algorithm** $\hat{=}$ Solving of complex computional problems
- **Datastructure** $\hat{=}$ Representation of data on computer

Example 1: Sorting

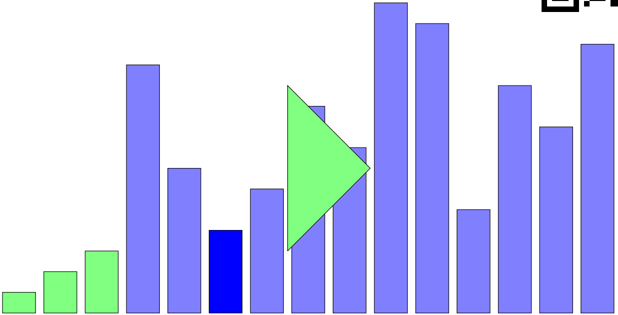


Figure: Sorting with *Minsort*

Example 2: Navigation

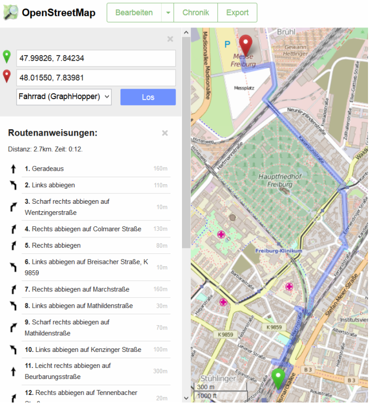
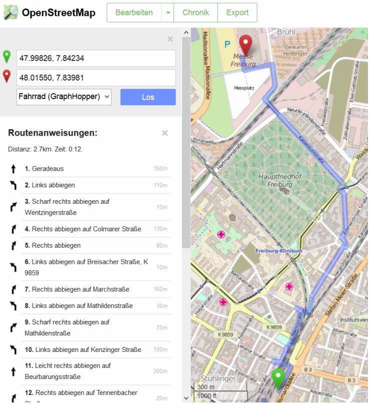


Figure: Navigationplan
© OpenStreetMap

Example 2: Navigation

- **Datastructures:** How to represent the map as data?



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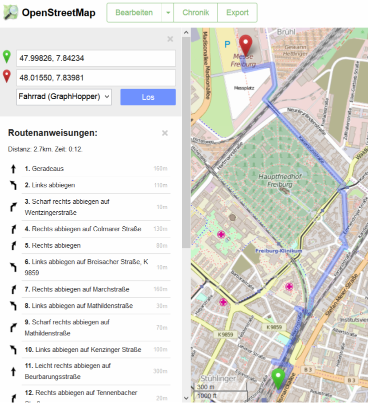
Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?



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Content of the Lecture 1 / 2

General:

- Most of you had a lecture on basic progamming . . .

  performance was not an issue

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- **Important** issues:
  - Most of the time: application runtime
  - Sometimes also: resource / space consumption

Content of the Lecture 2 / 2

Algorithms:

Content of the Lecture 2 / 2

Algorithms:

- Sorting
- Dynamic Arrays
- Associative Arrays
- Hashing
- Edit distance
- Priority Queue
- Linked Lists
- Pathfinding / Dijkstra Algorithm
- Search Trees

Content of the Lecture 2 / 2

Algorithms:

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Mathematics:

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- Dynamic Arrays
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- Priority Queue
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Mathematics:

- Runtime analysis
- $\mathcal{O}$-Notation
- Proof of correctness

After the lecture ...

- ... you should be able to understand the joke

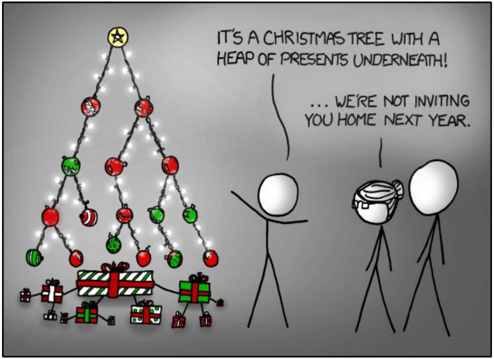

Figure: Comic 🅭 xkcd/835

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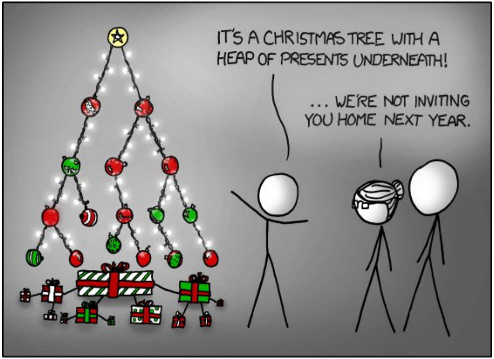

Figure: Comic © xkcd/835

- Hopefully your parents will still invite you

Structure

Links

Homepage:

- Exercise sheets
- Lectures
- Materials

Link to Homepage

Structure

Organisation 1 / 5

Lecture:

- Tuesday, 12:00 - 14:00, HS 00 006, Build. 082
- Recordings of the lecture will be uploaded to the webpage

Exercises:

- One exercise sheet per week
- Submission / Correction / Assistance online
- Tutorial: (if needed)
  Wednesday, 13:00-14:00 - HS 00 006, Build. 082

Exam:

- Planned: Sa. 23th March 2019, 10:00-12:00, Build. 101, Lec. theater 026 & 036

Organisation 2 / 5

Exercises:

- 80 % practical, 20 % theoretical

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- 50 % of all points from the exercise sheets are needed

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- We expect **everyone** to solve **every** exercise sheet

Exam:

- 50 % of all points from the exercise sheets are needed
- Content of exam: whole lecture **and all exercises**

Organisation - Exercises 3 / 5

Exercises:

- Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller

Organisation - Exercises 3 / 5

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- Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller
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Organisation - Exercises 3 / 5

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- Deadline: ESE: 1 week, IEMS: none

Organisation - Exercises 3 / 5

Exercises:

- Post questions into the forum (link later)

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Exercises:

- Post questions into the forum (link later)
- Submission via "`commit`" through `svn` and Daphne
- Feedback one week after deadline through "`update`" (`svn`)
- Unit test / checkstyle via Jenkins

Organisation - Exercises 4 / 5

Exercises - Points:

- Practical:
  - 60 % functionality
  - 20 % tests
  - 20 % documentation, Checkstyle, etc.
  - Program is not running $\Rightarrow$ 0 points

Organisation - Exercises 4 / 5

Exercises - Points:

- Practical:
  - 60 % functionality
  - 20 % tests
  - 20 % documentation, Checkstyle, etc.
  - Program is not running $\Rightarrow$ 0 points
- Theoretical (mathematical proof):
  - 40 % general idea / approach
  - 60 % clean / complete

Organisation 5 / 5

Effort:

- 4 ECTS (ESE), 6 ECTS (IEMS)
- 120 / 180 working hours per semester
- 14 Lectures each 6 h / 8 h + exam
- 4 h / 6 h per exercise sheet (one per week)

Structure

Daphne

Daphne:

- Provides the following information:
  - Name / contact information of your tutor
  - Download of / info needed for exercise sheets
  - Collected points of all exercise sheets
  - Links to:
    1. Coding standards
    2. Build system
    3. The other systems
- Link: Daphne

Forum

Forum:

- Please don't hesitate to ask if something is unclear
- Ask in the forum and not separate. Others might also be interested in the answer
- The tutors or the coordinators will reply as soon as possible
- Link: Forum

Checkstyle

Checkstyle / Linting (flake8):

- Installation: **python3** `-m pip install flake8`
- Check file: **python3** `-m flake8 path/to/files/*.py`
- Link: flake8

Unit Tests

Why unit tests?

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What is a good unit test?

- Unit test checks desired output for a given input
- At least one **typical** input
- At least one **critical** case
  E.g. double occurrence of a value in sorting

Testing (doctest):

```python
def subtract_one(n):
    """Subtracts 1 from n

    >>> subtract_one(5)
    4

    >>> subtract_one(3)
    2
    """
    return n-1

if __name__ == "__main__":
    print("2 - 1 = %d" % subtract_one(2))
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- Tests are contained in docstrings
- Module `doctest` runs them
- Run check with: **python3** *-m doctest path/to/files/*.py -v*

Version management (subversion):

- Keeps a history of code changes
- Initialize / update directory: **svn** `checkout <URL>`
- Add files / folders: **svn** `add <file> --all`
- Create snapshot: **svn** `commit -m "<Your Message>"`
  Data is uploaded to Jenkins automatically
- Link: Subversion

Jenkins

Jenkins:

- Provides our build system
- You can check if your uploded code runs
  - Especially whether all **unit test** pass
  - And if **checkstyle** (`flake8`) is statisfied
- Will be shown in the first exercise
- Link: Jenkins

Structure

Sorting 1 / 2

Problem:

- Input: n elements $x_1, \ldots, x_n$
- Transitive operator "**¡**" which returns **true** if the left value is smaller than the right one
  - Transitivity: $x < y,\ y < z \rightarrow x < z$
- Output: $x_1, \ldots, x_n$ sorted with operator

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Example

Input: 14, 4, 32, 19, 8, 44, 65
Output:

Sorting 2 / 2

Why do we need sorting?

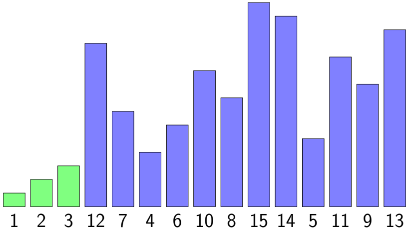
- Nearly every program needs a sorting algorithm
- **Examples:**
  - Index of a search engine
  - Listing filesystem in explorer / finder
  - (Music) library
  - Highscore list

Structure

Minsort - Algorithm

Informal description:

- Find the minimum and switch the value with the first position
- Find the minimum and switch the value with the second position
- ...



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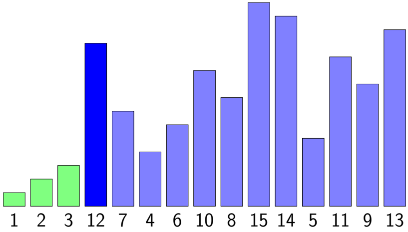

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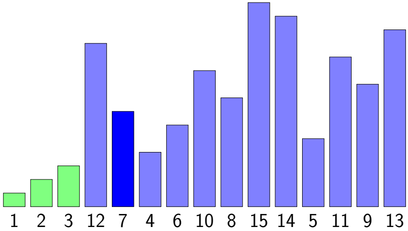

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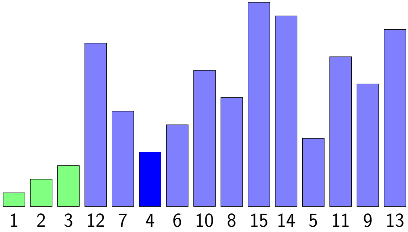

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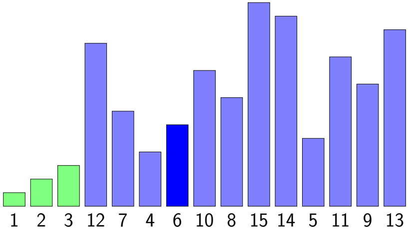

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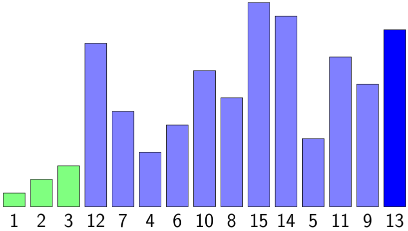

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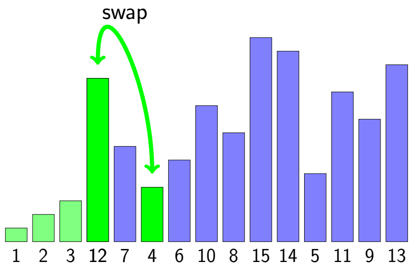

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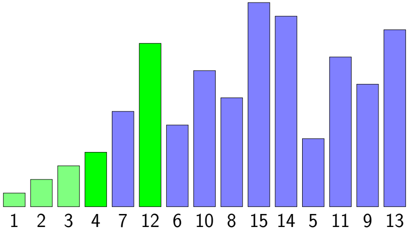

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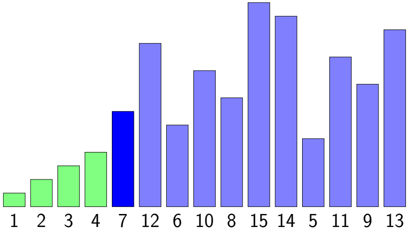

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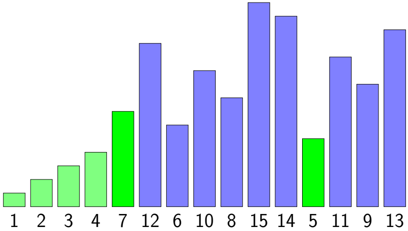

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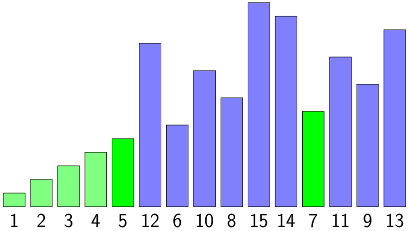

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Minsort - Algorithm

Minsort in Python:

```
def minsort(lst):
    for i in range(0, len(lst)-1):
        minimum = i

        for j in range(i+1, len(lst)):
            if lst[j] < lst[minimum]:
                minimum = j

        if minimum != i:
            lst[i], lst[minimum] = \
                lst[minimum], lst[i]

    return lst
```

MinSort - Runtime

How long does our program run?

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Table: Runtime for *Minsort*

n	Runtime / ms
2×10^3	5.24
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6×10^3	39.11
8×10^3	67.80
10×10^3	105.50
12×10^3	150.38
14×10^3	204.00
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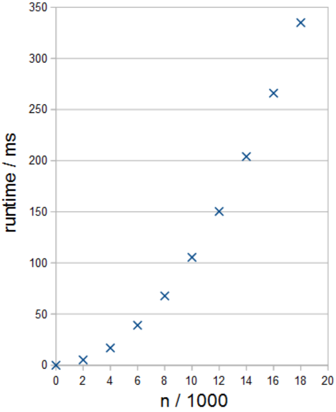

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Runtime analysis:

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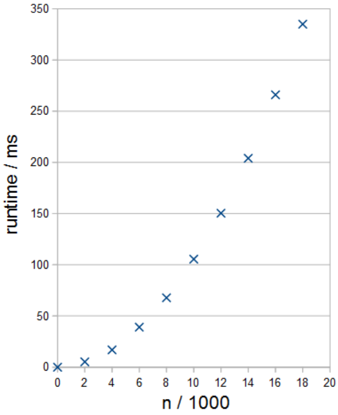
  - That is what you should do in the first exercise sheet



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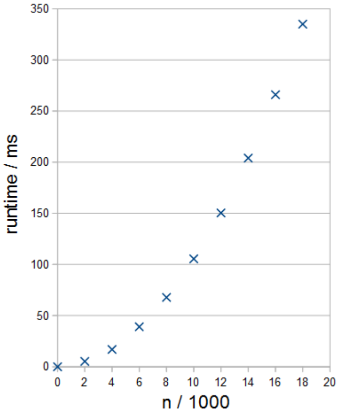

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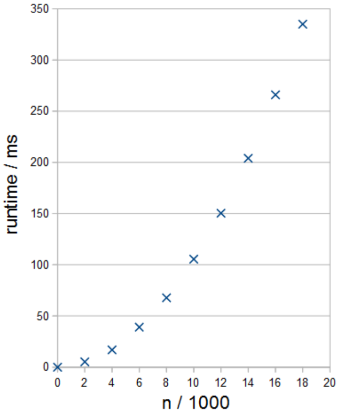

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- Next lecture we will analyze deeper with other methods

Structure

Heapsort - Algorithm 1 / 10

Heapsort:

- The principle stays the same
- Better structure for finding the smallest element quicker

Heapsort - Algorithm 1 / 10

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Binary heap:

- Preferably a complete binary tree
- **Heap property:** Each child is smaller (larger) than the parent element

Min heap:

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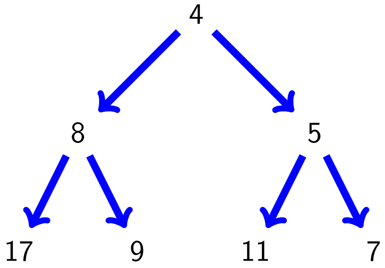
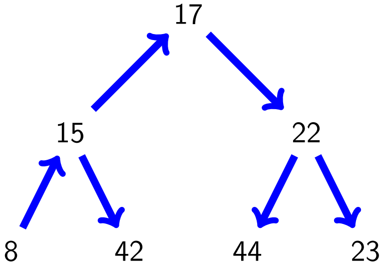


Figure: Valid min heap



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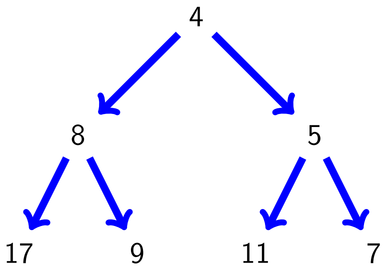
Heapsort - Algorithm 2 / 10

Min heap:

- **Heap property:** Each child is smaller (larger) than the parent element
- A valid heap fulfills the property at each node



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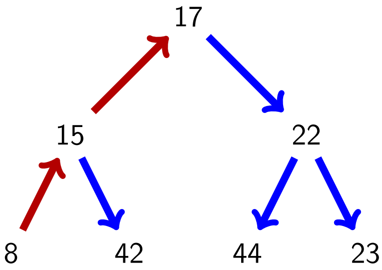

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Heapsort - Algorithm 3 / 10

How to save the heap?

- We number all nodes from top to bottom and left to right starting at 0
  - The children of node i are $2i + 1$ and $2i + 2$
  - The parent node of node i is $\mathrm{floor}\left(\frac{i-1}{2}\right)$

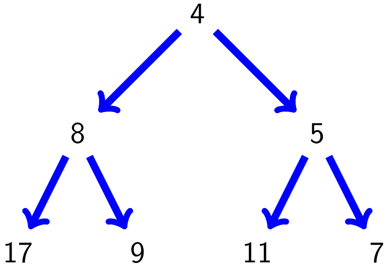

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Table: Elements can be stored in array

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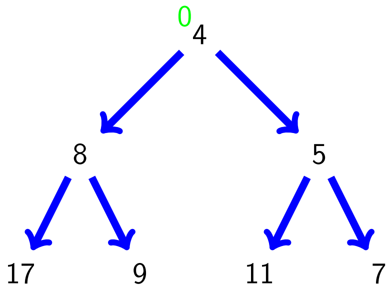

Figure: Min heap

Table: Elements can be stored in array

0						
4						

How to save the heap?

- We number all nodes from top to bottom and left to right starting at 0
  - The children of node i are $2i+1$ and $2i+2$
  - The parent node of node i is $\text{floor}\left(\frac{i-1}{2}\right)$

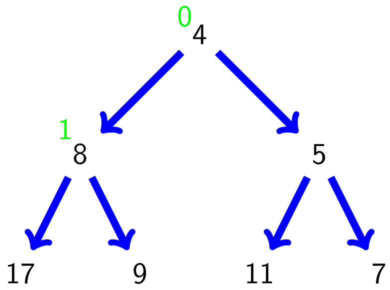

Figure: Min heap

Table: Elements can be stored in array

0	1					
4	8					

Heapsort - Algorithm 3 / 10

How to save the heap?

- We number all nodes from top to bottom and left to right starting at 0
  - The children of node i are $2i+1$ and $2i+2$
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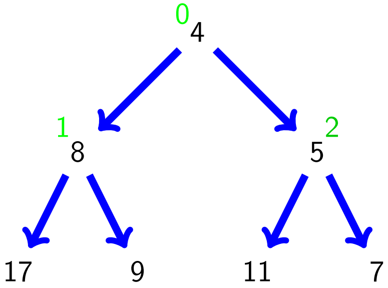

Figure: Min heap

Table: Elements can be stored in array

0	1	2				
4	8	5				

Heapsort - Algorithm 3 / 10

How to save the heap?

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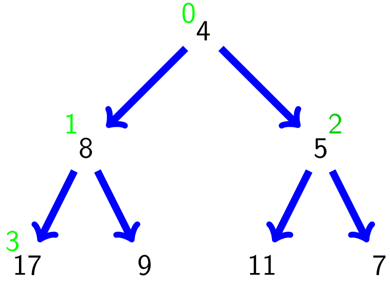

Figure: Min heap

Table: Elements can be stored in array

0	1	2	3			
4	8	5	17			

Heapsort - Algorithm 3 / 10

How to save the heap?

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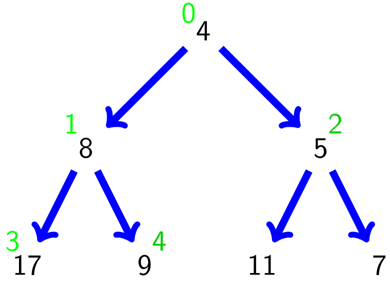

Figure: Min heap

Table: Elements can be stored in array

0	1	2	3	4		
4	8	5	17	9		

Heapsort - Algorithm 3 / 10

How to save the heap?

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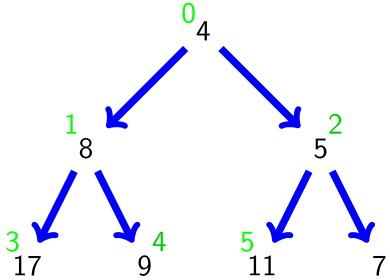

Figure: Min heap

Table: Elements can be stored in array

0	1	2	3	4	5	
4	8	5	17	9	11	

Heapsort - Algorithm 3 / 10

How to save the heap?

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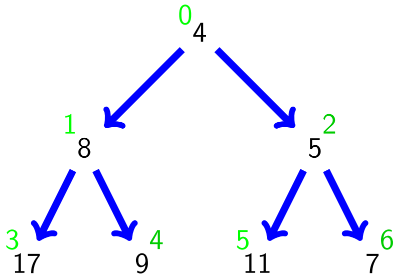

Figure: Min heap

Table: Elements can be stored in array

0	1	2	3	4	5	6
4	8	5	17	9	11	7

Heapsort - Algorithm 4 / 10

Repairing after taking the smallest element: `heap.pop()`

Heapsort - Algorithm 4 / 10

Repairing after taking the smallest element: `heap.pop()`

- Remove the smallest element (root node)

Heapsort - Algorithm 4 / 10

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- Replace the root with the last node

Heapsort - Algorithm 4 / 10

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- Remove the smallest element (root node)
- Replace the root with the last node
- Sift the new root node down until the heap property is satisfied

Heapsort - Algorithm 4 / 10

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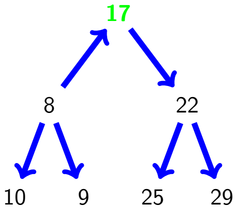

Figure: Repairing a min heap

Heapsort - Algorithm 4 / 10

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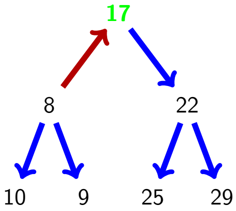

Figure: Repairing a min heap

Heapsort - Algorithm 4 / 10

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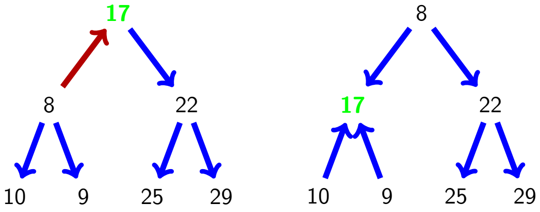

Figure: Repairing a min heap

Heapsort - Algorithm 4 / 10

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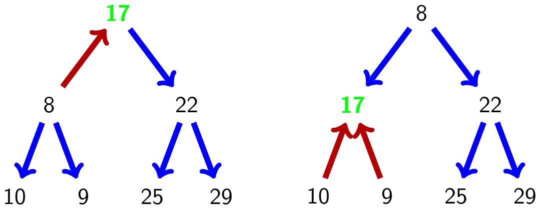

Figure: Repairing a min heap

Heapsort - Algorithm 4 / 10

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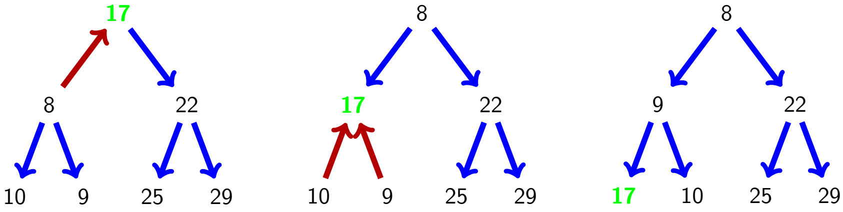

Figure: Repairing a min heap

HeapSort - Algorithm 5 / 10

Heapsort:

- Organize the n elements as heap
- While the heap still contains elements
  - Take the smallest element
  - Move the last node to the root
  - Repair the heap as described

HeapSort - Algorithm 5 / 10

Heapsort:

- Organize the n elements as heap
- While the heap still contains elements
  - Take the smallest element
  - Move the last node to the root
  - Repair the heap as described
- Output: 4

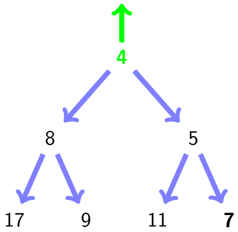

Figure: One iteration of Heapsort

HeapSort - Algorithm 5 / 10

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- Output: 4

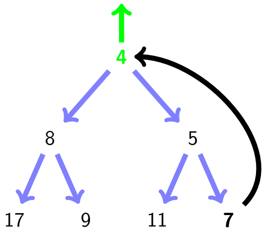


Figure: One iteration of Heapsort

Heapsort:

- Organize the n elements as heap
- While the heap still contains elements
  - Take the smallest element
  - Move the last node to the root
  - Repair the heap as described
- Output: 4

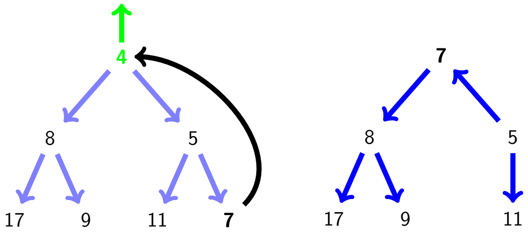
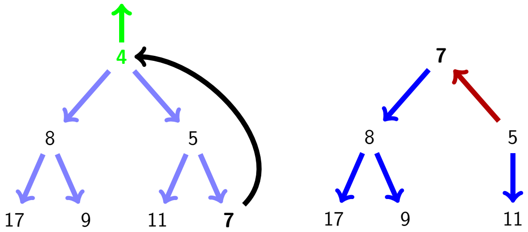


Figure: One iteration of Heapsort

HeapSort - Algorithm 5 / 10

Heapsort:

- Organize the n elements as heap
- While the heap still contains elements
  - Take the smallest element
  - Move the last node to the root
  - Repair the heap as described
- Output: 4



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HeapSort - Algorithm 5 / 10

Heapsort:

- Organize the n elements as heap
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  - Repair the heap as described
- Output: 4, 5, ...

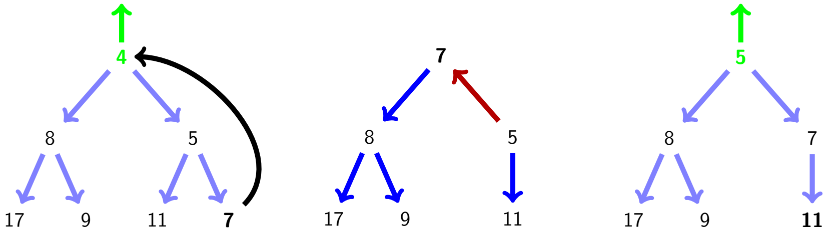

Figure: One iteration of Heapsort

HeapSort - Algorithm 5 / 10

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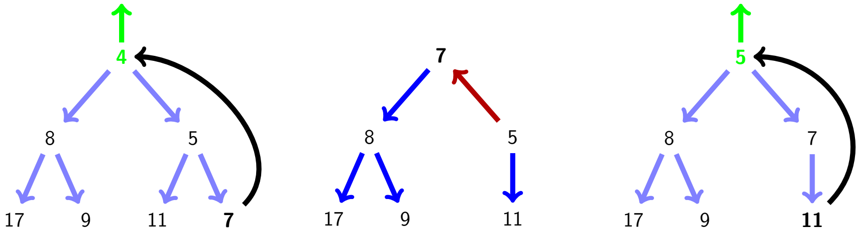

Figure: One iteration of Heapsort

Heapsort - Algorithm 6 / 10

Creating a heap:

- This operation is called heapify

Heapsort - Algorithm 6 / 10

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- The n elements are already stored in an array

Heapsort - Algorithm 6 / 10

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- Interpret the array as binary heap where the heap property is not yet satisfied

Heapsort - Algorithm 6 / 10

Creating a heap:

- This operation is called heapify
- The n elements are already stored in an array
- Interpret the array as binary heap where the heap property is not yet satisfied
- We repair the heap from bottom up (in layers) with sift

Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4

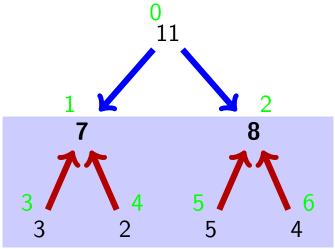

Figure: Heapify lower layer

Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4

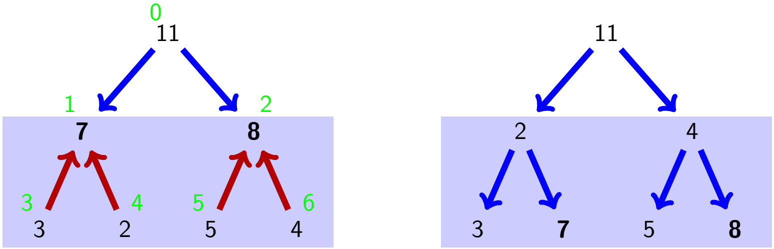

Figure: Heapify lower layer

Heapsort - Algorithm 8 / 10

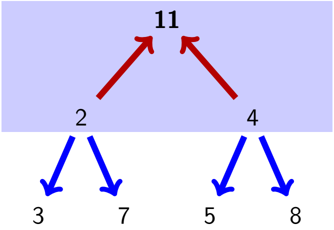

Figure: Heapify upper layer

Heapsort - Algorithm 8 / 10

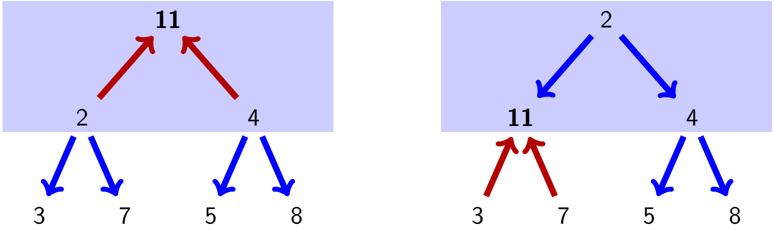

Figure: Heapify upper layer

Heapsort - Algorithm 9 / 10

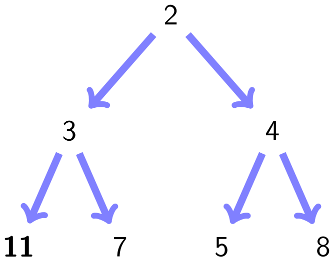

Figure: Resulting heap

Heapsort - Algorithm 10 / 10

Finding the minimum is intuitive:

- **Minsort:** Iterate through all non-sorted elements
- **Heapsort:** Finding the minimum is trivial (concept)

  Just take the root of the heap

Heapsort - Algorithm 10 / 10

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- **Minsort:** Iterate through all non-sorted elements
- **Heapsort:** Finding the minimum is trivial (concept)

  Just take the root of the heap

Removing the minimum in Heapsort:

- Repair the heap and restore the heap property
  - We don't have to repair the whole heap
- More of this in the next lecture

Further Literature

- **Course literature**

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.
Introduction to Algorithms.
MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.
Algorithms and Data Structures.
Springer, Berlin, 2008.
https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf.

Further Literature

- **Sorting**

[Wika] Wikipedia - Heapsort
https://en.wikipedia.org/wiki/Heapsort

[Wikb] Wikipedia - Selectionsort
https://de.wikipedia.org/wiki/Selectionsort

Further Literature

- **Subversion**

[Apa] Apache Subversion
https://subversion.apache.org/