

Algorithms and Datastructures

Runtime analysis Minsort / Heapsort, Induction

Prof. Dr. Rolf Backofen

Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, October 2018

Structure

Algorithms and Datastructures

- Structure

- Links

- Organisation

 - Daphne

 - Forum

 - Checkstyle

 - Unit Tests

 - Version management

 - Jenkins

Sorting

- Minsort

- Heapsort

Algorithms and Datastructures

Topics of this Lecture

Topics of the Lecture:

- ▶ Algorithms and Data Structures
- ▶ **Algorithm** $\hat{=}$ Solving of complex computational problems
- ▶ **Datastructure** $\hat{=}$ Representation of data on computer

Algorithms and Datastructures

Topics of this Lecture

Topics of the Lecture:

- ▶ Algorithms and Data Structures
Efficient data handling and processing
... for problems that occur in practical **any** larger program / project
- ▶ **Algorithm** $\hat{=}$ Solving of complex computational problems
- ▶ **Datastructure** $\hat{=}$ Representation of data on computer

Example 1: Sorting



Figure: Sorting with *Minsort*

Example 2: Navigation

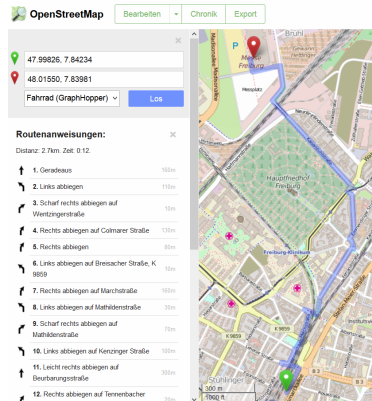


Figure: Navigationplan
© OpenStreetMap

Example 2: Navigation

- **Datastructures:** How to represent the map as data?

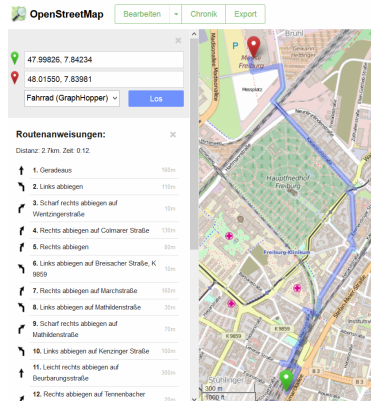


Figure: Navigationsplan
© OpenStreetMap

Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?

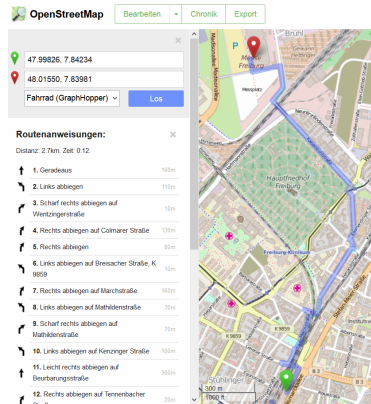


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Content of the Lecture 1 / 2

General:

- ▶ Most of you had a lecture on basic programming ...
performance was not an issue

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- ▶ Here it is going to be:
 1. How fast is our program?
 2. How can we make it faster?
 3. How can we proof that it will always be that fast?
- ▶ **Important** issues:
 - ▶ Most of the time: application runtime
 - ▶ Sometimes also: resource / space consumption

Content of the Lecture 2 / 2

Algorithms:

Content of the Lecture 2 / 2

Algorithms:

- ▶ Sorting
- ▶ Dynamic Arrays
- ▶ Associative Arrays
- ▶ Hashing
- ▶ Edit distance
- ▶ Priority Queue
- ▶ Linked Lists
- ▶ Pathfinding / Dijkstra Algorithm
- ▶ Search Trees

Content of the Lecture 2 / 2

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Mathematics:

Content of the Lecture 2 / 2

Algorithms:

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- ▶ Dynamic Arrays
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Mathematics:

- ▶ Runtime analysis
- ▶ Proof of correctness
- ▶ \mathcal{O} -Notation

After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](#)

After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](#)

- Hopefully your parents will still invite you

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Links



Homepage:

- ▶ Exercise sheets
- ▶ Lectures
- ▶ Materials

Link to [Homepage](#)

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Lecture:

- ▶ Tuesday, 12:00 - 14:00, HS 00 006, Build. 082
- ▶ Recordings of the lecture will be uploaded to the webpage

Exercises:

- ▶ One exercise sheet per week
- ▶ Submission / Correction / Assistance online
- ▶ Tutorial: (if needed)
Wednesday, 13:00-14:00 - HS 00 006, Build. 082

Exam:

- ▶ Planned: Sa. 23th March 2019, 10:00-12:00, Build. 101, Lec. theater 026 & 036

Organisation 2 / 5

Exercises:

- ▶ 80 % practical, 20 % theoretical

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Exam:

- ▶ 50 % of all points from the exercise sheets are needed
- ▶ Content of exam: whole lecture **and all exercises**

Organisation - Exercises 3 / 5

Exercises:

- ▶ Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller

Organisation - Exercises 3 / 5

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- ▶ Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller
- ▶ Coordinators: Michael Uhl, Florian Eggenhofer and Björn Grüning

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Exercises:

- ▶ Tutors: Tim Maffenbeier, Till Steinmann, Tobias Faller
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- ▶ Deadline: ESE: 1 week, IEMS: none

Organisation - Exercises 3 / 5

Exercises:

- ▶ Post questions into the forum (link later)

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Exercises:

- ▶ Post questions into the forum (link later)
- ▶ Submission via “commit” through svn and Daphne
- ▶ Feedback one week after deadline through “update” (svn)
- ▶ Unit test / checkstyle via Jenkins

Organisation - Exercises 4 / 5

Exercises - Points:

- ▶ Practical:
 - ▶ 60 % functionality
 - ▶ 20 % tests
 - ▶ 20 % documentation, Checkstyle, etc.
 - ▶ Program is not running \Rightarrow 0 points

Organisation - Exercises 4 / 5

Exercises - Points:

- ▶ Practical:
 - ▶ 60 % functionality
 - ▶ 20 % tests
 - ▶ 20 % documentation, Checkstyle, etc.
 - ▶ Program is not running \Rightarrow 0 points
- ▶ Theoretical (mathematical proof):
 - ▶ 40 % general idea / approach
 - ▶ 60 % clean / complete

Effort:

- ▶ 4 ECTS (ESE), 6 ECTS (IEMS)
- ▶ 120 / 180 working hours per semester
- ▶ 14 Lectures each 6 h / 8 h + exam
- ▶ 4 h / 6 h per exercise sheet (one per week)

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Daphne

Daphne:

- ▶ Provides the following information:
 - ▶ Name / contact information of your tutor
 - ▶ Download of / info needed for exercise sheets
 - ▶ Collected points of all exercise sheets
 - ▶ Links to:
 1. Coding standards
 2. Build system
 3. The other systems
- ▶ Link: [Daphne](#)

Forum:

- ▶ Please don't hesitate to ask if something is unclear
- ▶ Ask in the forum and not separate. Others might also be interested in the answer
- ▶ The [tutors](#) or the [coordinators](#) will reply as soon as possible
- ▶ Link: [Forum](#)

Checkstyle

flake8

Checkstyle / Linting (flake8):

- ▶ Installation: **python3** -m pip install flake8
- ▶ Check file: **python3** -m flake8 path/to/files/*.py
- ▶ Link: [flake8](#)

Unit Tests

Why unit tests?

1. A non-trivial method without a unit test is probably wrong

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- ▶ At least one **typical** input

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2. Simplifies debugging
3. We and you can automatically check correctness of code

What is a good unit test?

- ▶ Unit test checks desired output for a given input
 - ▶ At least one **typical** input
 - ▶ At least one **critical** case
- E.g. double occurrence of a value in sorting

Unit Tests

doctest

Testing (doctest):

```
def subtract_one(n):  
    """Subtracts 1 from n
```

- ▶ Tests are contained in docstrings

```
>>> subtract_one(5)  
4
```

```
>>> subtract_one(3)  
2  
"""
```

```
return n-1
```

```
if __name__ == "__main__":  
    print("2 - 1 = %d" % subtract_one(2))
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- ▶ Tests are contained in docstrings
- ▶ Module doctest runs them
- ▶ Run check with:
python3 -m doctest
path/to/files/.py -v*

Version management

Subversion

Version management (subversion):

- ▶ Keeps a history of code changes
- ▶ Initialize / update directory: **svn** checkout <URL>
- ▶ Add files / folders: **svn** add <file> --all
- ▶ Create snapshot: **svn** commit -m "<Your Message>"
Data is uploaded to Jenkins automatically
- ▶ Link: [Subversion](#)

Jenkins

Jenkins:

- ▶ Provides our build system
- ▶ You can check if your uploaded code runs
 - ▶ Especially whether all **unit test** pass
 - ▶ And if **checkstyle** (flake8) is statisfied
- ▶ Will be shown in the first exercise
- ▶ Link: [Jenkins](#)

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Sorting

- Minsort

- Heapsort

Sorting 1 / 2

Problem:

- ▶ Input: n elements x_1, \dots, x_n
- ▶ Transitive operator “**i**” which returns **true** if the left value is smaller than the right one
 - ▶ Transitivity: $x < y, y < z \rightarrow x < z$
- ▶ Output: x_1, \dots, x_n sorted with operator

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Example

Input: 14, 4, 32, 19, 8, 44, 65

Output:

Why do we need sorting?

- ▶ Nearly **every** program needs a sorting algorithm
- ▶ **Examples:**
 - ▶ Index of a search engine
 - ▶ Listing filesystem in explorer / finder
 - ▶ (Music) library
 - ▶ Highscore list

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Minsort - Algorithm

Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...

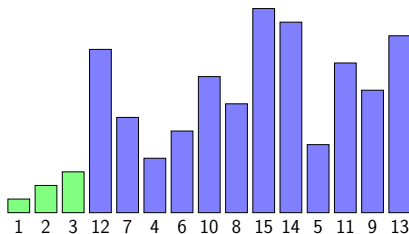


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Figure: *Minsort*

Minsort - Algorithm

Minsort in Python:

```
def minsort(lst):  
    for i in range(0, len(lst)-1):  
        minimum = i  
  
        for j in range(i+1, len(lst)):  
            if lst[j] < lst[minimum]:  
                minimum = j  
  
        if minimum != i:  
            lst[i], lst[minimum] = \  
                lst[minimum], lst[i]  
  
    return lst
```

MinSort - Runtime

How long does our program run?

- ▶ We test it for different input sizes

MinSort - Runtime

How long does our program run?

Table: Runtime for *Minsort*

- We test it for different input sizes

n	Runtime / ms
2×10^3	5.24
4×10^3	16.92
6×10^3	39.11
8×10^3	67.80
10×10^3	105.50
12×10^3	150.38
14×10^3	204.00
16×10^3	265.98
18×10^3	334.94

MinSort - Runtime

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Table: Runtime for *Minsort*

- ▶ We test it for different input sizes
- ▶ **Observation:**
It is going to be “disproportionately” slower the more numbers are being sorted

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Runtime analysis:

- ▶ *Minsort* runtime depicted in a diagram
 - ▶ That is what you should do in the first exercise sheet



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Runtime analysis:

- ▶ *Minsort* runtime depicted in a diagram
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- ▶ **We observe:**
 - ▶ The runtime **grows faster than linear**
 - ▶ With double the input size we need four times the time

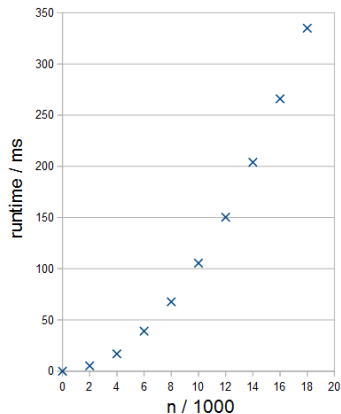


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Runtime analysis:

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- ▶ **We observe:**
 - ▶ The runtime **grows faster than linear**
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Figure: Runtime of *Minsort*

- ▶ Next lecture we will analyze deeper with other methods

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Heapsort - Algorithm 1 / 10

Heapsort:

- ▶ The principle stays the same
- ▶ Better structure for finding the smallest element quicker

Heapsort - Algorithm 1 / 10

Heapsort:

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Binary heap:

- ▶ Preferably a complete binary tree
- ▶ **Heap property:** Each child is **smaller** (larger) than the parent element

Heapsort - Algorithm 2 / 10

Min heap:

- **Heap property:** Each child is **smaller** (larger) than the parent element



Figure: Valid min heap



Figure: Invalid min heap

Heapsort - Algorithm 2 / 10

Min heap:

- ▶ **Heap property:** Each child is **smaller** (larger) than the parent element
- ▶ A valid heap fulfills the property at each node



Figure: Valid min heap



Figure: Invalid min heap

Heapsort - Algorithm 3 / 10

How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
 - ▶ The children of node i are $2i + 1$ and $2i + 2$
 - ▶ The parent node of node i is $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array



Figure: Min heap

Heapsort - Algorithm 3 / 10

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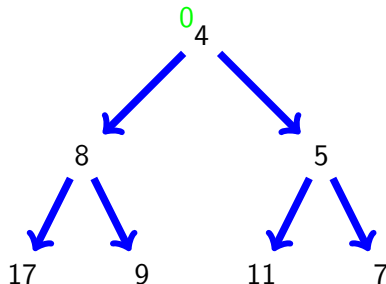


Table: Elements can be stored in array

0						
4						

Figure: Min heap

Heapsort - Algorithm 3 / 10

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Table: Elements can be stored in array

0	1					
4	8					

Figure: Min heap

Heapsort - Algorithm 3 / 10

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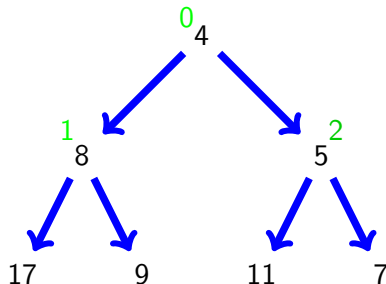


Table: Elements can be stored in array

0	1	2				
4	8	5				

Figure: Min heap

Heapsort - Algorithm 3 / 10

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Table: Elements can be stored in array

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Table: Elements can be stored in array

0	1	2	3	4	5	6
4	8	5	17	9	11	7

Figure: Min heap

Heapsort - Algorithm 4 / 10

Repairing after taking the smallest element: `heap.pop()`

Heapsort - Algorithm 4 / 10

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- ▶ Remove the smallest element (root node)

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- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node

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- ▶ **Sift** the new root node down until the **heap property** is satisfied

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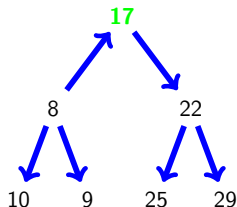


Figure: Repairing a min heap

Heapsort - Algorithm 4 / 10

Repairing after taking the smallest element: `heap.pop()`

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- ▶ Replace the root with the last node
- ▶ **Sift** the new root node down until the **heap property** is satisfied

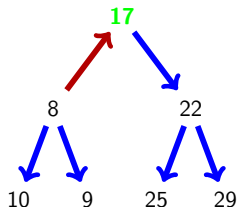


Figure: Repairing a min heap

Heapsort - Algorithm 4 / 10

Repairing after taking the smallest element: `heap.pop()`

- ▶ Remove the smallest element (root node)
- ▶ Replace the root with the last node
- ▶ **Sift** the new root node down until the **heap property** is satisfied

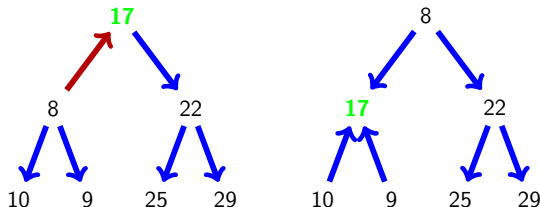


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Heapsort - Algorithm 4 / 10

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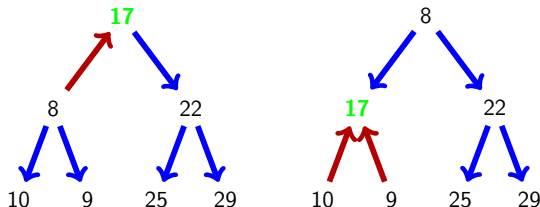


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Heapsort - Algorithm 4 / 10

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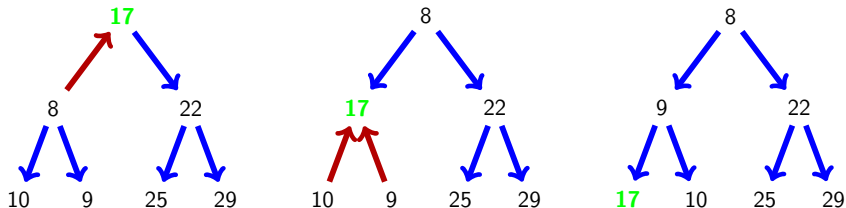


Figure: Repairing a min heap

HeapSort - Algorithm 5 / 10

Heapsort:

- ▶ Organize the n elements as heap
- ▶ While the heap still contains elements
 - ▶ Take the smallest element
 - ▶ Move the last node to the root
 - ▶ Repair the heap as described

HeapSort - Algorithm 5 / 10

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 - ▶ Repair the heap as described
- ▶ Output: 4

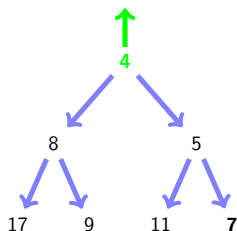


Figure: One iteration of Heapsort

HeapSort - Algorithm 5 / 10

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 - ▶ Move the last node to the root
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- ▶ Output: 4

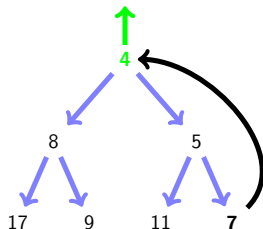


Figure: One iteration of Heapsort

HeapSort - Algorithm 5 / 10

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HeapSort - Algorithm 5 / 10

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 - ▶ Move the last node to the root
 - ▶ Repair the heap as described
- ▶ Output: 4

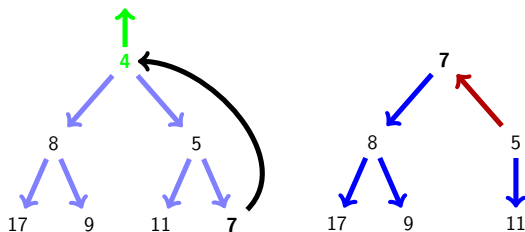


Figure: One iteration of Heapsort

HeapSort - Algorithm 5 / 10

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HeapSort - Algorithm 5 / 10

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- ▶ While the heap still contains elements
 - ▶ Take the smallest element
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 - ▶ Repair the heap as described
- ▶ Output: 4, 5, ...

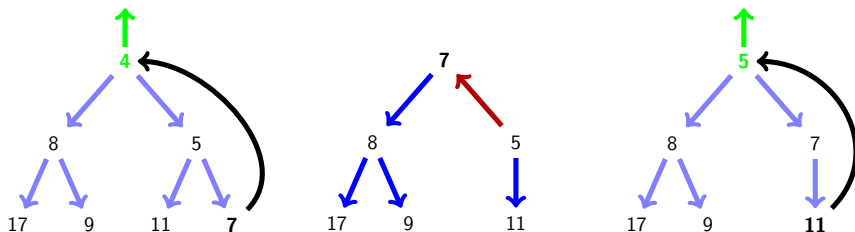


Figure: One iteration of Heapsort

Heapsort - Algorithm 6 / 10

Creating a heap:

- ▶ This operation is called **heapify**

Heapsort - Algorithm 6 / 10

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Heapsort - Algorithm 6 / 10

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Heapsort - Algorithm 6 / 10

Creating a heap:

- ▶ This operation is called **heapify**
- ▶ The n elements are already stored in an array
- ▶ Interpret the array as binary heap where the **heap property** is not yet satisfied
- ▶ We repair the heap from bottom up (in layers) with **sift**

Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4

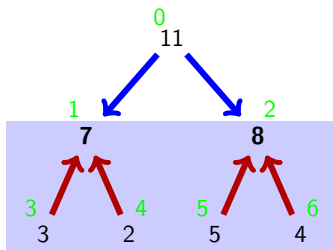


Figure: Heapify lower layer

Heapsort - Algorithm 7 / 10

Table: Input in array

0	1	2	3	4	5	6
11	7	8	3	2	5	4



Figure: Heapify lower layer

Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

Heapsort - Algorithm 9 / 10



Figure: Resulting heap

Heapsort - Algorithm 10 / 10

Finding the minimum is intuitive:

- ▶ **Minsort:** Iterate through all non-sorted elements
- ▶ **Heapsort:** Finding the minimum is trivial (concept)
Just take the root of the heap

Heapsort - Algorithm 10 / 10

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- ▶ **Minsort:** Iterate through all non-sorted elements
- ▶ **Heapsort:** Finding the minimum is trivial (concept)

Just take the root of the heap

Removing the minimum in Heapsort:

- ▶ Repair the heap and restore the **heap property**
 - ▶ We don't have to repair the whole heap
- ▶ More of this in the next lecture

Further Literature

► Course literature

- [CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.

Introduction to Algorithms.

MIT Press, Cambridge, Mass, 2001.

- [MS08] Kurt Mehlhorn and Peter Sanders.

Algorithms and Data Structures.

Springer, Berlin, 2008.

<https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf>.

Further Literature

► **Sorting**

[Wika] [Wikipedia - Heapsort](https://en.wikipedia.org/wiki/Heapsort)

`https://en.wikipedia.org/wiki/Heapsort`

[Wikb] [Wikipedia - Selectionsort](https://de.wikipedia.org/wiki/Selectionsort)

`https://de.wikipedia.org/wiki/Selectionsort`

Further Literature

- ▶ **Subversion**

[Apa] [Apache Subversion](https://subversion.apache.org/)

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