Algorithms and Datastructures

Graphs, Depth-/Breadth-first Search, Graph-Connectivity

Prof. Dr. Rolf Backofen

Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, January 2017

Structure

Graphs - Overview:

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- Breadth first search (BFS)
- Depth first search (DFS)
- Connected components of a graph

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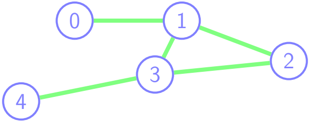

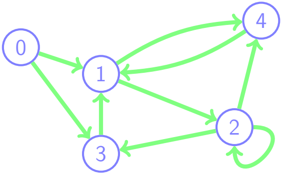

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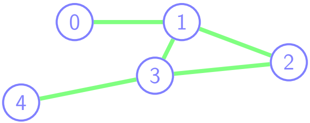

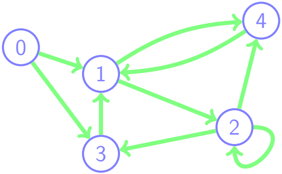

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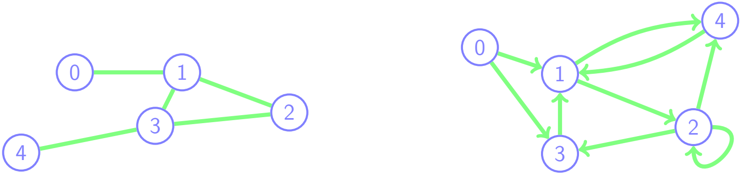

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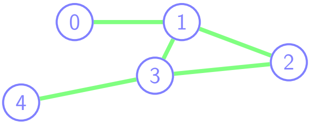

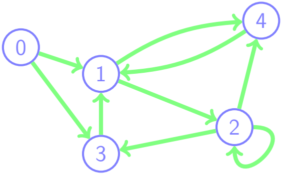

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Graphs

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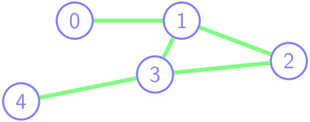

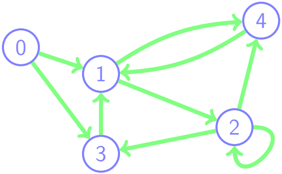

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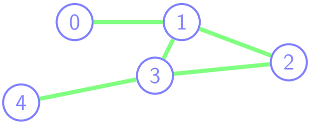

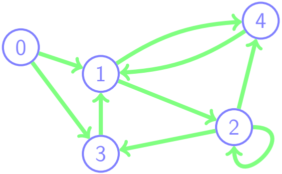

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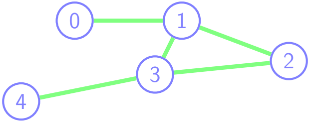

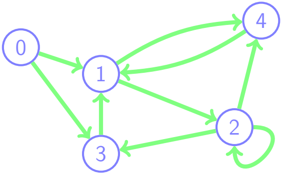

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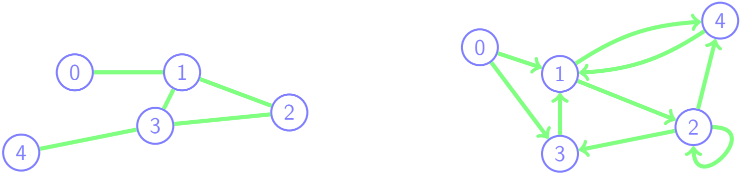

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  - Undirected edge: $e = \{u, v\}$ (set)
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- Self-loops are also possible: $e = (u, u)$ or $e = \{u, u\}$

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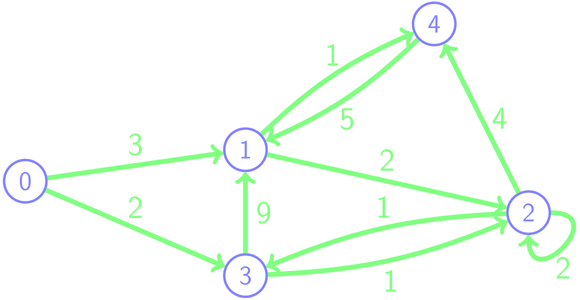

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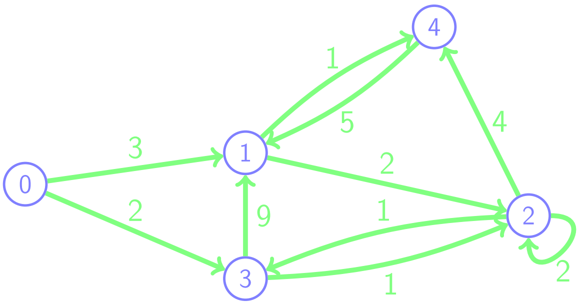

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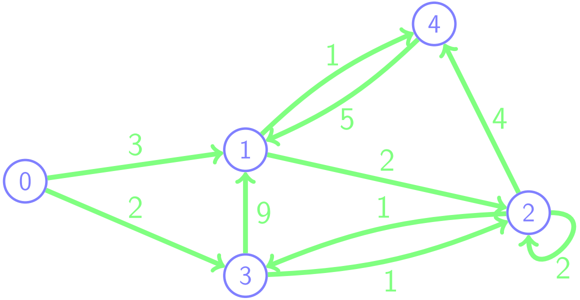

- Each edge is marked with a real number named weight
- The weight is also named length or cost of the edge depending on the application

Example: Road network

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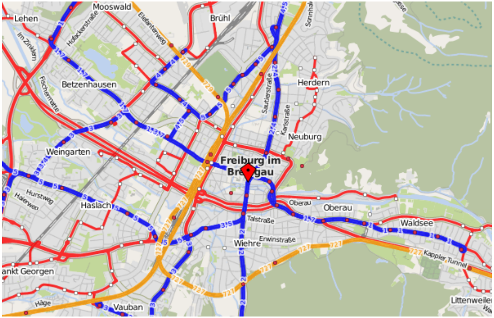

Figure: Map of Freiburg © OpenStreetMap

Structure

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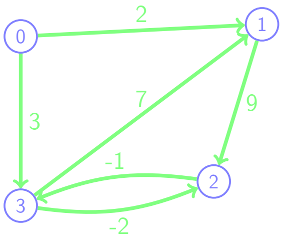

Figure: Weighted graph with $|V| = 4$, $|E| = 6$

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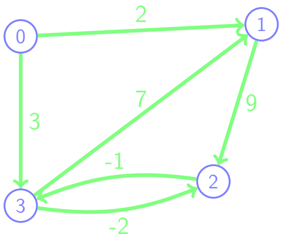

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end-vertice

start-vertice	0	1	2	3
0		2		3
1			9	
2				-1
3		7	-2	

Figure: Adjacency matrix

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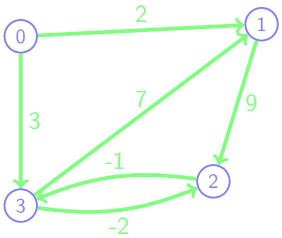

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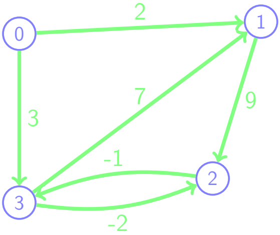

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start-vertice		
0	1, 2	3, 3
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Figure: Adjacency list

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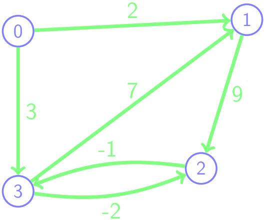

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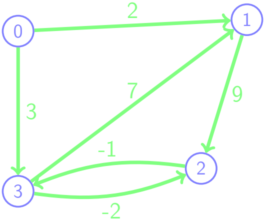

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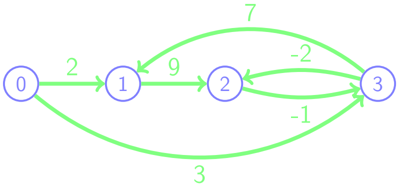

Figure: Same graph ordered by number - outer planar graph

Graphs

Implementation - Python

```python
class Graph:
    def __init__(self):
        self.vertices = []
        self.edges = []

    def addVertice(self, vert):
        self.vertices.append(vert)

    def addEdge(self, fromVert, toVert, cost):
        self.edges.append( \
            (fromVert, toVert, cost))

    ...
```

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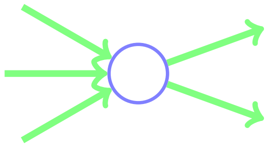

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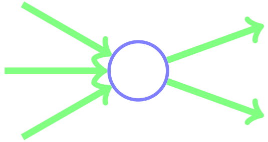

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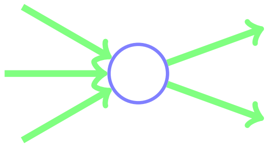

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- Outdegree of a vertex u is the number of edge tail ends adjacent to the vertex

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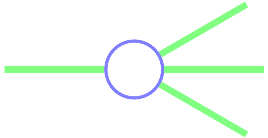

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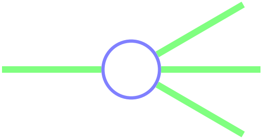

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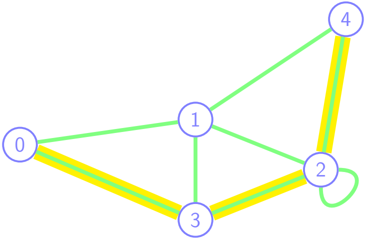
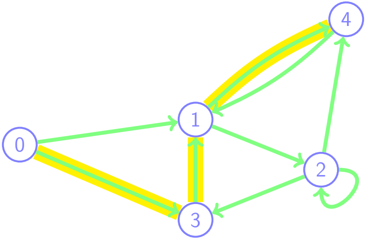


Figure: Undirected path of length 3
$P = (0, 3, 2, 4)$



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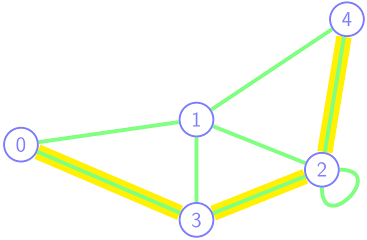

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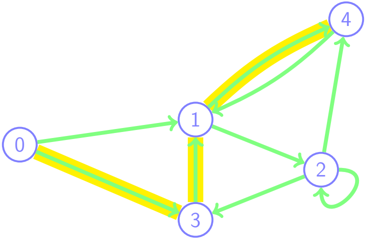

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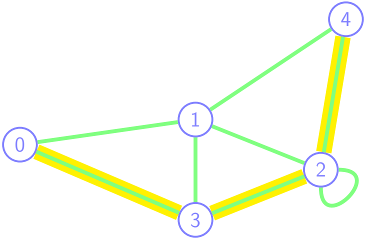

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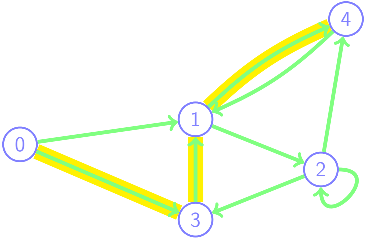

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  - Directed graph: $(u_1, u_2), (u_2, u_3), \ldots, (u_{i-1}, u_i) \in E$

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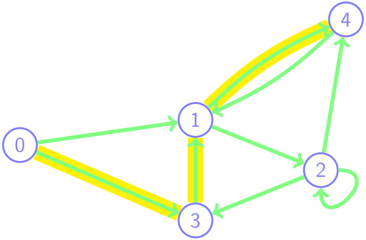

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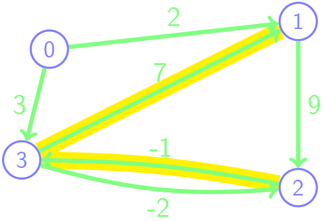

Figure: Weighted path with cost 6
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Graphs
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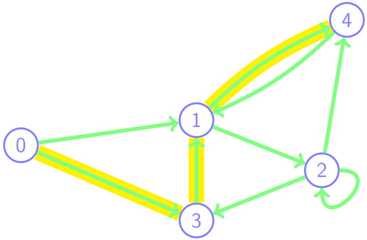

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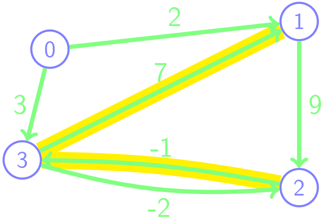

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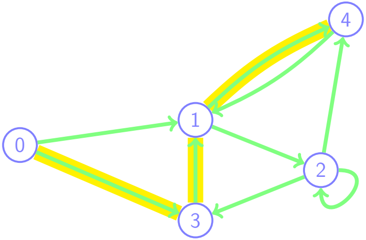

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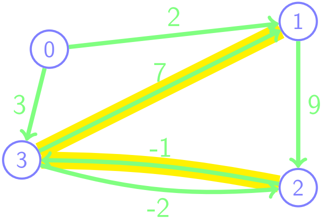

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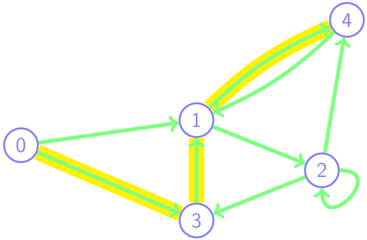

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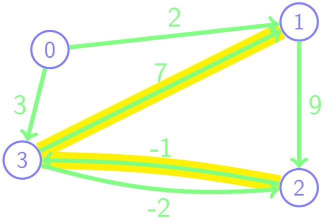

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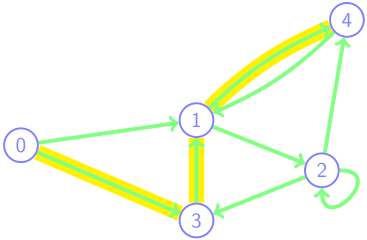

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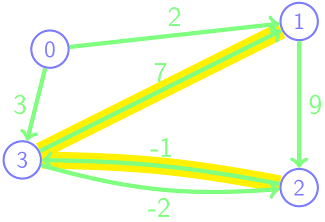

Figure: Weighted path with cost 6
$P = (2, 3, 1)$

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  - Without weights: number of edges taken

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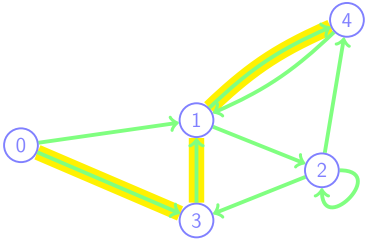

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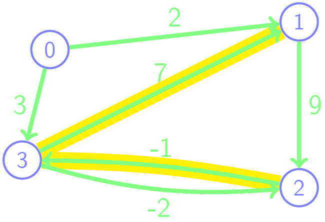

Figure: Weighted path with cost 6
$P = (2, 3, 1)$

- The length of a path is: (also costs of a path)
  - Without weights: number of edges taken
  - With weights: sum of weigths of edges taken

Graphs

Paths

Shortest path in a graph: $G = (V, E)$

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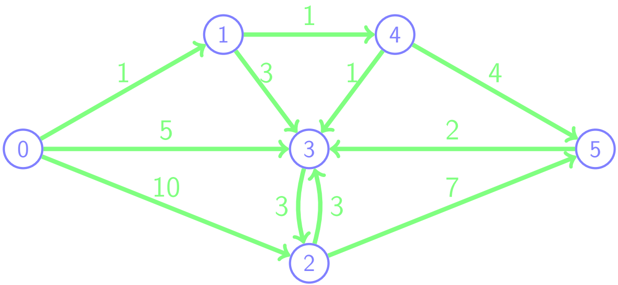

Figure: Shortest path from 0 to 2 with cost / distance $d(0, 2) = ?$

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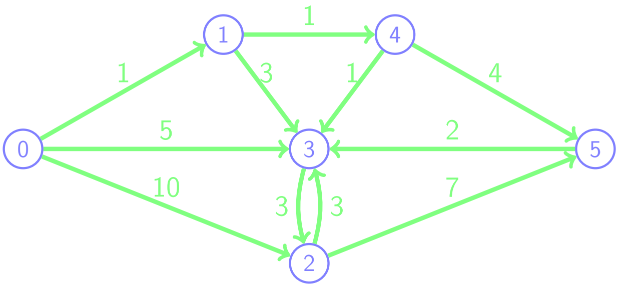

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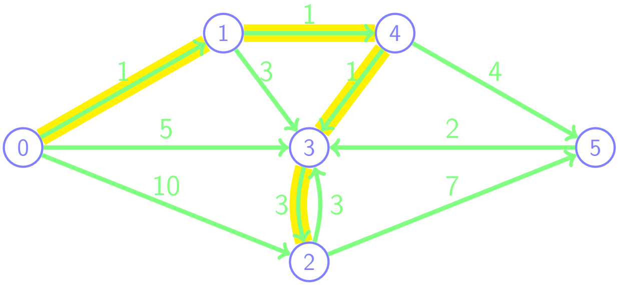

Figure: Shortest path from 0 to 2 with cost / distance $d(0, 2) = 6$
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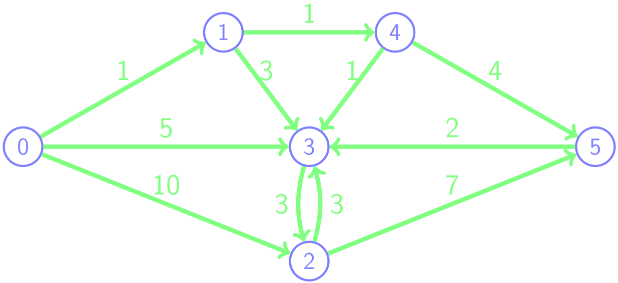

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Graphs

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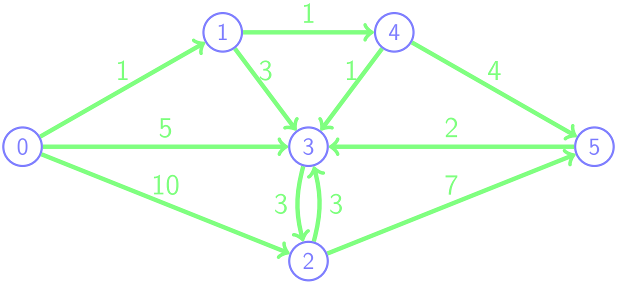

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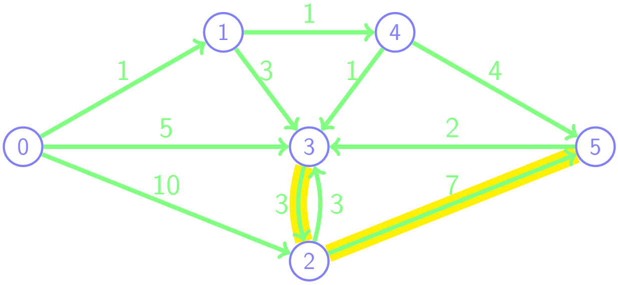

Figure: Diameter of graph is $d = 10,\ P = (3, 2, 5)$

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Connected components: $G = (V, E)$

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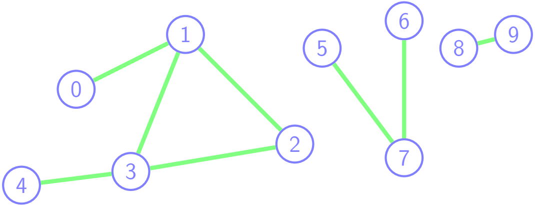
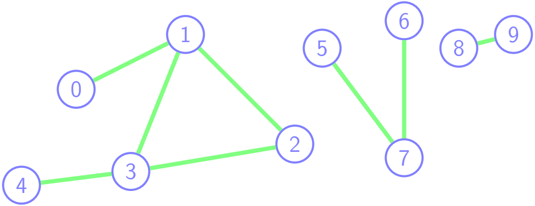


Figure: Three connected components

- Undirected graph:

Connected components: $G = (V, E)$



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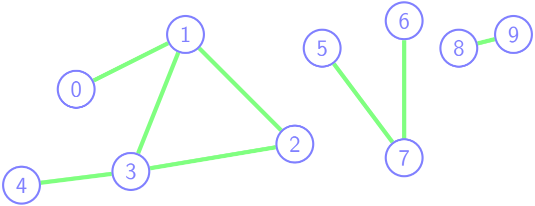

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6. All connected nodes are now marked and in the same connected component as the start vertex s

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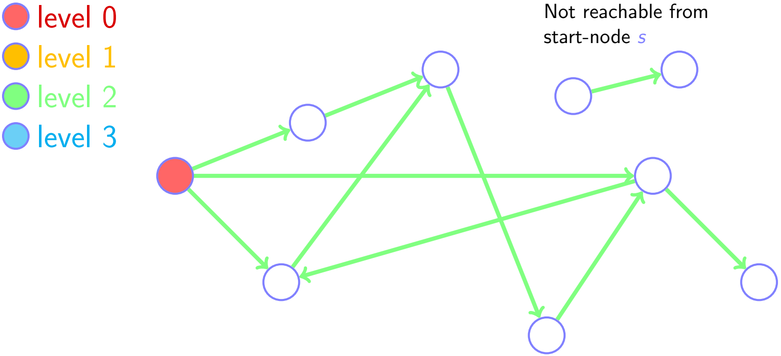

Figure: spanning tree of a breadth-first search

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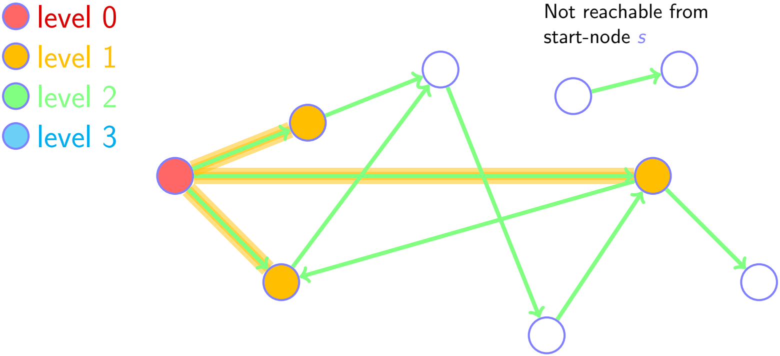

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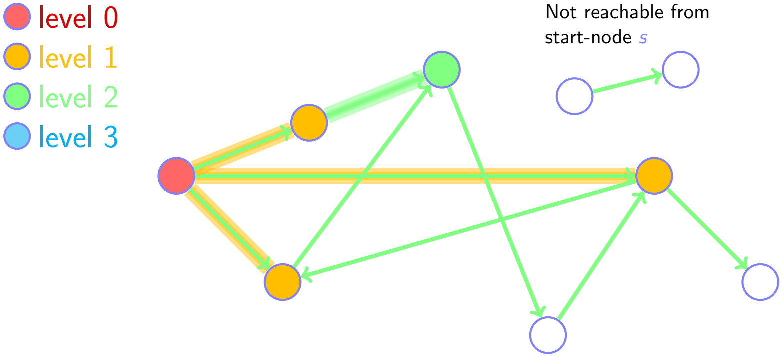

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Graphs

Connected Components - Breadth-First Search

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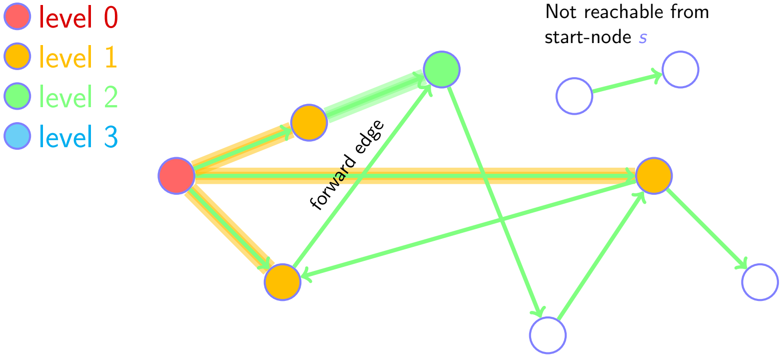

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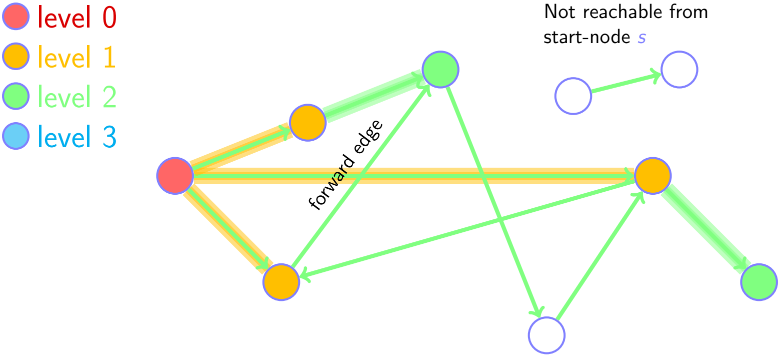

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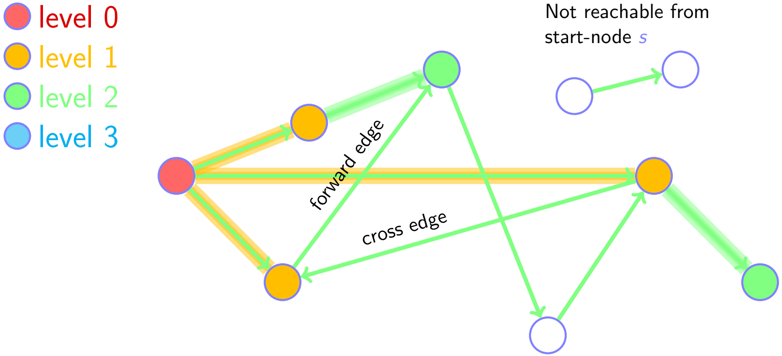

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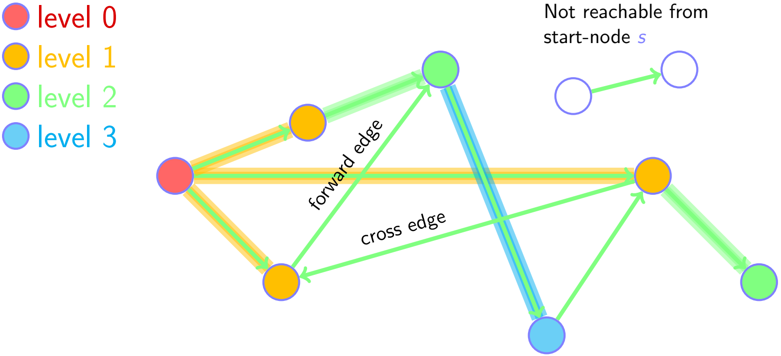

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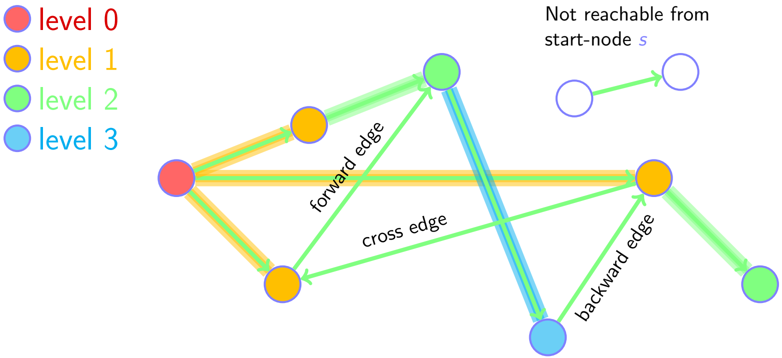

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Graphs

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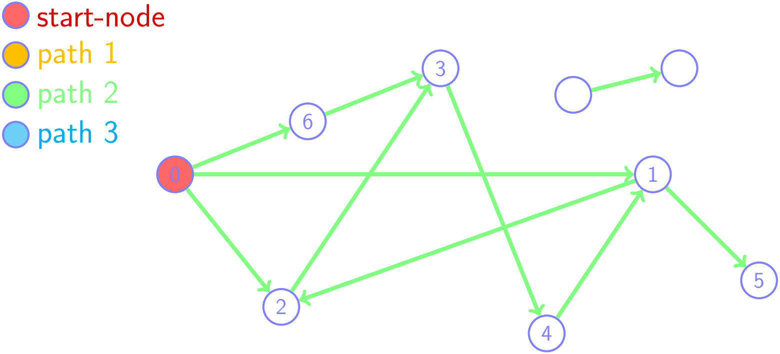

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Graphs

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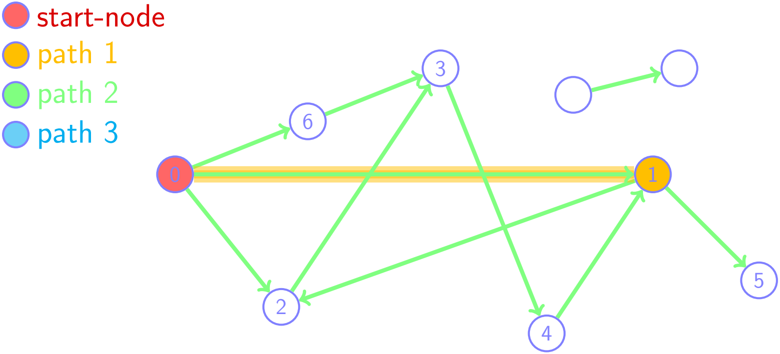

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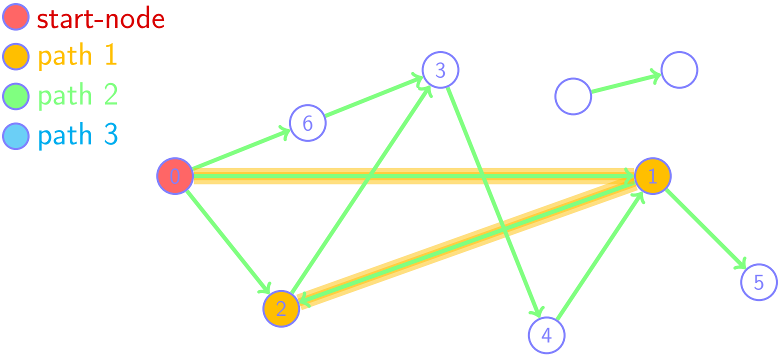

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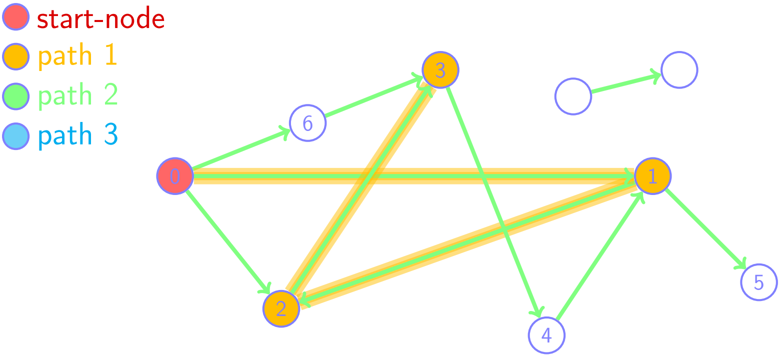

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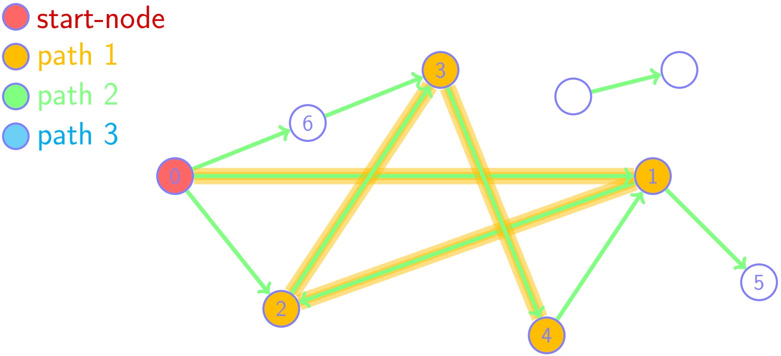

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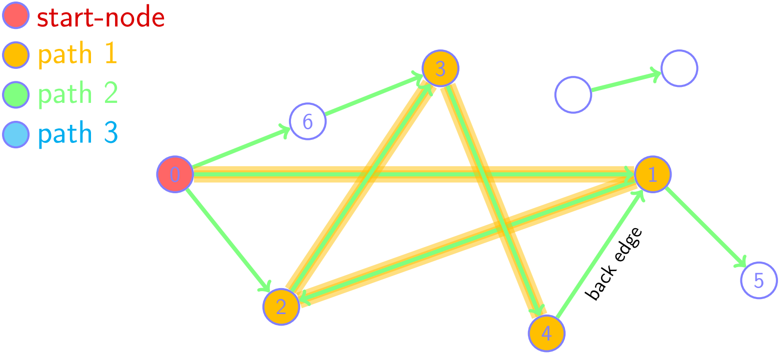

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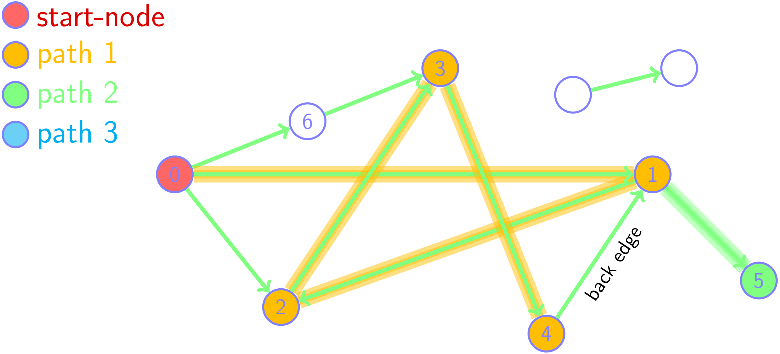

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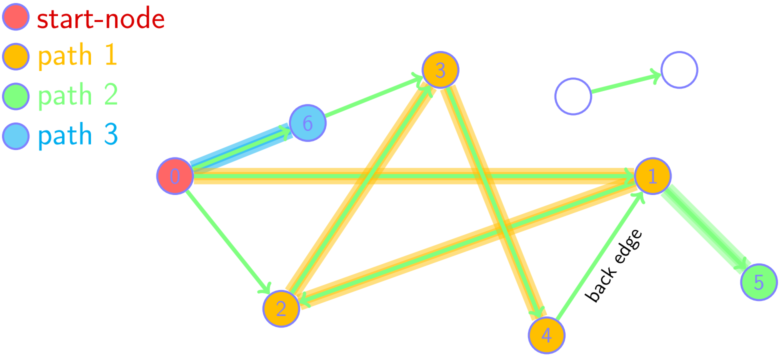

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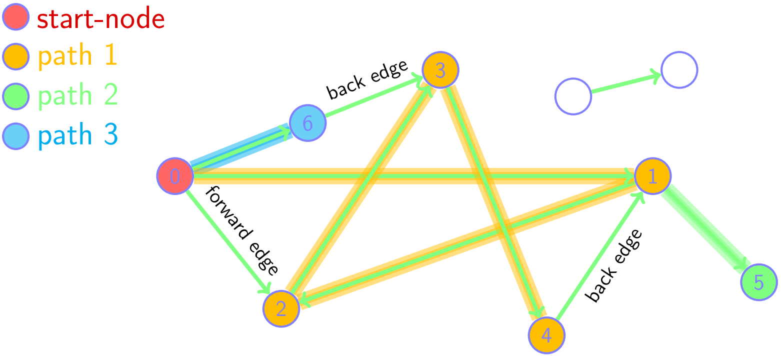

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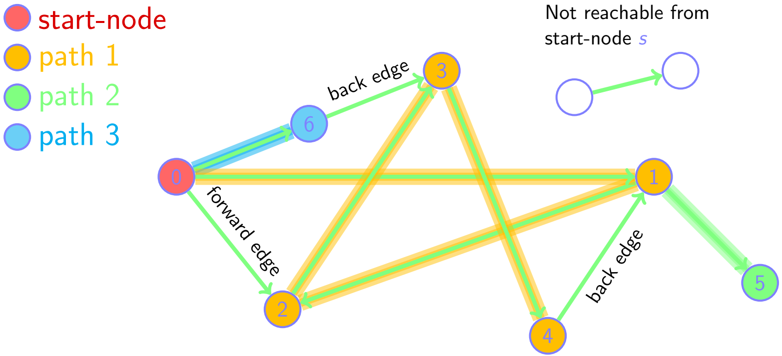

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Why is this called Breadth - and Depth First Search?

Runtime complexity:

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- This can only be improved by a constant factor

Structure

Application example

Image processing

Application example

Image processing

- Connected component labeling

Application example

Image processing

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- Counting of objects in an image

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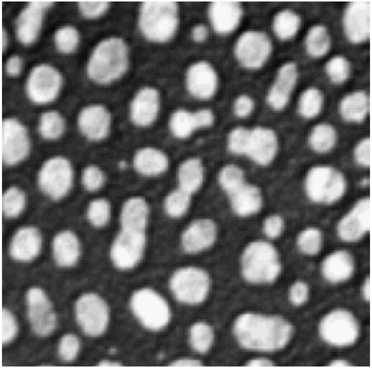
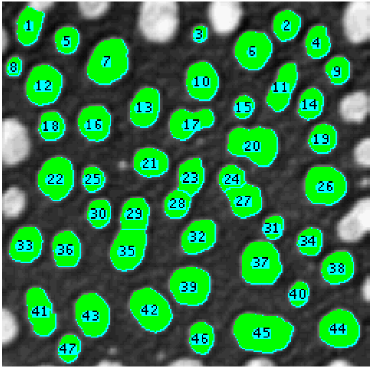


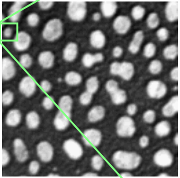
$\Rightarrow$



Application example

Image processing

What's object, what's background?



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
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36	80	64	48	32	16	16	16	24	32	40	40	40	40	40	40	40	32	32	24	24	24
37	56	48	32	24	8	16	16	32	40	48	48	48	40	40	40	40	32	32	24	24	24
38	40	32	24	24	16	32	48	64	72	80	80	72	56	56	48	48	40	40	32	32	32
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40	16	16	24	40	56	88	120	128	136	144	144	120	96	88	72	56	48	48	40	40	40
41	8	16	24	56	80	120	160	168	168	168	168	144	120	104	80	64	48	48	40	40	32
42	16	32	40	80	112	144	176	176	176	176	168	152	128	112	88	64	48	40	32	32	24
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48	48	72	96	128	168	184	200	200	200	192	184	160	128	96	64	48	24	32	32	56	72
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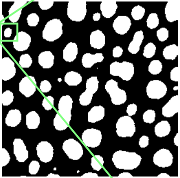
53	40	48	48	56	64	72	72	80	80	80	72	64	48	40	24	32	32	72	104	144	184
54	48	48	48	48	48	56	56	56	64	56	56	48	40	32	24	40	48	88	128	160	200

Application example

Image processing

Convert to black white using threshold:

`value = 255 if value > 100 else 0`



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
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36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
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46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Application example

Image processing

Interpret image as graph:

Application example

Image processing

Interpret image as graph:

- Each white pixel is a node

Application example

Image processing

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)

Application example

Image processing

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)
- Edges are not saved externally, algorithm works directly on array

Application example

Image processing

Interpret image as graph:

- Each white pixel is a node
- Edges between adjacent pixels (normally 4 or 8 neighbors)
- Edges are not saved externally, algorithm works directly on array
- Breadth- / depth-first search find all connected components (particles)

Application example

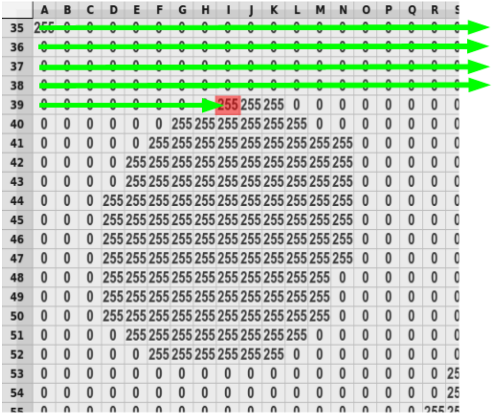
Image processing

Find connected components:

Application example

Image processing

Find connected components:

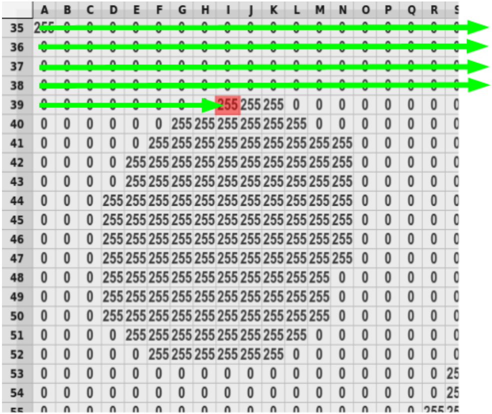


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

Application example

Image processing

Find connected components:



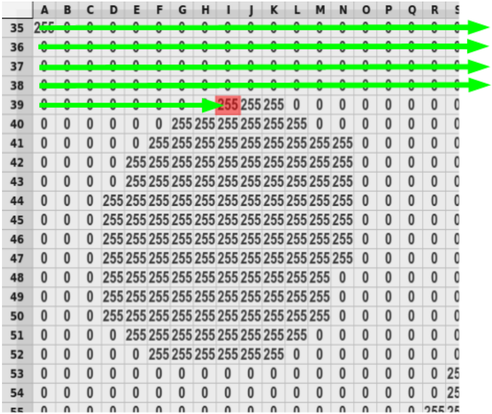
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25

- Search pixel-by-pixel for non-zero intensity

Application example

Image processing

Find connected components:



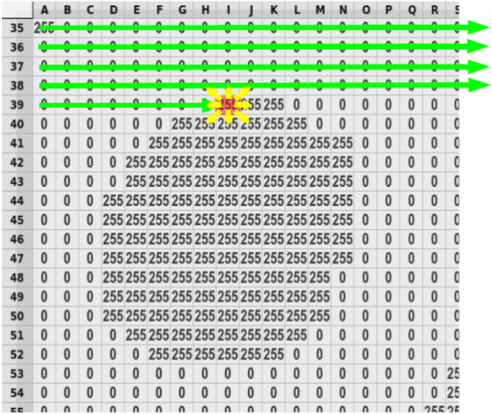
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1

Application example

Image processing

Find connected components:



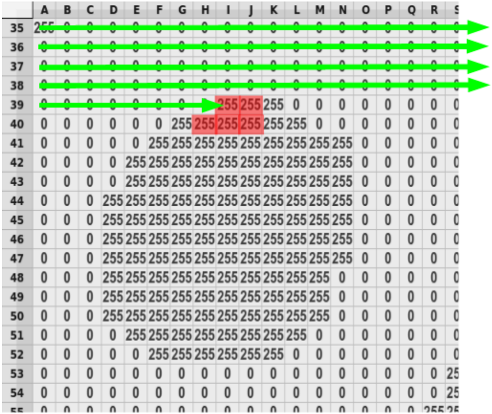
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels

Application example

Image processing

Find connected components:



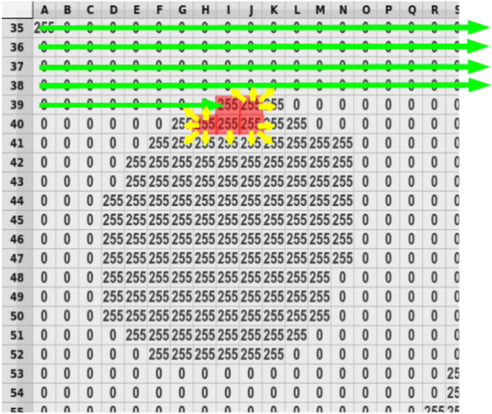
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



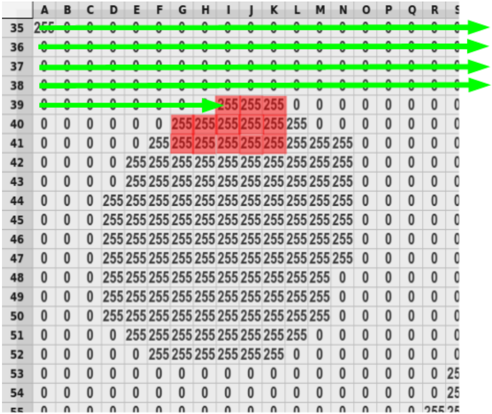
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



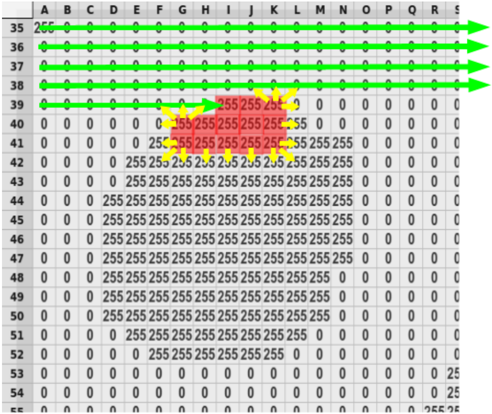
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



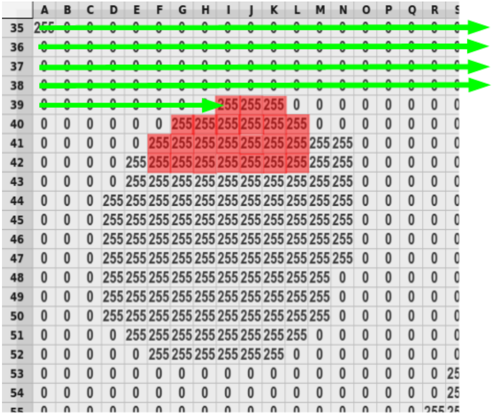
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	[illegible]	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



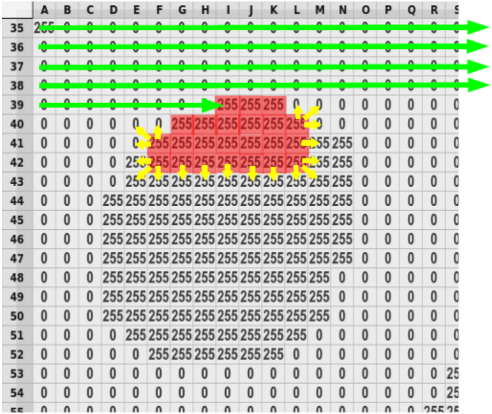
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

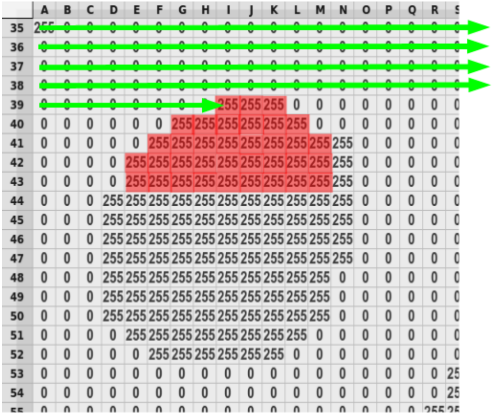


- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



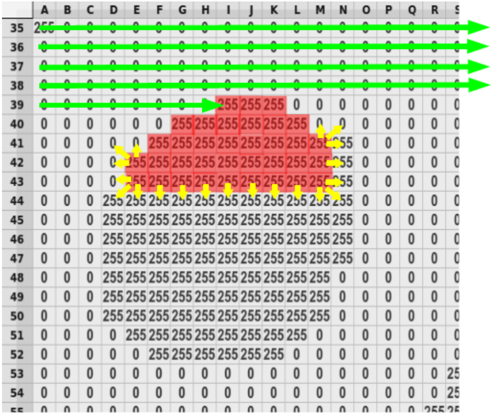
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



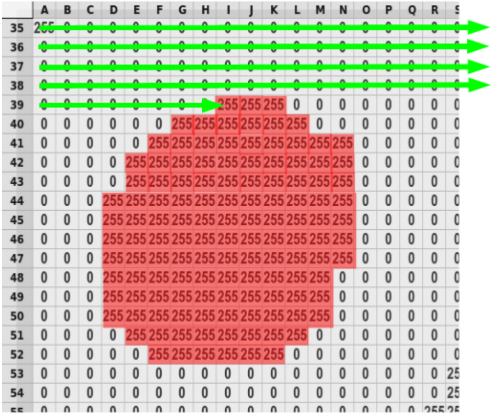
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	25

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 1
- Check neighbors of all new labeled pixels
- Label non-zero pixels as component 1

Application example

Image processing

Find connected components:

- Search pixel-by-pixel for non-zero intensity
- Label found pixel as component 2
- . . .

Application example

Image processing

Result of connected component labeling:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
35	255	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
44	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
45	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	0
46	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	0	25
47	0	0	0	255	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	25
48	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	25
49	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	25
50	0	0	0	255	255	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	255	255	25
51	0	0	0	0	255	255	255	255	255	255	255	255	0	0	0	0	0	0	0	255	255	255	255	25
52	0	0	0	0	0	255	255	255	255	255	255	0	0	0	0	0	0	0	0	255	255	255	255	25
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	25
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	255	25
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	255	255	255	255	255	255	25

$\Rightarrow$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
35	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	13	13	13	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
42	0	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
43	0	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
44	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
45	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
46	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	0
47	0	0	0	13	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17
48	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17
49	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17
50	0	0	0	13	13	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17	17
51	0	0	0	0	13	13	13	13	13	13	13	13	0	0	0	0	0	0	0	17	17	17	17
52	0	0	0	0	0	13	13	13	13	13	13	0	0	0	0	0	0	0	0	17	17	17	17
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17	17
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17	17	17	17	17	17

Figure: Result: particle indices instead of intensities

Further Literature

- **General**

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.
Introduction to Algorithms.
MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.
Algorithms and data structures, 2008.
https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf.

Further Literature

- **Graph-Search**

[Wika] Breadth-first search
https://en.wikipedia.org/wiki/Breadth-first_search

[Wikb] Depth-first search
https://en.wikipedia.org/wiki/Depth-first_search

- **Graph-Connectivity**

[Wik] Connectivity (graph theory)
https://en.wikipedia.org/wiki/Connectivity_(graph_theory)