

# Algorithms and Datastructures

Runtime analysis Minsort / Heapsort, Induction

Prof. Dr. Rolf Backofen

Bioinformatics Group / Department of Computer Science

Algorithms and Datastructures, October 2017

# Structure

## Algorithms and Datastructures

- Structure

- Links

- Organisation

  - Daphne

  - Forum

  - Checkstyle

  - Unit Tests

  - Version management

  - Jenkins

## Sorting

- Minsort

- Heapsort

# Algorithms and Datastructures

## Topics of this Lecture

### Topics of the Lecture:

- ▶ Algorithms and Data Structures
- ▶ **Algorithm**  $\hat{=}$  Solving of complex computational problems
- ▶ **Datastructure**  $\hat{=}$  Representation of data on computer

# Algorithms and Datastructures

## Topics of this Lecture

### Topics of the Lecture:

- ▶ Algorithms and Data Structures  
Efficient data handling and processing  
... for problems that occur in practical **any** larger program / project
- ▶ **Algorithm**  $\hat{=}$  Solving of complex computational problems
- ▶ **Datastructure**  $\hat{=}$  Representation of data on computer

## Example 1: Sorting



Figure: Sorting with *Minsort*

## Example 2: Navigation

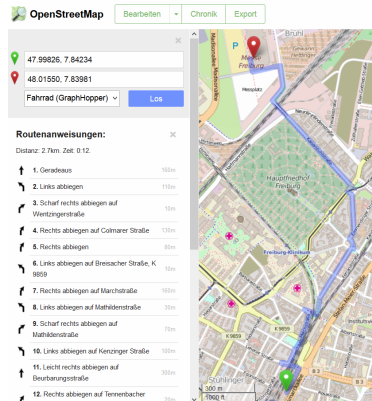


Figure: Navigationplan  
© OpenStreetMap

## Example 2: Navigation

- **Datastructures:** How to represent the map as data?

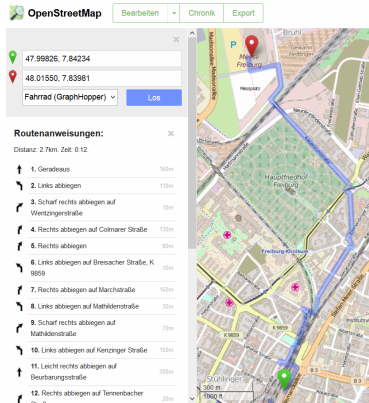


Figure: Navigationplan  
© OpenStreetMap

## Example 2: Navigation

- **Datastructures:** How to represent the map as data?
- **Algorithms:** How to find the shortest / fastest way?



Figure: Navigationplan  
© OpenStreetMap



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# Content of the Lecture 1 / 2

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performance was not an issue

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performance was not an issue
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  2. How can I make it faster?
  3. How can I proof that it will always be that fast?
- ▶ **Important** issues:
  - ▶ Most of the time: application runtime
  - ▶ Sometimes also: resource / space consumption

## Content of the Lecture 2 / 2

**Algorithms:**



# Content of the Lecture 2 / 2

## **Algorithms:**

- ▶ Sorting
- ▶ Dynamic Arrays
- ▶ Associative Arrays
- ▶ Hashing
- ▶ Priority Queue
- ▶ Linked Lists
- ▶ Pathfinding / Dijkstra Algorithm
- ▶ Search-Trees

# Content of the Lecture 2 / 2

## **Algorithms:**

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- ▶ Dynamic Arrays
- ▶ Associative Arrays
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- ▶ Priority Queue
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## **Mathematics:**

# Content of the Lecture 2 / 2

## Algorithms:

- ▶ Sorting
- ▶ Dynamic Arrays
- ▶ Associative Arrays
- ▶ Hashing
- ▶ Priority Queue
- ▶ Linked Lists
- ▶ Pathfinding / Dijkstra Algorithm
- ▶ Search-Trees

## Mathematics:

- ▶ Runtime analysis
- ▶ Proof of correctness
- ▶  $\mathcal{O}$ -Notation

## After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](#)

## After the lecture ...

- ... you should be able to understand the joke



Figure: Comic © [xkcd/835](#)

- Hopefully your parents will still invite you

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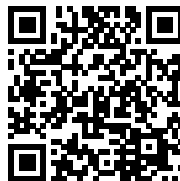
Jenkins

## Sorting

Minsort

Heapsort

# Links



## Homepage:

- ▶ Exercise sheets
- ▶ Lectures
- ▶ Materials

Link to [Homepage](#)

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# Organisation 1 / 5

## Lecture:

- ▶ Tuesday, 12:00 - 14:00, SR 00 010/014, Build. 101
- ▶ Recordings of the lecture will be uploaded to the webpage

## Exercises:

- ▶ One exercise sheet per week
- ▶ Submission / Correction / Assistance online
- ▶ Tutorial: (if needed)  
Wednesday, 12:00-13:00 - SR 00 010/014, Build. 101

## Exam:

- ▶ Planned: Sa. 24th March 2018, 10:00-12:00, Build. 101, Lec. theater 026 & 036

# Organisation 2 / 5

## **Exercises:**

- ▶ 80 % practical, 20 % theoretical

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- ▶ 50 % of all points from the exercise sheets are needed
- ▶ Content of exam: Whole lecture **and all exercises**

# Organisation - Exercises 3 / 5

## Exercises:

- ▶ Tutors: Tim Maffenbeier, Abderrahmen Rakez, Tobias Faller

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- ▶ Tutors: Tim Maffenbeier, Abderrahmen Rakez, Tobias Faller
- ▶ Coordinators: Michael Uhl, Stefan Mautner, Florian Eggenhofer and Björn Grüning
- ▶ Deadline: ESE: 1 week, IEMS: none

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## **Exercises:**

- ▶ Post questions into the forum (link later)

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## Exercises:

- ▶ Post questions into the forum (link later)
- ▶ Submission via “commit” through svn and Daphne
- ▶ Feedback one week after deadline through “update” (svn)
- ▶ Unit test / checkstyle via Jenkins

# Organisation - Exercises 4 / 5

## Exercises - Points:

- ▶ Practical:
  - ▶ 60 % functionality
  - ▶ 20 % tests
  - ▶ 20 % documentation, Checkstyle, etc.
  - ▶ Program is not running  $\Rightarrow$  0 points

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- ▶ Practical:
  - ▶ 60 % functionality
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  - ▶ 20 % documentation, Checkstyle, etc.
  - ▶ Program is not running  $\Rightarrow$  0 points
- ▶ Theoretical (mathematical proof):
  - ▶ 40 % general idea / approach
  - ▶ 60 % clean / complete

## Effort:

- ▶ 4 ECTS (ESE), 6 ECTS (IEMS)
- ▶ 120 / 180 working hours per semester
- ▶ 14 Lectures each 6 h / 8 h + exam
- ▶ 4 h / 6 h per exercise-sheet (one per week)



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# Daphne

## Daphne:

- ▶ Provides the following information:
  - ▶ Name / contact information of your tutor
  - ▶ Download of / info needed for exercise sheets
  - ▶ Collected points of all exercise sheets
  - ▶ Links to:
    1. Coding standards
    2. Build system
    3. The other systems
- ▶ Link: [Daphne](#)

## Forum:

- ▶ Please don't hesitate to ask if something is unclear
- ▶ Ask in the forum and not separate. Others might also be interested in the answer
- ▶ I, [Claudis Korzen](#) or one of the [tutors](#) will reply as fast as possible
- ▶ Link: [Forum](#)

# Checkstyle

flake8

## Checkstyle / Linting (flake8):

- ▶ Installation: **python3** -m pip install flake8
- ▶ Check file: **python3** -m flake8 path/to/files/\*.py
- ▶ Link: [flake8](#)

# Unit Tests

## **Why unit tests?**

1. A non-trivial method without an unit test is probably wrong

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- ▶ At least one **typical** input

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3. We and you can automatic check correctness of code

## What is a good unit test?

- ▶ Unit test checks desired output for a given input
  - ▶ At least one **typical** input
  - ▶ At least one **critical** case
- E.g. double occurrence of a value in sorting

# Unit Tests

## doctest

### Testing (doctest):

```
def subOne(n):  
    """Subtracts 1 from n
```

- Tests are contained in docstrings

```
>>> subOne(5)  
4
```

```
>>> subOne(3)  
2  
"""
```

```
return n-2
```

```
if __name__ == "__main__":  
    print("2 minus 1: %d" % subOne(2))
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# Unit Tests

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- ▶ Module doctest runs them

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- ▶ Tests are contained in docstrings
- ▶ Module doctest runs them
- ▶ Run check with:  
**python3 -m doctest**  
*path/to/files/\*.py -v*

# Version management

## Subversion

### Version management (subversion):

- ▶ Keeps a history of code changes
- ▶ Initialize / update directory: **svn** checkout <URL>
- ▶ Add files / folders: **svn** add <file> --all
- ▶ Create snapshot: **svn** commit -m "<Your Message>"  
Data is uploaded to Jenkins automatically
- ▶ Link: [Subversion](#)

# Jenkins

## Jenkins:

- ▶ Provides our build system
- ▶ You can check if your uploaded code runs
  - ▶ Especially whether all **unit test** pass
  - ▶ And if **checkstyle** (flake8) is statisfied
- ▶ Will be shown in the first exercise
- ▶ Link: [Jenkins](#)



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## Sorting

- Minsort

- Heapsort

# Sorting 1 / 2

## Problem:

- ▶ Input:  $n$  elements  $x_1, \dots, x_n$
- ▶ Transitive operator “**i**” which returns **true** if the left value is smaller than the right one
  - ▶ Transitivity:  $x < y, y < z \rightarrow x < z$
- ▶ Output:  $x_1, \dots, x_n$  sorted with operator

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## Example

Input: 14, 4, 32, 19, 8, 44, 65

Output:

### Why do we need sorting?

- ▶ Nearly **every** program needs a sorting-algorithm
- ▶ **Examples:**
  - ▶ Index of a search engine
  - ▶ Listing filesystem in explorer / finder
  - ▶ (Music-) Library
  - ▶ Highscore list

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## Sorting

Minsort

Heapsort

# Minsort - Algorithm

## Informal description:

- ▶ Find the minimum and switch the value with the **first** position
- ▶ Find the minimum and switch the value with the **second** position
- ▶ ...



Figure: *Minsort*

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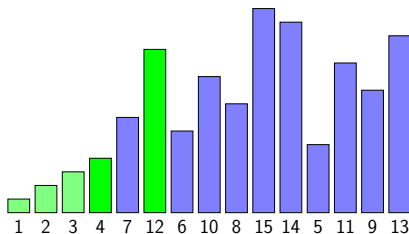


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Figure: *Minsort*



# Minsort - Algorithm

## Minsort in Python:

```
def minsort(lst):  
    for i in range(0, len(lst)-1):  
        minimum = i  
  
        for j in range(i+1, len(lst)):  
            if lst[j] < lst[minimum]:  
                minimum = j  
  
        if minimum != i:  
            lst[i], lst[minimum] = \  
                lst[minimum], lst[i]  
  
    return lst
```

# MinSort - Runtime

**How long does our program run?**

- ▶ We test it for different input sizes

# MinSort - Runtime

How long does our program run?

Table: Runtime for *Minsort*

- We test it for different input sizes

| $n$              | Runtime / ms |
|------------------|--------------|
| $2 \times 10^3$  | 5.24         |
| $4 \times 10^3$  | 16.92        |
| $6 \times 10^3$  | 39.11        |
| $8 \times 10^3$  | 67.80        |
| $10 \times 10^3$ | 105.50       |
| $12 \times 10^3$ | 150.38       |
| $14 \times 10^3$ | 204.00       |
| $16 \times 10^3$ | 265.98       |
| $18 \times 10^3$ | 334.94       |

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- ▶ **Observation:**  
It is going to be “disproportional”  
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- ▶ As a first example serves this diagram for *Minsort*
  - ▶ That's what you should do in the first exercise sheet



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## Runtime analysis:

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- ▶ **We observe:**
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  - ▶ With double the input size we need four times the time

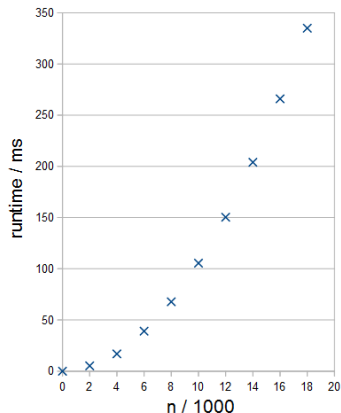


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  - ▶ With double the input size we need four times the time



Figure: Runtime of *Minsort*

- ▶ Next lecture we will analyze deeper with other methods



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# Heapsort - Algorithm 1 / 10

## **Heapsort:**

- ▶ The principle stays the same
- ▶ Better structure for finding the smallest element quicker

# Heapsort - Algorithm 1 / 10

## Heapsort:

- ▶ The principle stays the same
- ▶ Better structure for finding the smallest element quicker

## Binary heap:

- ▶ Preferably a complete binary tree
- ▶ **Heap property:** Each child is **smaller** (larger) than the parent element

## Heapsort - Algorithm 2 / 10

### Min heap:

- **Heap property:** Each child is **smaller** (larger) than the parent element

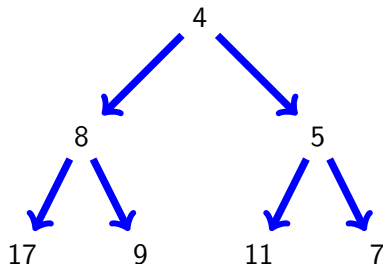


Figure: Valid min heap

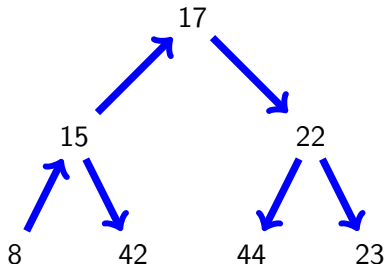


Figure: Invalid min heap

# Heapsort - Algorithm 2 / 10

## Min heap:

- ▶ **Heap property:** Each child is **smaller** (larger) than the parent element
- ▶ A valid heap fulfills the property at each node



Figure: Valid min heap



Figure: Invalid min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array



Figure: Min heap

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Table: Elements can be stored in array

|   |  |  |  |  |  |  |
|---|--|--|--|--|--|--|
| 0 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |

Figure: Min heap

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| 0 | 1 |  |  |  |  |  |
|---|---|--|--|--|--|--|
| 4 | 8 |  |  |  |  |  |

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Table: Elements can be stored in array

| 0 | 1 | 2 |  |  |  |  |
|---|---|---|--|--|--|--|
| 4 | 8 | 5 |  |  |  |  |

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Table: Elements can be stored in array

| 0 | 1 | 2 | 3  | 4 |  |  |
|---|---|---|----|---|--|--|
| 4 | 8 | 5 | 17 | 9 |  |  |

Figure: Min heap

# Heapsort - Algorithm 3 / 10

## How to save the heap?

- ▶ We number all nodes from top to bottom and left to right starting at 0
  - ▶ The children of node  $i$  are  $2i + 1$  and  $2i + 2$
  - ▶ The parent node of node  $i$  is  $\text{floor}(\frac{i-1}{2})$



Table: Elements can be stored in array

| 0 | 1 | 2 | 3  | 4 | 5  |  |
|---|---|---|----|---|----|--|
| 4 | 8 | 5 | 17 | 9 | 11 |  |

Figure: Min heap

# Heapsort - Algorithm 3 / 10

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Table: Elements can be stored in array

| 0 | 1 | 2 | 3  | 4 | 5  | 6 |
|---|---|---|----|---|----|---|
| 4 | 8 | 5 | 17 | 9 | 11 | 7 |

Figure: Min heap

## Heapsort - Algorithm 4 / 10

**Repairing after taking the smallest element:** `heap.pop()`

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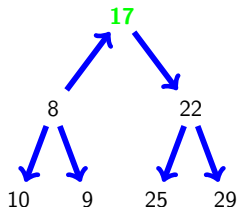


Figure: Repair of a min heap

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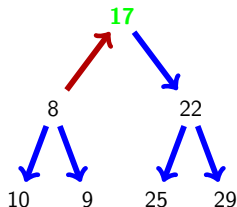


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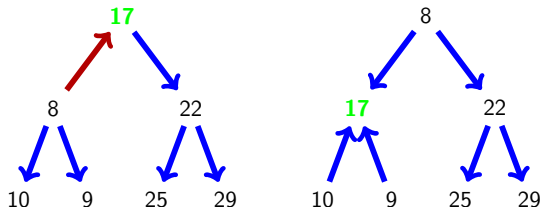


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## Heapsort - Algorithm 4 / 10

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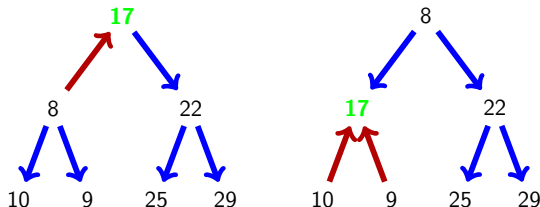


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## Heapsort - Algorithm 4 / 10

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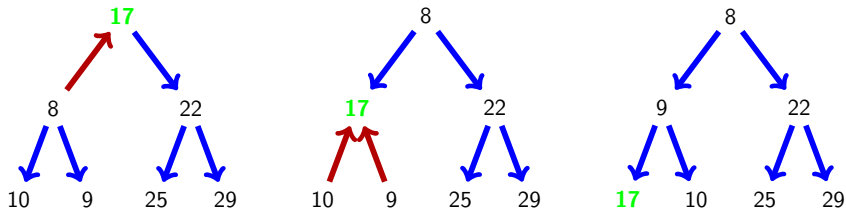


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# HeapSort - Algorithm 5 / 10

## Heapsort:

- ▶ Organize the  $n$  elements as heap
- ▶ While the heap still contains elements
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# HeapSort - Algorithm 5 / 10

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- ▶ Output: 4

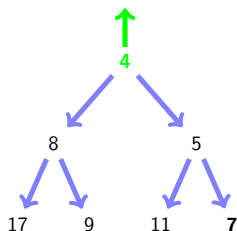


Figure: One iteration of Heapsort



# HeapSort - Algorithm 5 / 10

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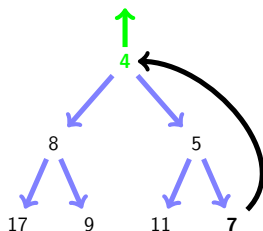


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# HeapSort - Algorithm 5 / 10

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# HeapSort - Algorithm 5 / 10

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Figure: One iteration of Heapsort

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# Heapsort - Algorithm 6 / 10

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# Heapsort - Algorithm 6 / 10

## Creation of a heap:

- ▶ This operation is called **heapify**
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- ▶ Interpret this field als binary heap where the **heap property** is not yet statisfied
- ▶ We repair the heap from bottom up (in layers) with **sift**

# Heapsort - Algorithm 7 / 10

Table: Input in array

| 0  | 1 | 2 | 3 | 4 | 5 | 6 |
|----|---|---|---|---|---|---|
| 11 | 7 | 8 | 3 | 2 | 5 | 4 |



Figure: Heapify lower layer

# Heapsort - Algorithm 7 / 10

Table: Input in array

|    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 0  | 1 | 2 | 3 | 4 | 5 | 6 |
| 11 | 7 | 8 | 3 | 2 | 5 | 4 |



Figure: Heapify lower layer

## Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

## Heapsort - Algorithm 8 / 10



Figure: Heapify upper layer

## Heapsort - Algorithm 9 / 10



Figure: Resulting heap

# Heapsort - Algorithm 10 / 10

## Finding the minimum is intuitive:

- ▶ **Minsort:** Iterate through all non-sorted elements
- ▶ **Heapsort:** Finding the minimum is trivial (concept)  
*Just take the root of the heap*

# Heapsort - Algorithm 10 / 10

## Finding the minimum is intuitive:

- ▶ **Minsort:** Iterate through all non-sorted elements
- ▶ **Heapsort:** Finding the minimum is trivial (concept)

*Just take the root of the heap*

## Removing the minimum in Heapsort:

- ▶ Repair the heap and restore the **heap property**
  - ▶ We don't have to repair the whole heap
- ▶ More of this in the next lecture



# Further Literature

## ► General for this Lecture

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.

*Introduction to Algorithms.*

MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.

*Algorithms and Data Structures.*

Springer, Berlin, 2008.

<https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf>.

# Further Literature

## ► **Sorting**

[Wika] [Wikipedia - Heapsort](https://en.wikipedia.org/wiki/Heapsort)

`https://en.wikipedia.org/wiki/Heapsort`

[Wikb] [Wikipedia - Selectionsort](https://de.wikipedia.org/wiki/Selectionsort)

`https://de.wikipedia.org/wiki/Selectionsort`

# Further Literature

- ▶ **Subversion**

[Apa] [Apache Subversion](https://subversion.apache.org/)

`https://subversion.apache.org/`