

Algorithms and Datastructures

Open Addressing, Priority Queue

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Algorithms and Datastructures, November 2017

Structure

Hashing

- Recapitulation

- Treatment of hash collisions

- Open Addressing

- Summary

Priority Queue

- Introduction

Hashing

Recapitulation

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- ▶ Then the random keys make sure that it is distributed evenly
- ▶ To find a good hash function for every key set universal hashing is needed
 - ▶ Then however, for a fixed set of keys not every hash function is suitable, but only some

Hashing

Recapitulation

Rehashing:

Hashing

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How to rehash?

- ▶ New hash table with a new random hash function

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 - ▶ Expensive but happens not often

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How to rehash?

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- ▶ Copy elements into the new table
 - ▶ Expensive but happens not often
 - ▶ Therefore the average cost is low
 - ▶ Look at **amortized analysis** in the next lecture

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Recapitulation

Treatment of hash collisions

Open Addressing

Summary

Priority Queue

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Linked List

Buckets as linked list:

Hashing

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Buckets as linked list:

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- ▶ Operations in $O(1)$ are possible if a suitable table size and hashfunction is selected

Hashing

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- ▶ Each bucket is a linked list
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- ▶ Worst case $O(n)$, e.g. table size of 1

Hashing

Linked List

Buckets as linked list:

- ▶ Each bucket is a linked list
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- ▶ Operations in $O(1)$ are possible if a suitable table size and hash function is selected
- ▶ Worst case $O(n)$, e.g. table size of 1
- ▶ Dynamic number of elements is possible

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Open Addressing

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- ▶ Static, fixed number of elements

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Open Addressing

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- ▶ For colliding keys we choose a new free entry
- ▶ Static, fixed number of elements
- ▶ The **probe sequence** determines for each key, in which sequence all hash table entries are searched for a free bucket
 - ▶ If a entry is already occupied, then iteratively the following entry can be checked. If a free entry is found the element is inserted

Hashing

Open Addressing

- ▶ For colliding keys we choose a new free entry
- ▶ Static, fixed number of elements
- ▶ The **probe sequence** determines for each key, in which sequence all hash table entries are searched for a free bucket
 - ▶ If an entry is already occupied, then iteratively the following entry can be checked. If a free entry is found the element is inserted
 - ▶ If element is not found at the corresponding table entry, even if the entry is occupied, then probing has to be performed until the element or a free entry have been found

Hashing

Open Addressing

Definitions:

Hashing

Open Addressing

Definitions:

$h(s)$ Hash function for key s

Hashing

Open Addressing

Definitions:

$h(s)$ Hash function for key s

$g(s, j)$ Probing function for key s with overflow positions
 $j \in \{0, \dots, m - 1\}$ e.g. $g(s, j) = j$

Hashing

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Definitions:

$h(s)$ Hash function for key s

$g(s, j)$ Probing function for key s with overflow positions
 $j \in \{0, \dots, m-1\}$ e.g. $g(s, j) = j$

- The **probe sequence** is calculated by

$$h(s, j) = (h(s) - g(s, j)) \bmod m \in \{0, \dots, m-1\}$$



Hashing

Open Addressing - Python

```
def insert(s, value):  
    j = 0  
  
    while t[(h(s) - g(s, j)) mod m] \  
           is not None:  
        j += 1  
  
    t[(h(s) - g(s, j)) mod m] \  
      = (s, value)
```

Hashing

Open Addressing - Python

```
def lookup(s):  
    j = 0  
  
    while t[(h(s) - g(s, j)) mod m] \  
           is not None:  
  
        if t[(h(s) - g(s, j)) mod m][0] == s:  
            return t[(h(s) - g(s, j)) mod m]  
  
        j += 1  
  
    return None
```

Hashing

Open Addressing - Linear Probing



Figure: Linear probe sequence

Hashing

Open Addressing - Linear Probing



Figure: Linear probe sequence

- Check the element with lower index: $g(s, j) := j$
⇒ Hash function: $h(s, j) = (h(s) - j) \bmod m$

Hashing

Open Addressing - Linear Probing



Figure: Linear probe sequence

- ▶ Check the element with lower index: $g(s, j) := j$
 \Rightarrow Hash function: $h(s, j) = (h(s) - j) \bmod m$
- ▶ This leads to the following probe sequence

$$h(s), h(s) - 1, h(s) - 2, \dots, \underbrace{0, m - 1}_{\text{clipping}}, m - 2, \dots, h(s) + 1$$

Hashing

Open Addressing - Linear Probing



Figure: Linear probe sequence

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Figure: Linear probe sequence

- Can result in primary clustering

Hashing

Open Addressing - Linear Probing



Figure: Linear probe sequence

- ▶ Can result in primary clustering
- ▶ Dealing with a hash collision will result in a higher probability of hash collisions in close entries

Hashing

Open Addressing - Linear Probing

Example:

Hashing

Open Addressing - Linear Probing

Example:

- Keys: {12, 53, 5, 15, 2, 19}

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Keys: {12, 53, 5, 15, 2, 19}
- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Keys: {12, 53, 5, 15, 2, 19}
- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$
- ▶ t. insert (12, "A"), $h(12, 0) = 5$

0	1	2	3	4	5	6
					12, A	

Hashing

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Example:

- ▶ Keys: {12, 53, 5, 15, 2, 19}
- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$
- ▶ t.insert (12, "A"), $h(12, 0) = 5$

0	1	2	3	4	5	6
					12, A	

- ▶ t.insert (53, "B"), $h(53, 0) = 4$

				53, B	12, A	
--	--	--	--	-------	-------	--

Figure: Probe/Insertion sequence on a hash map

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$
- ▶ t. insert (5, "C"), $h(5, 0) = 5$, $h(5, 1) = 4$, $h(5, 2) = 3$

0	1	2	3	4	5	6
			5, C	53, B	12, A	

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$
- ▶ t. insert (5, "C"), $h(5, 0) = 5$, $h(5, 1) = 4$, $h(5, 2) = 3$

0	1	2	3	4	5	6
			5, C	53, B	12, A	

- ▶ t. insert (15, "D"), $h(15, 0) = 1$

	15, D		5, C	53, B	12, A	
--	-------	--	------	-------	-------	--

Figure: Probe/Insertion sequence on a hash map

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$

Hashing

Open Addressing - Linear Probing

Example:

- ▶ Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$
- ▶ t. insert (2, "E"), $h(2, 0) = 2$

0	1	2	3	4	5	6
	15, D	2, E	5, C	53, B	12, A	

Hashing

Open Addressing - Linear Probing

Example:

► Hash function: $h(s, j) = (s \bmod 7 - j) \bmod 7$

► t. insert (2, "E"), $h(2, 0) = 2$

0	1	2	3	4	5	6
	15, D	2, E	5, C	53, B	12, A	

► t. insert (19, "F"), $h(19, 0) = 5$, $h(19, 1) = 4$,
 $h(19, 2) = 3$, $h(19, 3) = 2$, $h(19, 4) = 1$, $h(19, 5) = 0$

19, F	15, D	2, E	5, C	53, B	12, A	
-------	-------	------	------	-------	-------	--

Figure: Probe/Insertion sequence on a hash map

Hashing

Open Addressing - Squared Probing

Squared probing:

Hashing

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Squared probing:

- Motivation: Avoid local clustering

$$g(s, j) := (-1)^j \left\lceil \frac{j}{2} \right\rceil^2$$

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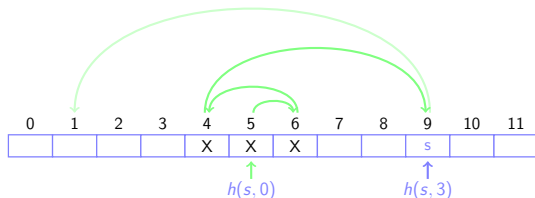


Figure: Squared probe sequence

Hashing

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$$g(s, j) := (-1)^j \left\lceil \frac{j}{2} \right\rceil^2$$



Figure: Squared probe sequence

- This leads to the following probe sequence

$$h(s), h(s) + 1, h(s) - 1, h(s) + 4, h(s) - 4, h(s) + 9, h(s) - 9, \dots$$

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Open Addressing - Squared Probing

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- If m is a prime number for which $m = 4 \cdot k + 3$ then the probe sequence is a permutation of the indices of the hash tables

Hashing

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Squared probing:

$$g(s, j) := (-1)^j \left\lceil \frac{j}{2} \right\rceil^2$$

- ▶ If m is a prime number for which $m = 4 \cdot k + 3$ then the probe sequence is a permutation of the indices of the hash tables
- ▶ Alternatively: $h(s, j) := (h(s) - c_1 \cdot j + c_2 \cdot j^2) \bmod m$

Hashing

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Squared probing:

$$g(s, j) := (-1)^j \left\lceil \frac{j}{2} \right\rceil^2$$

- ▶ If m is a prime number for which $m = 4 \cdot k + 3$ then the probe sequence is a permutation of the indices of the hash tables
- ▶ Alternatively: $h(s, j) := (h(s) - c_1 \cdot j + c_2 \cdot j^2) \bmod m$
- ▶ Problem of secondary clustering
No local clustering anymore, but keys with same hash value have similar probe sequence

Hashing

Open Addressing - Uniform Probing

Uniform Probing:

Hashing

Open Addressing - Uniform Probing

Uniform Probing:

- ▶ Motivation: So far uses function $g(s, j)$ only the step counter j for linear and squared probing
⇒ The probe sequence is independent of the key s

Hashing

Open Addressing - Uniform Probing

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- ▶ Motivation: So far uses function $g(s, j)$ only the step counter j for linear and squared probing
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- ▶ Uniform probing computes the sequence $g(s, j)$ of permutations of all possible indices in dependency on key s

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⇒ The probe sequence is independent of the key s
- ▶ Uniform probing computes the sequence $g(s, j)$ of permutations of all possible indices in dependency on key s
- ▶ **Advantage:** Prevents clustering because different keys with the same hash value do not produce the same probe sequence

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Uniform Probing:

- ▶ Motivation: So far uses function $g(s, j)$ only the step counter j for linear and squared probing
⇒ The probe sequence is independent of the key s
- ▶ Uniform probing computes the sequence $g(s, j)$ of permutations of all possible indices in dependency on key s
- ▶ **Advantage:** Prevents clustering because different keys with the same hash value do not produce the same probe sequence
- ▶ **Disadvantage:** Hard to implement

Hashing

Open Addressing - Double Hashing

Double Hashing:



Figure: Double hashing probe sequence

Hashing

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Figure: Double hashing probe sequence

Hashing

Open Addressing - Double Hashing

Double Hashing:



Figure: Double hashing probe sequence

- Motivation: Consider key s in probe sequence
- Use two independent hash functions $h_1(s)$, $h_2(s)$

Hashing

Open Addressing - Double Hashing

Double Hashing:



Figure: Double hashing probe sequence

- ▶ Motivation: Consider key s in probe sequence
- ▶ Use two independent hash functions $h_1(s)$, $h_2(s)$
- ▶ Hash function: $h(s, j) = (h_1(s) + j \cdot h_2(s)) \bmod m$

Hashing

Open Addressing - Double Hashing

Double Hashing:

- ▶ Hash function: $h(s, j) = (h_1(s) + j \cdot h_2(s)) \bmod m$

Hashing

Open Addressing - Double Hashing

Double Hashing:

- ▶ Hash function: $h(s, j) = (h_1(s) + j \cdot h_2(s)) \bmod m$
- ▶ probe sequence:

$$h_1(s), h_1(s) + h_2(s), h_1(s) + 2 \cdot h_2(s), h_1(s) + 3 \cdot h_2(s), \dots$$

Hashing

Open Addressing - Double Hashing

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- ▶ Works well in practical use

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Open Addressing - Double Hashing

Double Hashing:

- ▶ Hash function: $h(s, j) = (h_1(s) + j \cdot h_2(s)) \bmod m$
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- ▶ Works well in practical use
- ▶ This method is an approximation of uniform probing

Hashing

Open Addressing - Double Hashing - Example

Example:

Hashing

Open Addressing - Double Hashing - Example

Example:

$$h_1(s) = s \bmod 7$$

$$h_2(s) = (s \bmod 5) + 1$$

$$h(s, j) = h_1(s) + j \cdot h_2(s) \bmod 7$$

Hashing

Open Addressing - Double Hashing - Example

Example:

$$h_1(s) = s \bmod 7$$

$$h_2(s) = (s \bmod 5) + 1$$

$$h(s, j) = h_1(s) + j \cdot h_2(s) \bmod 7$$

Table: Comparing both hash functions

s	10	19	31	22	14	16
$h_1(s)$	3	5	3	1	0	2
$h_2(s)$	1	5	2	3	5	2

- The efficiency of double hashing is dependent on $h_1(s) \neq h_2(s)$

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing

Double hashing by Brent:

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing

Double hashing by Brent:

- Motivation:

Because different keys have different probe sequences, the sequence of the insertions has impact on efficiency of a successful search

Hashing

Open Addressing - Double Hashing - Optimization



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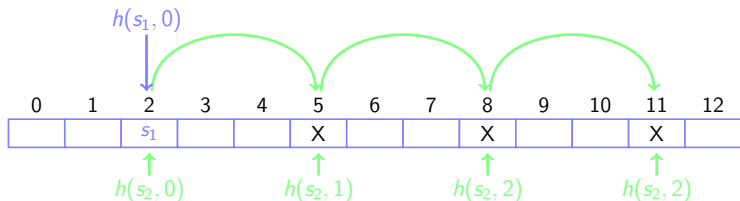


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Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing

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Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing

Example:

- ▶ The key s_1 is inserted at position $p_1 = h(s_1, 0)$

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing

Example:

- ▶ The key s_1 is inserted at position $p_1 = h(s_1, 0)$
- ▶ The hash function for s_2 also results in $p_2 = h(s_2, 0) = p_1$

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing

Example:

- ▶ The key s_1 is inserted at position $p_1 = h(s_1, 0)$
- ▶ The hash function for s_2 also results in $p_2 = h(s_2, 0) = p_1$
- ▶ The locations $h(s_2, j)$, $j \in \{1, \dots, n\}$ are also occupied

Hashing

Open Addressing - Double Hashing - Optimization



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Example:

- ▶ The key s_1 is inserted at position $p_1 = h(s_1, 0)$
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- ▶ The key s_1 is inserted at position $p_1 = h(s_1, 0)$
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Open Addressing - Double Hashing - Optimization



Figure: Double hashing

Example:

- ▶ The key s_1 is inserted at position $p_1 = h(s_1, 0)$
- ▶ The hash function for s_2 also results in $p_2 = h(s_2, 0) = p_1$
- ▶ The locations $h(s_2, j)$, $j \in \{1, \dots, n\}$ are also occupied
- ▶ If we insert s_2 at position $h(s_2, n + 1)$ the search will be inefficient

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

- Reversed sequence of keys would have been better

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

- ▶ Reversed sequence of keys would have been better
- ▶ **Brents Idea:**

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

- ▶ Reversed sequence of keys would have been better
- ▶ **Brents Idea:**
 - ▶ Test if location $h(s_1, 1)$ is free

Hashing

Open Addressing - Double Hashing - Optimization



Figure: Double hashing by Brent

- ▶ Reversed sequence of keys would have been better
- ▶ **Brents Idea:**
 - ▶ Test if location $h(s_1, 1)$ is free
 - ▶ If yes, move s_1 from $h(s_1, 0)$ to $h(s_1, 1)$ and insert s_2 at $h(s_2, 0)$

Hashing

Open Addressing - Ordered Hashing

Idea:

- ▶ Motivation: Colliding elements are inserted in the hashtable sorted.
- ▶ Therefore, in case of an unsuccessful search of elements in combination with linear probing or double hashing, aborting is earlier possible because single probing steps have a fixed length

Implementation:

- ▶ Compare both keys if a collision occurs
- ▶ Insert the smaller key at p_1
- ▶ Search a position based on the diversion order for the bigger key

Hashing

Open Addressing - Ordered Hashing

Example:

- ▶ The key 12 is saved at position $p_1 = h(12, 0)$
- ▶ We insert the key 5 into the hash map
- ▶ We assume $h(5, 0)$ results in location p_1
- ▶ Because $5 < 12$ we insert the key 5 at position p_1
- ▶ For the key 12 we iterate through the sequence

$$h(12, 1), h(12, 2), h(12, 3), \dots$$

Hashing

Open Addressing - Robin-Hood Hashing

Motivation:

Hashing

Open Addressing - Robin-Hood Hashing

Motivation:

- ▶ Having similar length of probe sequences for all elements.
Total costs stay the same, but they are distributed evenly.
Results in approximately similar search times for all elements

Hashing

Open Addressing - Robin-Hood Hashing

Motivation:

- ▶ Having similar length of probe sequences for all elements.
Total costs stay the same, but they are distributed evenly.
Results in approximately similar search times for all elements

Implementation:

Hashing

Open Addressing - Robin-Hood Hashing

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- ▶ If two keys s_1, s_2 collide ($p_1 = h(s_1, j_1) = h(s_2, j_2)$) we compare the length of the sequence (j_1 or j_2)

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- ▶ If two keys s_1, s_2 collide ($p_1 = h(s_1, j_1) = h(s_2, j_2)$) we compare the length of the sequence (j_1 or j_2)
- ▶ The key with the bigger search sequence is inserted at p_1 . The other key is assigned a new location based on the sequence

Hashing

Open Addressing - Robin-Hood Hashing

Example:

- ▶ The key 12 is saved at position $p_1 = h(12, 7)$
- ▶ We insert the key 5 into the hash map
- ▶ We assume $h(5, 0)$ results in location p_1
- ▶ Because $j_1 < j_2$ ($0 < 7$) the key 12 stays at position p_1
- ▶ For the key 5 we iterate through the sequence

$$h(5, 1), h(5, 2), h(5, 3), \dots$$

Hashing

Open Addressing - Implement Insert / Remove

Problem:

- ▶ The key s_1 is inserted at position p_1
- ▶ The key s_2 returns the same hash value, but is inserted at position p_2 because of the probing order
- ▶ If s_1 is removed, it is impossible to find s_2

Hashing

Open Addressing - Implement Insert / Remove

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Solution:

Hashing

Open Addressing - Implement Insert / Remove

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Solution:

- ▶ **Remove:** Elements are marked as removed, but not deleted

Hashing

Open Addressing - Implement Insert / Remove

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- ▶ If s_1 is removed, it is impossible to find s_2

Solution:

- ▶ **Remove:** Elements are marked as removed, but not deleted
- ▶ **Inserting:** Elements marked as removed will be overwritten

Structure

Hashing

Recapitulation

Treatment of hash collisions

Open Addressing

Summary

Priority Queue

Introduction

Hashing

Open Addressing - Summary Collision Handling

Bucket as linked list: (dynamic, number of elements variable)

- ▶ Save colliding elements as linked list

Open hashing: (static, number of elements fixed)

- ▶ Determine a probe sequence, permutation of all hash values
- ▶ Linear, quadratic probing:
 - ▶ Easy to implement
 - ▶ Raise the probability of collisions because probing order does not depend on the key

Hashing

Open Addressing - Summary Collision Handling

Open hashing: (static, number of elements fixed)

Hashing

Open Addressing - Summary Collision Handling

Open hashing: (static, number of elements fixed)

- ▶ Uniform probing, double hashing:
 - ▶ Different probing orders for different keys
 - ▶ Avoids clustering of elements

Hashing

Open Addressing - Summary Collision Handling

Open hashing: (static, number of elements fixed)

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Improving efficiency: (Brent, Ordered Hashing)

- ▶ Improve search efficiency by sorting colliding insertions
 - ▶ Abortion of unsuccessful search
 - ▶ Search sequence length balancing

Hashing

Open Addressing - Summary Hashing

Hashing:

Hashing

Open Addressing - Summary Hashing

Hashing:

- Efficient for dictionary operations:

Insert: $O(1) \dots O(n)$

Search: $O(1) \dots O(n)$

Remove: $O(1) \dots O(n)$

Hashing

Open Addressing - Summary Hashing

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- ▶ Efficient for dictionary operations:
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Open Addressing - Summary Hashing

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- ▶ Direct access of all elements in a hash table
- ▶ Using a hash function to find the position (hash value) in the hash table
- ▶ Hash function, size of the hash table and strategy to avoid hash collisions influence the efficiency of the datastructure

Structure

Hashing

- Recapitulation

- Treatment of hash collisions

- Open Addressing

- Summary

Priority Queue

- Introduction

Priority Queue

Introduction

Definition:

Priority Queue

Introduction

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- ▶ A priority queue saves a set of elements

Priority Queue

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- ▶ A priority queue saves a set of elements
- ▶ Each element contains a key and a value like a map

Priority Queue

Introduction

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- ▶ A priority queue saves a set of elements
- ▶ Each element contains a key and a value like a map
- ▶ There is a total order (like \leq) defined on the keys

Priority Queue

Introduction

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Priority Queue

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- ▶ The priority queue supports the following operations:

Priority Queue

Introduction

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- ▶ The priority queue supports the following operations:

`insert(key, value):` Inserts a new element into the queue

Priority Queue

Introduction

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`getMin():` Returns the element with the smallest key

`deleteMin():` Removes the element with the smallest key

Priority Queue

Introduction

Definition:

- ▶ The priority queue supports the following operations:

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- ▶ Sometimes additional operations are defined:

Priority Queue

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Priority Queue

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`changeKey(item, key):` Changes the key of the element

`remove(item):` Removes the element from the queue

Priority Queue

Introduction

Special features:

Priority Queue

Introduction

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- ▶ Multiple elements with the same key

Priority Queue

Introduction

Special features:

- ▶ Multiple elements with the same key
 - ▶ No problem and for many applications necessary
 - ▶ If there is more than one element with the smallest key
 - `getMin()`: Returns just one of the possible elements
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Priority Queue

Introduction

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- ▶ Argument of `changeKey` and `remove` operations

Priority Queue

Introduction

Special features:

- ▶ Multiple elements with the same key
 - ▶ No problem and for many applications necessary
 - ▶ If there is more than one element with the smallest key
 - `getMin()`: Returns just one of the possible elements
 - `deleteMin()`: Deletes the element returned by `getMin`
- ▶ Argument of `changeKey` and `remove` operations
 - ▶ There is no **quick-access** to a element in the queue
 - ▶ That's why `insert` and `getMin` return a reference (handle, accessor object)
 - ▶ `changeKey` and `remove` take this reference as argument
 - ▶ Therefore each element has to store its current position in the heap.

Priority Queue

Python

```
from queue import PriorityQueue

q = PriorityQueue()

e1 = (5, "A") # element with priority 5
q.put(e1); # insert element e1

# remove and return the lowest item
e2 = q.get()
```

Priority Queue

Application Example

Example 1:

- ▶ Calculation of the sorted union of k sorted lists
(multi-way merge or k -way merge)

L_1 :


3	5	8	12	...
---	---	---	----	-----

L_3 :

1	10	11	24	...
---	----	----	----	-----

L_2 :

4	5	6	7	...
---	---	---	---	-----


 $\Rightarrow R$:

1	3	4	5	5	6	7	8	10	...
---	---	---	---	---	---	---	---	----	-----

Figure: 3-way merge

Priority Queue

Application Example

Example 1:

Priority Queue

Application Example

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Priority Queue

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- ▶ Runtime: N = length of resulting list

Priority Queue

Application Example

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 - ▶ Trivial: $\Theta(N \cdot k)$, minimum calculation $\Theta(k)$

Priority Queue

Application Example

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 - ▶ Priority queue: $\Theta(N \cdot \log k)$, minimum calculation $\Theta(\log k)$

Priority Queue

Application Example

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Example 2:

Priority Queue

Application Example

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- ▶ Calculation of the sorted union of k sorted lists (multi-way merge or k -way merge)
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Example 2:

- ▶ For example Dijkstra's algorithm for computing the shortest path (\leftarrow following lecture)

Priority Queue

Application Example

Example 1:

- ▶ Calculation of the sorted union of k sorted lists (multi-way merge or k -way merge)
- ▶ Runtime: N = length of resulting list
 - ▶ Trivial: $\Theta(N \cdot k)$, minimum calculation $\Theta(k)$
 - ▶ Priority queue: $\Theta(N \cdot \log k)$, minimum calculation $\Theta(\log k)$

Example 2:

- ▶ For example Dijkstra's algorithm for computing the shortest path (\leftarrow following lecture)
- ▶ Among other applications it can be used for sorting

Priority Queue

Implementation

Idea:

Priority Queue

Implementation

Idea:

- Save elements as tuples in a binary heap

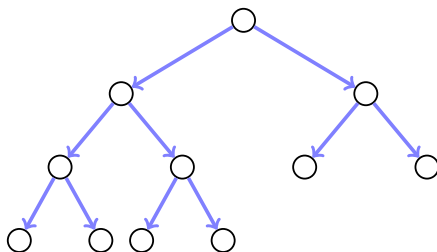


Figure: Heap with 11 nodes

Priority Queue

Implementation

Idea:

- ▶ Save elements as tuples in a binary heap
- ▶ Summary from lecture 1 (*HeapSort*):
 - ▶ Nearly complete binary tree
 - ▶ **Heap condition:**
The key of each node \leq the keys of the children



Figure: Heap with 11 nodes

Priority Queue

Implementation



Figure: Min heap stored in array

Priority Queue

Implementation



Figure: Min heap stored in array

Storing a binary heap:

Priority Queue

Implementation



Figure: Min heap stored in array

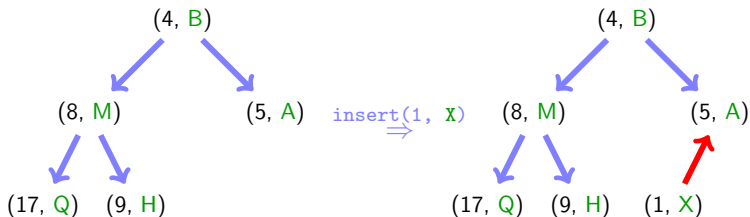
Storing a binary heap:

- ▶ Number nodes from top to bottom and left to right starting with 0 and store entries in array
- ▶ Children of node i are the nodes $2i + 1$ and $2i + 2$
- ▶ Parent node of node i is $\text{floor}((i - 1)/2)$

Priority Queue

Implementation - Insertion

Inserting an element: `insert(key, item)`



Priority Queue

Implementation - Insertion

Inserting an element: `insert(key, item)`

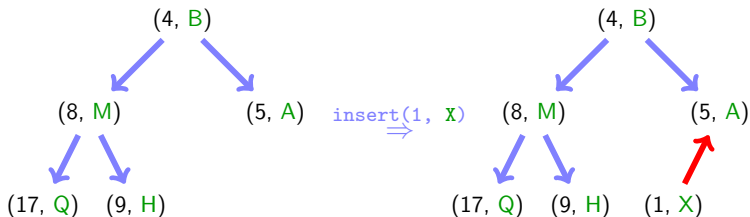


- Append the element at the end of the array

Priority Queue

Implementation - Insertion

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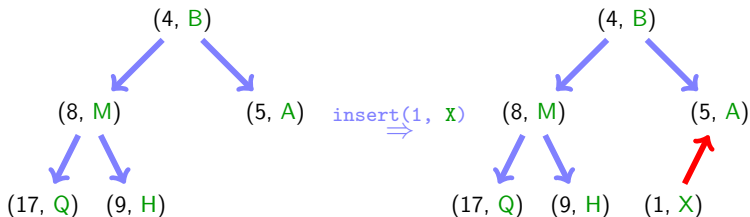


- ▶ Append the element at the end of the array
- ▶ The **heap condition** may be violated, but only at the last index

Priority Queue

Implementation - Insertion

Inserting an element: `insert(key, item)`



- ▶ Append the element at the end of the array
- ▶ The **heap condition** may be violated, but only at the last index
- ▶ Repair **heap condition** \Rightarrow We will see later how to do this

Priority Queue

Implementation

Returning the minimum: `getMin()`



Priority Queue

Implementation

Returning the minimum: `getMin()`



- Else return the first element

Priority Queue

Implementation

Returning the minimum: `getMin()`

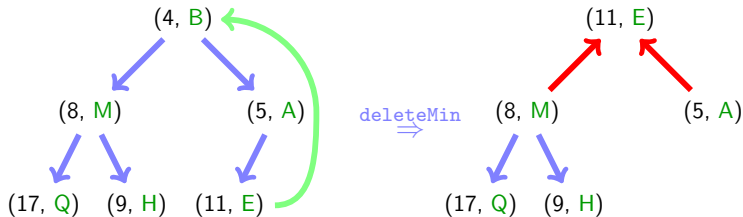


- ▶ Else return the first element
- ▶ If the heap is empty return `None`

Priority Queue

Implementation

Removing the minimum: `deleteMin()`



Priority Queue

Implementation

Removing the minimum: `deleteMin()`



- Deleting the element with the lowest key

Priority Queue

Implementation

Removing the minimum: `deleteMin()`



- ▶ Deleting the element with the lowest key
- ▶ Swap the last element with the first element and shrink the heap by one

Priority Queue

Implementation

Removing the minimum: `deleteMin()`



- ▶ Deleting the element with the lowest key
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Removing the minimum: `deleteMin()`



- ▶ Deleting the element with the lowest key
- ▶ Swap the last element with the first element and shrink the heap by one
- ▶ The **heap condition** may be violated, but only at the first index
- ▶ Repair **heap condition**

Priority Queue

Implementation

Changing the key (priority): `changeKey(item, key)`

- ▶ The element (queue item) is given as argument
- ▶ Replace the key of the element
- ▶ The **heap condition** may be violated, but only at the element index and only in one direction (up / down)
- ▶ Repair **heap condition**



Priority Queue

Implementation

Changing the key (priority): `changeKey(item, key)`



Priority Queue

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- The **heap condition** may be violated, but only at the element index and only in one direction (up / down)

Priority Queue

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Priority Queue

Implementation

Removing an element: `remove(item)`



Priority Queue

Implementation

Removing an element: `remove(item)`



- The element (queue item) is given as argument

Priority Queue

Implementation

Removing an element: `remove(item)`



- ▶ The element (queue item) is given as argument
- ▶ Replace the element with the last element and shrink the heap by one

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- ▶ Replace the element with the last element and shrink the heap by one
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- ▶ Repair **heap condition**

Priority Queue

Implementation - Repairing the Heap

Repairing after modifying operations:

Priority Queue

Implementation - Repairing the Heap

Repairing after modifying operations:

- ▶ The heap condition can be violated after using `insert`, `deleteMin`, `changeKey`, `remove`, but only at one known position with index i

Priority Queue

Implementation - Repairing the Heap

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- ▶ The heap condition can be violated after using `insert`, `deleteMin`, `changeKey`, `remove`, but only at one known position with index i
- ▶ Heap conditions can be violated in two directions:

Priority Queue

Implementation - Repairing the Heap

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- ▶ The heap condition can be violated after using `insert`, `deleteMin`, `changeKey`, `remove`, but only at one known position with index i
- ▶ Heap conditions can be violated in two directions:
 - ▶ Downwards: The key at index i is not \leq than the value of its children

Priority Queue

Implementation - Repairing the Heap

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- ▶ Heap conditions can be violated in two directions:
 - ▶ Downwards: The key at index i is not \leq than the value of its children
 - ▶ Upwards: The key at index i is not \geq than the value of its parent

Priority Queue

Implementation - Repairing the Heap

Repairing after modifying operations:

- ▶ The heap condition can be violated after using `insert`, `deleteMin`, `changeKey`, `remove`, but only at one known position with index i
- ▶ Heap conditions can be violated in two directions:
 - ▶ Downwards: The key at index i is not \leq than the value of its children
 - ▶ Upwards: The key at index i is not \geq than the value of its parent
- ▶ We need two repair methods: `repairHeapUp`, `repairHeapDown`

Priority Queue

Implementation - Repairing the Heap

`repairHeapDown:`

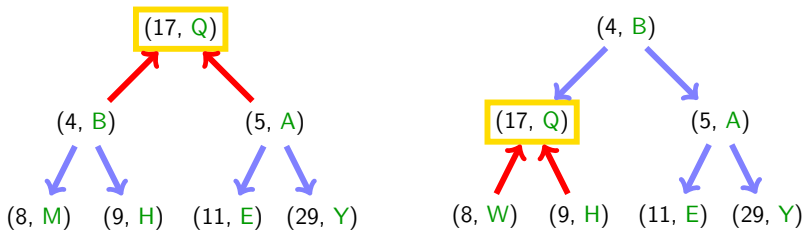


Figure: Repairing the heap downwards

Priority Queue

Implementation - Repairing the Heap

`repairHeapDown:`

- Sift the element until the **heap condition** is valid



Figure: Repairing the heap downwards

Priority Queue

Implementation - Repairing the Heap

repairHeapDown:

- ▶ Sift the element until the **heap condition** is valid
 - ▶ Change node with child, which has the lower key of both children



Figure: Repairing the heap downwards

Priority Queue

Implementation - Repairing the Heap

repairHeapDown:

- Sift the element until the **heap condition** is valid
 - Change node with child, which has the lower key of both children
 - If the **heap condition** is violated repeat for the child node



Figure: Repairing the heap downwards

Priority Queue

Implementation - Repairing the Heap

`repairHeapDown:`

- ▶ Sift the element until the **heap condition** is valid
 - ▶ Change node with child, which has the lower key of both children
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Figure: Repairing the heap downwards

Priority Queue

Implementation - Repairing the Heap

repairHeapUp:

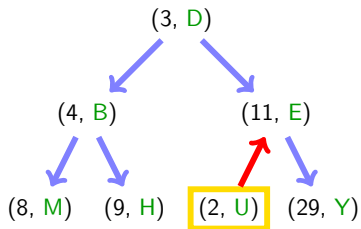


Figure: Repairing the heap upwards

Priority Queue

Implementation - Repairing the Heap

`repairHeapUp:`

- Change node with parent



Figure: Repairing the heap upwards

Priority Queue

Implementation - Repairing the Heap

`repairHeapUp:`

- Change node with parent
- If the **heap condition** is violated repeat for parent node



Figure: Repairing the heap upwards

Priority Queue

Implementation - Repairing the Heap

`repairHeapUp:`

- Change node with parent
- If the **heap condition** is violated repeat for parent node

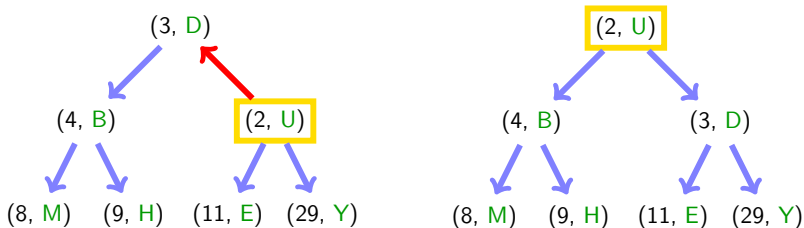


Figure: Repairing the heap upwards

Priority Queue

Implementation - Priority Queue Item

Index of a priority queue item:

Priority Queue

Implementation - Priority Queue Item

Index of a priority queue item:

- ▶ **Attention:** For `changeKey` and `remove` the item has to “know” where it is located in the heap

Priority Queue

Implementation - Priority Queue Item

Index of a priority queue item:

- ▶ **Attention:** For `changeKey` and `remove` the item has to “know” where it is located in the heap
- ▶ Remember for `repairHeapUp` and `repairHeapDown`:
Update the index if moving an heap element

Priority Queue

Implementation - Priority Queue Item - Python

```
class PriorityQueueItem:

    """Provides a handle for a queue item.

    This handle can be used to remove or
    update the queue item.
    """

    def __init__(self, key, value, index):
        self.key = key
        self.value = value
        self.index = index
```

Priority Queue

Complexity

Summary lecture 1:

Priority Queue

Complexity

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- ▶ A full binary tree with n elements, has a depth of $O(\log n)$

Priority Queue

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We have only one path to traverse: $O(\log n)$

Priority Queue

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Runtime for methods

Priority Queue

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Priority Queue

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We have only one path to traverse: $O(\log n)$

Runtime for methods

- ▶ `insert`, `deleteMin`, `changeKey`, `remove`:
We have to repair the heap: $O(\log n)$
- ▶ `getMin`: Return the element at index 0: $O(1)$

Priority Queue

Complexity

Improvements (Fibonacci heaps):

Priority Queue

Complexity

Improvements (Fibonacci heaps):

- ▶ `getMin`, `insert` and `decreaseKey` in amortized time of $O(1)$

Priority Queue

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Priority Queue

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Practical experience:

Priority Queue

Complexity

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Practical experience:

- ▶ The binary heap is simpler: Costs for managing the structure are low

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- ▶ `deleteMin` in amortized time $O(\log n)$

Practical experience:

- ▶ The binary heap is simpler: Costs for managing the structure are low
- ▶ If the number of elements is relatively small so the difference is negligible
- ▶ Example:
 - ▶ For $n = 2^{10} \approx 1,000$ is the the `depth` $\log_2 n$ only 10
 - ▶ For $n = 2^{20} \approx 1,000,000$ is the `depth` $\log_2 n$ only 20

Further Literature

► General

[CRL01] Thomas H. Cormen, Ronald L. Rivest, and Charles E. Leiserson.

Introduction to Algorithms.

MIT Press, Cambridge, Mass, 2001.

[MS08] Kurt Mehlhorn and Peter Sanders.

Algorithms and data structures, 2008.

<https://people.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf>.

Further Literature

► Priority Queue - Implementations / API

[Cpp] [C++ - priority_queue](#)

`http:`

`//www.sgi.com/tech/stl/priority_queue.html`

[Jav] [Java - PriorityQueue](#)

`https://docs.oracle.com/javase/7/docs/api/
java/util/PriorityQueue.html`

[Pyt] [Python - PriorityQueue](#)

`https://docs.python.org/3/library/queue.
html#queue.PriorityQueue`