1a) $\frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/\sqrt{2} \end{pmatrix} = |\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$\langle\psi| = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

$\langle\psi|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = 1/2 + 1/2 = 1$

1b) $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle = |\psi\rangle$

$\langle\psi|\psi\rangle = \left|\frac{\sqrt{3}}{2}\right|^2 + \left|\frac{1}{2}\right| = \frac{3}{4} + \frac{1}{4} = 1$

1c) $0{,}7|0\rangle + 0{,}3|1\rangle = |\psi\rangle$

$\langle\psi|\psi\rangle = |0{,}7|^2 + |0{,}3|^2 = 0{,}58 \neq 1$

1d) $0{,}8|0\rangle + 0{,}6|1\rangle = |\psi\rangle$

$\langle\psi|\psi\rangle = |0{,}8|^2 + |0{,}6|^2 = 1$

1e) $\cos(\theta)|0\rangle + i\,\mathrm{sen}(\theta)|1\rangle$

$\langle\psi|\psi\rangle = |\cos(\theta)|^2 + |i\,\mathrm{sen}(\theta)|^2 = 1$

1f) $\cos^2(\theta)|0\rangle - \mathrm{sen}^2(\theta)|1\rangle$

$\langle\psi|\psi\rangle = \cos^4(\theta) + \mathrm{sen}^4(\theta) = 1 \quad , \quad \theta_1 = 2\pi k \qquad \forall k = 0,1,2,$

$\theta_2 = \pi(2k + 1/2)$

$P(x) = |\langle x|\psi\rangle|^2$

1a) $P(0) = 1/2 \qquad P(1) = 1/2$

$P(+) = \langle +|\psi\rangle = \frac{\langle 0| + \langle 1|}{\sqrt{2}} \cdot (|0\rangle + |1\rangle)\frac{1}{\sqrt{2}} = 1$

$P(-) = 0$

1b) $P(+) = \langle +|\psi\rangle = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \left|\frac{\sqrt{3}-1}{2\sqrt{2}}\right|^2$

$P(-) = \left|\frac{\sqrt{3}+1}{2\sqrt{2}}\right|^2$

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1 d) $P(+) = \left| \frac{0,8}{\sqrt{2}} + \frac{0,6}{\sqrt{2}} \right|^2 = \left| \frac{1,4}{\sqrt{2}} \right|^2 = 0,98$

$P(-) = \left| \frac{0,8}{\sqrt{2}} - \frac{0,6}{\sqrt{2}} \right|^2 = \left| \frac{0,2}{\sqrt{2}} \right|^2 = 0,02$

1 e) $P(+) \left| \frac{\cos(\theta)}{\sqrt{2}} + \frac{i\, sen(\theta)}{\sqrt{2}} \right|^2 = \left| \frac{e^{i\theta}}{\sqrt{2}} \right|^2 = 1/2$

$P(-) = \left| \frac{\cos(\theta)}{\sqrt{2}} - i \frac{sen(\theta)}{\sqrt{2}} \right|^2 = \left| \frac{e^{-i\theta}}{\sqrt{2}} \right| = 1/2$

III 1

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad B^{\dagger} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$B B^{\dagger} = \frac{1}{2} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & -i+i \\ i-1 & 1+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

a) Si es unitaria

b)

$$|0\rangle \; \underbrace{- \boxed{B} - \boxed{S} - \boxed{H} -}_{U} \; |1\rangle \qquad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$\underbrace{H \, S \, B}_{U} |0\rangle = |1\rangle$$

$$U \, |0\rangle = |1\rangle$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1-1 & i+i \\ 1+1 & i-i \end{pmatrix} = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} \qquad U |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

I) 2) $|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$

$$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle = a\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + B\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$$\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle = \left(\frac{a}{\sqrt{2}} + \frac{B}{\sqrt{2}}\right)|0\rangle + \left(\frac{a}{\sqrt{2}} - \frac{B}{\sqrt{2}}\right)|1\rangle$$

$$\begin{cases} \frac{a+B}{\sqrt{2}} = \frac{3}{5} \to a = \frac{3\sqrt{2}}{5} - B \to \boxed{a = \frac{7}{10}\sqrt{2}} \\ \frac{a-B}{\sqrt{2}} = \frac{4}{5} = \end{cases}$$

$$\frac{\frac{3\sqrt{2}}{5} - B - B}{\sqrt{2}} = \frac{4}{5} \to \frac{3}{5} - \frac{2B}{\sqrt{2}} = \frac{4}{5} \to B = \frac{1}{5}\cdot\frac{-\sqrt{2}}{2} \boxed{= \frac{-\sqrt{2}}{10}}$$

II) 2) a)

$$M = (\alpha|0\rangle + \beta|1\rangle)\langle 0| + (\beta^*|0\rangle - \alpha^*|1\rangle)\langle 1|$$

$\cancel{M = \alpha|0\rangle + \alpha^*|1\rangle}$

$M|0\rangle = \alpha|0\rangle + \beta|1\rangle$

$M|1\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$

b)

$$\begin{pmatrix} \alpha & \\ \beta & \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\begin{pmatrix} & \beta^* \\ & -\alpha^* \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \beta^* \\ -\alpha^* \end{pmatrix}$$

$$M = \begin{pmatrix} \alpha & \beta^* \\ \beta & -\alpha^* \end{pmatrix}$$

c) $M^\dagger = \begin{pmatrix} \alpha^* & \beta^* \\ \beta^* & -\alpha \end{pmatrix}$

$$MM^\dagger = \begin{pmatrix} \alpha & \beta^* \\ \beta & -\alpha^* \end{pmatrix}\begin{pmatrix} \alpha^* & \beta^* \\ \beta & -\alpha \end{pmatrix} = \begin{pmatrix} \alpha\alpha^* + \beta\beta^* & \alpha\beta^* - \alpha\beta^* \\ \alpha^*\beta - \alpha^*\beta & \beta\beta^* + \alpha\alpha^* \end{pmatrix} =$$

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$$MM^{\dagger} = \begin{pmatrix} |\alpha|^2+|\beta|^2 & 0 \\ 0 & |\alpha|^2+|\beta|^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\underbrace{|\alpha|^2}_{P(0)} + \underbrace{|\beta|^2}_{P(1)} = 1 \qquad M \text{ es unitaria}$$

3) $W = U_x (H \otimes I)$, con $H = \frac{X+Z}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$W = U_x \frac{1}{\sqrt{2}} \begin{pmatrix} I & I \\ I & -I \end{pmatrix}$$

$$W = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{U_x} \cdot \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}}_{H \otimes I} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

III 1) a) $(X \otimes I) = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$X \otimes I = |10\rangle + |11\rangle + |00\rangle + |01\rangle$

b) $I \otimes X = \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$I \otimes X = |01\rangle + |00\rangle + |11\rangle + |10\rangle$

c) $X \otimes X = \begin{pmatrix} 0 & X \\ X & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

d) $U_x = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$

$U_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$U_x|00\rangle = |00\rangle \quad U_x|01\rangle = |01\rangle$

$U_x|10\rangle = |11\rangle \quad U_x|11\rangle = |10\rangle$

Como X e I son unitarios un producto tensorial entre ellos, se preserva la unitariedad

$U_x U_x^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = I$

U_x es unitario

como son unitarios $U^{-1} = U^\dagger$

$$U_S = U_X U_{\bar{X}} U_X$$

$$\bar{U}_X U_X = \left(|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X\right)\left(I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|\right)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$U_X \bar{U}_X U_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = U_S$$