Flyball Governor

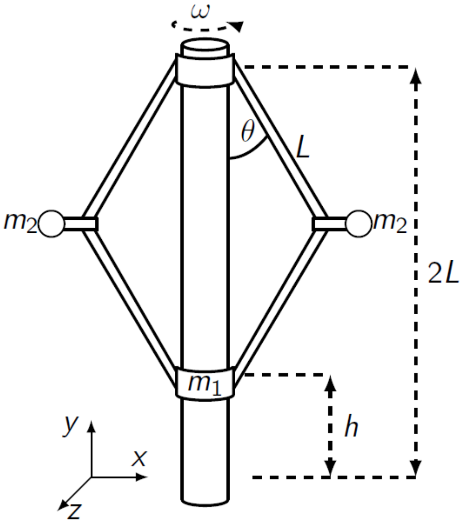
Eda Sevim
Tobias Karg

Dynamic Models & Simulation

February 19, 2017

System Description

System Overview



Eda Sevim
Tobias Karg

System Description

Position Vectors

$$\vec{r}_{m2,1} = \begin{pmatrix} L\sin(\theta)\cos(\beta) \\ L(2-\cos(\theta)) \\ -L\sin(\theta)\sin(\beta) \end{pmatrix}$$

$$\vec{r}_{m2,2} = \begin{pmatrix} -L\sin(\theta)\cos(\beta) \\ L(2-\cos(\theta)) \\ L\sin(\theta)\sin(\beta) \end{pmatrix}$$

$$\vec{r}_{m1} = \begin{pmatrix} 0 \\ 2L(1-\cos(\theta)) \\ 0 \end{pmatrix}$$

$$\text{mit } \beta = \int \omega dt$$

System Dynamics

Kinetic Energy T and Potential Energy V

$$T = \frac{m_1 \| \dot{\vec{r}}_{m1} \|^2}{2} + \frac{m_2 \| \dot{\vec{r}}_{m2,1} \|^2}{2} + \frac{m_2 \| \dot{\vec{r}}_{m2,2} \|^2}{2}$$

$$T = 2 m_1 L^2 \sin^2(\theta) \dot{\theta}^2 + m_2 L^2 \dot{\theta}^2 + m_2 L^2 \sin^2(\theta) \omega^2$$

$$V = m_1 g h_1 + m_2 g h_{2,1} + m_2 g h_{2,2}$$

$$V = 2 m_1 g L (1 - \cos(\theta)) + 2 m_2 g L (2 - \cos(\theta))$$

Eda Sevim
Tobias Karg

System Dynamics

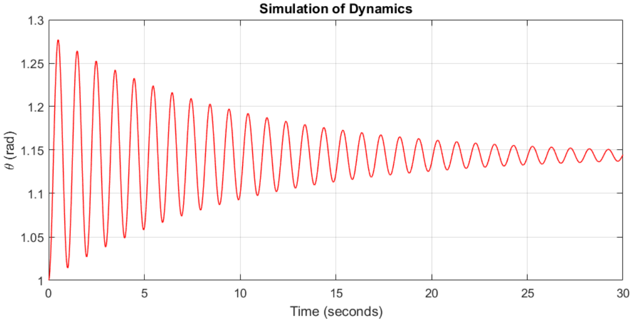
Result

$$\ddot{\theta} = \frac{\sin(2\theta)(\frac{m_2}{2}\omega^2 - m_1\dot{\theta}^2) - \frac{g}{L}\sin(\theta)(m_1 + m_2)}{m_2 + 2m_1\sin(\theta)^2}$$

Eda Sevim
Tobias Karg

System Dynamics
Simulation

$$m_1 = 1\ [\mathrm{kg}],\ m_2 = 2\ [\mathrm{kg}],\ L = 0.4\ [\mathrm{m}],\ \omega = 3\pi\ \left[\frac{\mathrm{rad}}{\mathrm{s}}\right],$$
$$x_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}^T$$



Task 1: $h(\omega)$

Approach

$$\text{Steady-State} \rightarrow \dot{\theta} = \ddot{\theta} = 0$$

$$\frac{m_2}{2}\sin(2\theta)\omega^2 - \frac{g}{L}(m_1 + m_2)\sin(\theta) = 0$$

$$\theta_e = 0 \quad \theta_e = \arccos\left(\frac{1}{\omega^2}\frac{g}{L}\frac{(m_1 + m_2)}{m_2}\right)$$

which can be substituted into $\vec{r}_{m1}$:

Task 1: $h(\omega)$

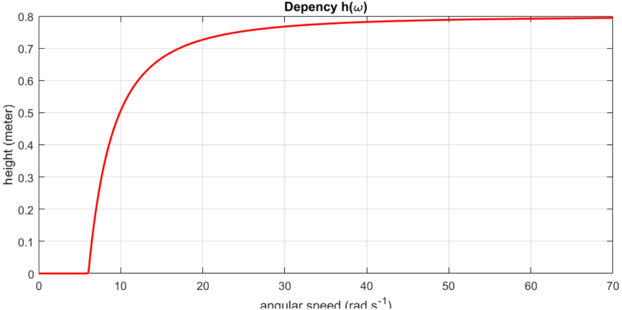
Result

$$h_{m1} = \begin{cases} 2L\left(1 - \frac{1}{\omega^2}\frac{g}{L}\frac{(m_1+m_2)}{m_2}\right) & \omega > \sqrt{\frac{g}{L}\frac{m_1+m_2}{m_2}} \\ 0 & \text{else} \end{cases}$$

Task 1: $h(\omega)$

Dependency

$$m_1 = 1\ [\mathrm{kg}],\ m_2 = 2\ [\mathrm{kg}],\ L = 0.4\ [\mathrm{m}]$$



Task 2: Frequency

Ansatz

Small Oscillations around Steady-State

$$\theta_e = 0 \quad \theta_e = \arccos\left(\frac{1}{\omega^2}\frac{g}{L}\frac{(m_1 + m_2)}{m_2}\right)$$

Approximation with spring-mass-equation

$$m\ddot{x} = -kx$$

Eda Sevim
Tobias Karg

Task 2: Frequency

Result

$$\Delta\ddot{\theta} = \frac{m_2 \cos(2\theta_e)\omega^2 - \frac{g}{L}\cos(\theta_e)(m_1 + m_2)}{m_1(1 - \cos(2\theta_e)) + m_2} \Delta\theta$$

$$\omega_e = \sqrt{\frac{m_2\omega^2(g^2(m_1 + m_2)^2 - L^2 m_2^2 \omega^4)}{2g^2 m_1(m_1 + m_2)^2 - L^2 m_2^2 \omega^4 (2m_1 + m_2)}}$$

Edge Case: $m_1 = 0$

$$\omega_e = \sqrt{\frac{L^2\omega^4 - g^2}{L^2\omega^2}}$$

Eda Sevim
Tobias Karg

Task 2: Frequency

Verification with simulation

$$m_1 = 1\ [\mathrm{kg}],\ m_2 = 2\ [\mathrm{kg}],\ L = 0.4\ [\mathrm{m}],\ \omega = 3\pi\ \left[\frac{\mathrm{rad}}{\mathrm{s}}\right],$$

$$x_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}^T \quad \text{Matlab: } 1.01\ [\mathrm{Hz}]$$