Eda Sevim Tobias Karg

Dynamic Models & Simulation

February 19, 2017

# Flyball Governor Eda Sevim Tobias Karg

### System Description

Position Vectors

#### System Dyna

Result Simulation

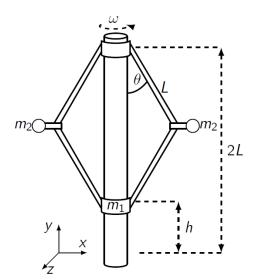
#### Task 1: $h(\omega)$

Approach Result

#### Task 2: Frequency

# System Description

System Overview



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Task 1:  $h(\omega)$ 

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Task 2: Frequency

$$\vec{r}_{m2,1} = \begin{pmatrix} L\sin(\theta)\cos(\beta) \\ L(2-\cos(\theta)) \\ -L\sin(\theta)\sin(\beta) \end{pmatrix}$$

$$\vec{r}_{m2,2} = \begin{pmatrix} -L\sin(\theta)\cos(\beta) \\ L(2-\cos(\theta)) \\ L\sin(\theta)\sin(\beta) \end{pmatrix}$$

$$\vec{r}_{m1} = \begin{pmatrix} 0 \\ 2L(1-\cos(\theta)) \\ 0 \end{pmatrix}$$

$$\text{mit } \beta = \int \omega dt$$

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$$T = \frac{m_1 \|\dot{\vec{r}}_{m1}\|^2}{2} + \frac{m_2 \|\dot{\vec{r}}_{m2,1}\|^2}{2} + \frac{m_2 \|\dot{\vec{r}}_{m2,2}\|^2}{2}$$
$$T = 2m_1 L^2 \sin^2(\theta) \dot{\theta}^2 + m_2 L^2 \dot{\theta}^2 + m_2 L^2 \sin^2(\theta) \omega^2$$

$$V = m_1 g h_1 + m_2 g h_{2,1} + m_2 g h_{2,2}$$
$$V = 2 m_1 g L (1 - \cos(\theta)) + 2 m_2 g L (2 - \cos(\theta))$$

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$$\ddot{\theta} = \frac{\sin(2\theta)(\frac{m_2}{2}\omega^2 - m_1\dot{\theta}^2) - \frac{g}{L}\sin(\theta)(m_1 + m_2)}{m_2 + 2m_1\sin(\theta)^2}$$

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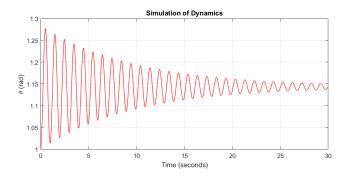
Simulation

### Task 1: $h(\omega)$

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$$m_1=1$$
 [kg],  $m_2=2$  [kg],  $L=0.4$  [m],  $\omega=3\pi$  [ $\frac{\mathrm{rad}}{\mathrm{s}}$ ],  $x_0=\left(\begin{array}{cc} 1 & 0 \end{array}\right)^T$ 



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Steady-State 
$$ightarrow \dot{ heta} = \ddot{ heta} = 0$$

$$\frac{m_2}{2}\sin(2\theta)\omega^2 - \frac{g}{L}(m_1 + m_2)\sin(\theta) = 0$$

$$\theta_e = 0$$
  $\theta_e = \arccos\left(\frac{1}{\omega^2}\frac{g}{L}\frac{(m_1 + m_2)}{m_2}\right)$ 

which can be substituted into  $\vec{r}_{m1}$ :

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$$h_{m1} = \begin{cases} 2L \left(1 - \frac{1}{\omega^2} \frac{g}{L} \frac{(m_1 + m_2)}{m_2}\right) & \omega > \sqrt{\frac{g}{L} \frac{m_1 + m_2}{m_2}} \\ 0 & \text{else} \end{cases}$$

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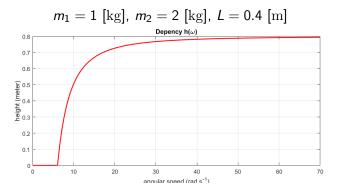
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# Small Oscillations around Steady-State

$$\theta_e = 0$$
  $\theta_e = \arccos\left(\frac{1}{\omega^2}\frac{g}{L}\frac{(m_1 + m_2)}{m_2}\right)$ 

Approximation with spring-mass-equation

$$m\ddot{x} = -kx$$

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$$\Delta \ddot{\theta} = \frac{m_2 \cos(2\theta_e) \omega^2 - \frac{g}{L} \cos(\theta_e) (m_1 + m_2)}{m_1 (1 - \cos(2\theta_e)) + m_2} \Delta \theta$$

$$\omega_e = \sqrt{\frac{m_2\omega^2(g^2(m_1 + m_2)^2 - L^2m_2^2\omega^4)}{2g^2m_1(m_1 + m_2)^2 - L^2m_2^2\omega^4(2m_1 + m_2)}}$$

Edge Case:  $m_1 = 0$ 

$$\omega_{\mathsf{e}} = \sqrt{rac{L^2 \omega^4 - g^2}{L^2 \omega^2}}$$

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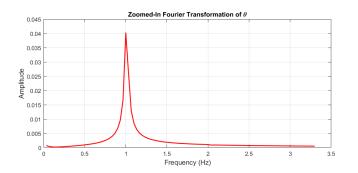
Energies Result Simulation

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$$m_1 = 1 \; [\mathrm{kg}], \; m_2 = 2 \; [\mathrm{kg}], \; L = 0.4 \; [\mathrm{m}], \; \omega = 3\pi \; \left[\frac{\mathrm{rad}}{\mathrm{s}}\right], \ x_0 = \begin{pmatrix} 1 & 0 \end{pmatrix}^T \; \mathsf{Matlab:} \; 1.01 \; [\mathrm{Hz}]$$



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Verification with simulation