CT-LIKE CONTROLLER WITH DISTURBANCE ESTIMATOR

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OUTLINE

- Introduction
- Decoupled CT-like controller
- Design of disturbance estimator
- Exercise

Introduction

- Computed torque like controller with disturbance estimator useful when (most of) the dynamic model is unknown.
- This chapter: Decoupled, very robust controller which works well for motion control of most of the robot mechanisms with gears (even with lower reduction ratio).
- PI disturbance estimator.

Decoupled CT-like controller

CT controller with proportional + derivative (PD) feedback :

$$\tau_c = M(q)\left(\ddot{q}_D + K_v\dot{e} + K_p e\right) + N(q,\dot{q}) =$$
$$= M(q)\ddot{q}_{calc} + N(q,\dot{q})$$

K_v and K_p are *nxn* matrices:

$K_v = diag\left(k_{v,i}\right)$, $K_p = diag\left(k_{p,i}\right)$

Decoupled CT-like controller

CT : $\tau_c = M(q)\ddot{q}_{calc} + N(q,\dot{q})$

If dynamic model is not well known conventional CT cannot be implemented.

Decoupled CT-like controller:

$$\tau_{c,i} = \bar{M}_{ii}\ddot{q}_{calc,i} + \hat{w}_i(\dot{q}_i, q_i)$$

$$\ddot{q}_{calc,i} = \ddot{q}_{D,i} + K_{v,i}\dot{e}_i + K_{p,i}e_i$$

- $\hat{w}_i(\dot{q}_i, q_i)$ is estimated disturbance (unknown dynamics) of i-th robot joint;
- $\bar{M}_{ii}$ estimated or average inertia of i-th robot joint.

What are advantages of decoupled controllers?

Design of disturbance estimator

$$\tau_{c,i} = \bar{M}_{ii}\ddot{q}_{calc,i} + \hat{w}_i(\dot{q}_i, q_i)$$

PI disturbance estimator:

$$\hat{w}_i(\dot{q}_i, q_i) = l_i\left(\dot{q}_{calc,i} - \dot{q}_i\right)$$

l_i estimator parameter for i-th robot joint

Error dynamics for decoupled control:

$$E_i(s) = Q_i^d(s) - Q_i(s) = \frac{\frac{1}{\bar{\bar{M}}_{ii}} \cdot s}{\left(s^2 + K_{v,i} \cdot s + K_{p,i}\right) \cdot \left(s + \frac{l_i}{\bar{\bar{M}}_{ii}}\right)} \cdot W_i(s)$$

Design of disturbance estimator

Error dynamics for decoupled control:

$$E_i(s) = Q_i^d(s) - Q_i(s) = \frac{\frac{1}{\overline{\overline{M}}_{ii}} \cdot s}{\left(s^2 + K_{v,i} \cdot s + K_{p,i}\right) \cdot \left(s + \frac{l_i}{\overline{\overline{M}}_{ii}} \right)} \cdot W_i(s)$$

Which part of transfer function determines time response?

The controller's poles are:

$$p_1 = \omega_{estim} = -\frac{l_i}{\overline{\overline{M}}_{ii}},$$

$$p_{2,3} = \frac{-K_{v,i} \pm \sqrt{K_{v,i}^2 - 4 \cdot K_{p,i}}}{2}.$$

$$K_{p,i} = \omega_{n,i}^2$$
$$K_{v,i} = 2\xi_i \omega_{n,i}$$

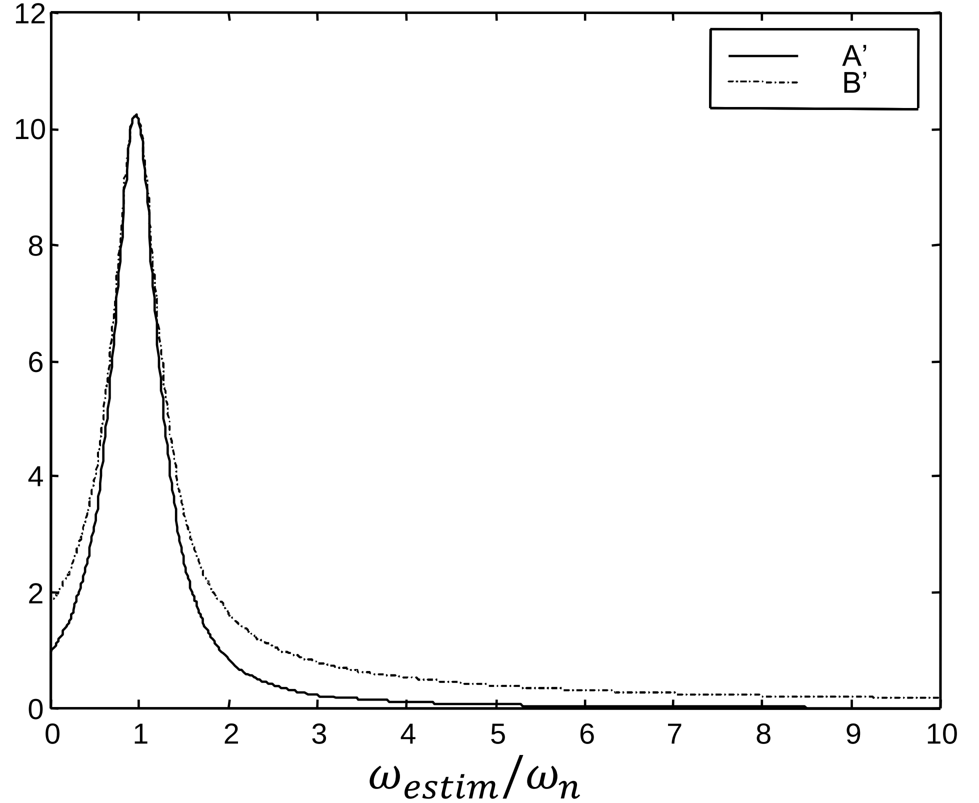
Design of disturbance estimator

Position error in the time domain for the damping ξ <1 and impulse disturbance *W(s)=1* (all frequencies included):

$$e(t) = Ae^{-\omega_{estim}t} + Be^{-\xi\omega_n t} cos\left(\omega_n\sqrt{1-\xi^2}t + arctg(\upsilon)\right)$$

$$A = \frac{1}{\overline{\overline{M}}_{ii}} \cdot \frac{1}{\omega_n^2} \cdot A'$$

$$B = \frac{1}{\overline{\overline{M}}_{ii}} \cdot \frac{1}{\omega_n^2} \cdot B'$$



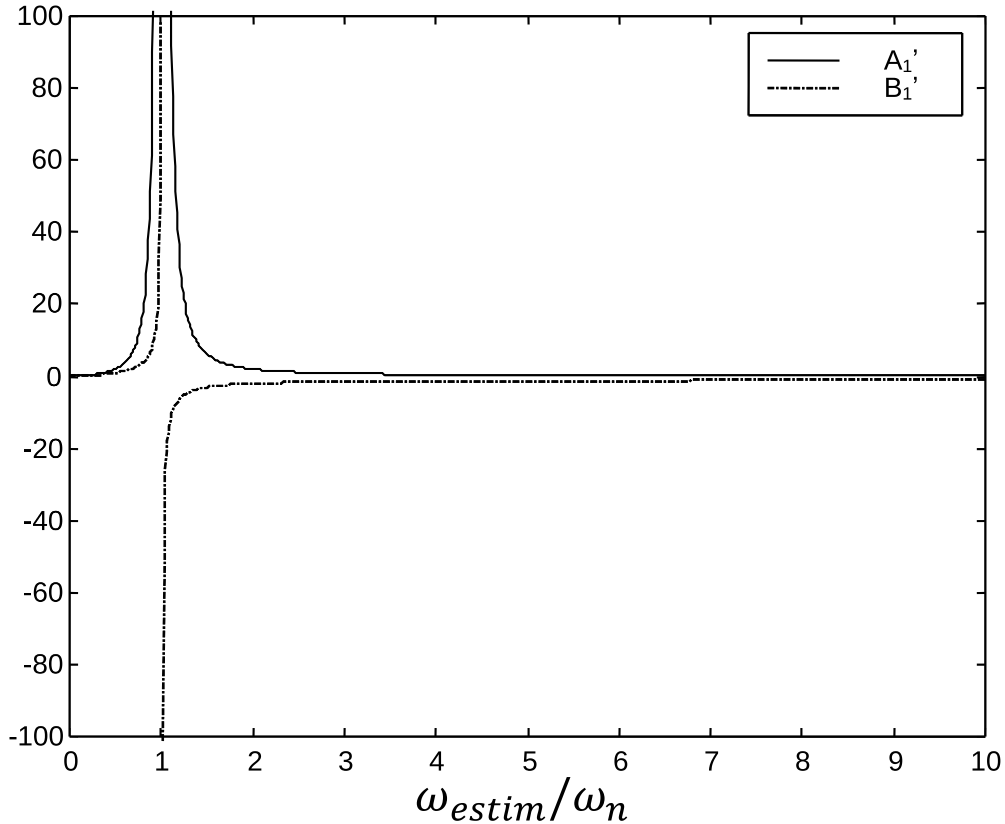
Design of disturbance estimator

Position error in the time domain for the damping ξ =1 and impulse disturbance *W(s)=1*

$$e(t) = -A_1\, e^{-\omega_{estim} t} + (A_1 + B_1 t) e^{-\omega_n t}$$

$$A = \frac{1}{\overline{\overline{M}}_{ii}} \cdot \frac{1}{\omega_n^2} \cdot A'$$

$$B = \frac{1}{\overline{\overline{M}}_{ii}} \cdot \frac{1}{\omega_n^2} \cdot B'$$



Design of disturbance estimator

$$A = \frac{1}{\overline{M}_{ii}} \cdot \frac{1}{\omega_n^2} \cdot A'$$

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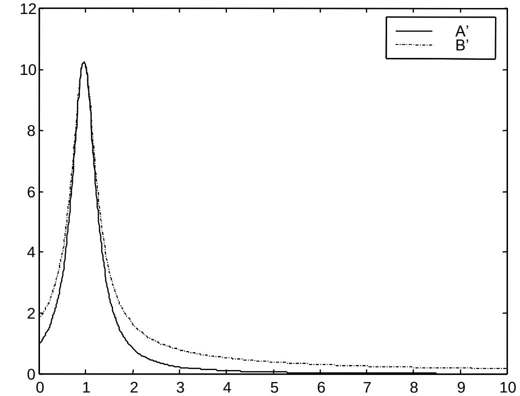
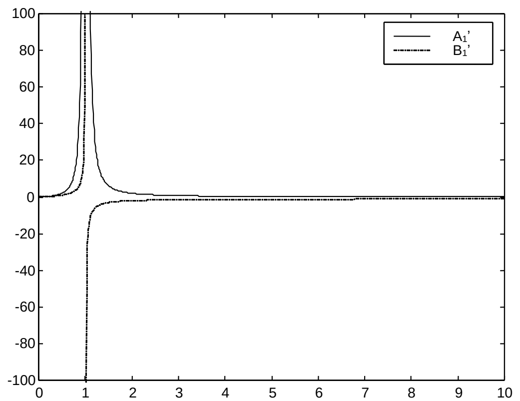
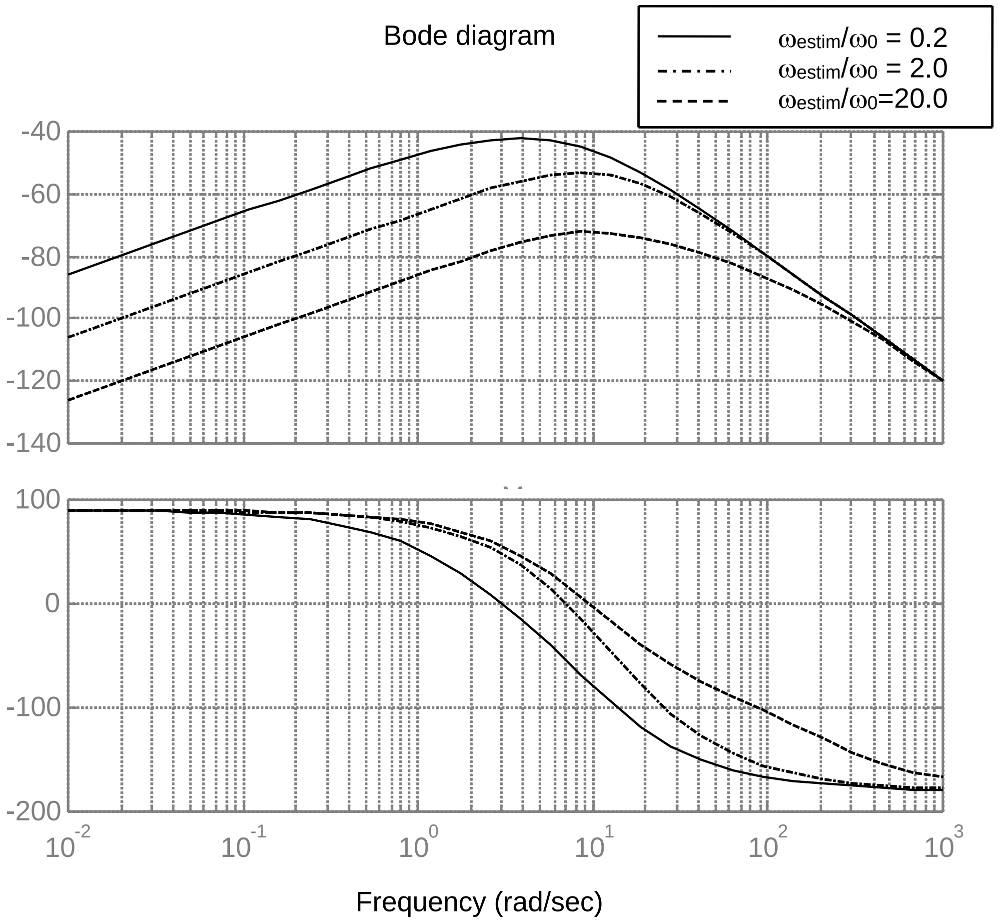




Conclusions:

- Peak position error decreases with increasing $\bar{M}_{ii}$.
- Increasing pole ratio ω_{estim}/ω_n also decreases peak position error.
- $\dfrac{\omega_{estim}}{\omega_n} = 2$ should be already fine.
- Method to position estimator poles: choosing suitable values of parameters l_i .

Design of disturbance estimator

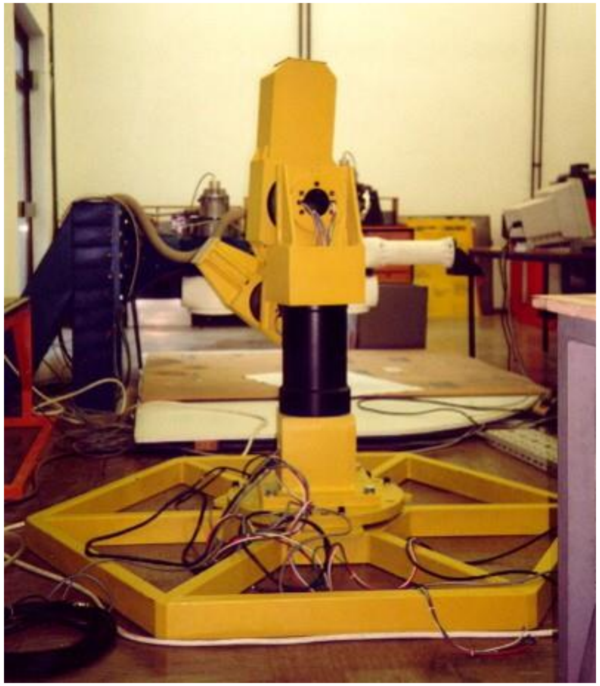


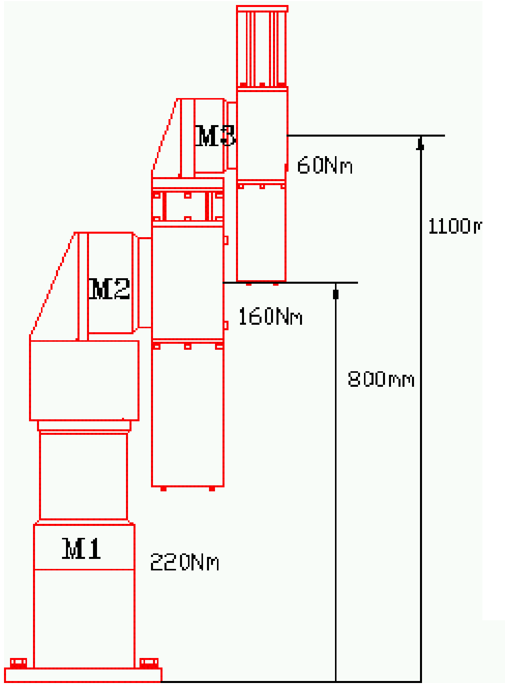
Conclusions

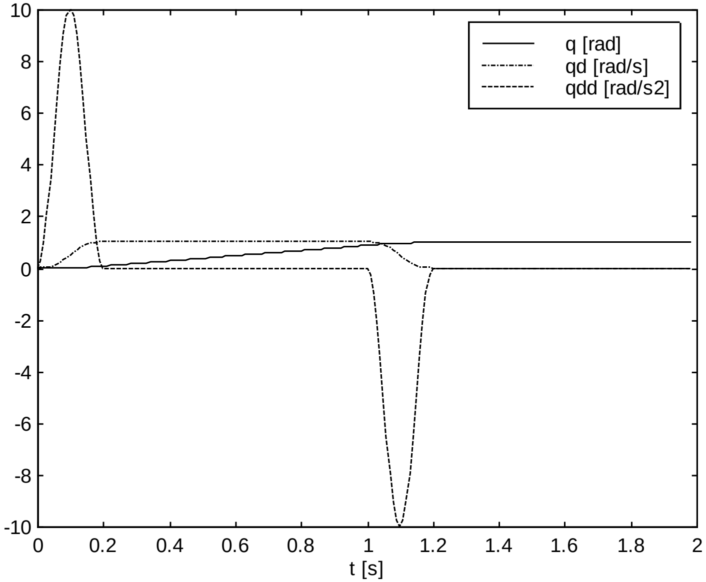
- Increasing pole ratio ω_{estim}/ω_n has a significant effect on suppressing of the low-pass frequency disturbances.
- No such action is observed in high-pass frequency domain.

Design of disturbance estimator

Experimental results 3-DOF direct drive robot

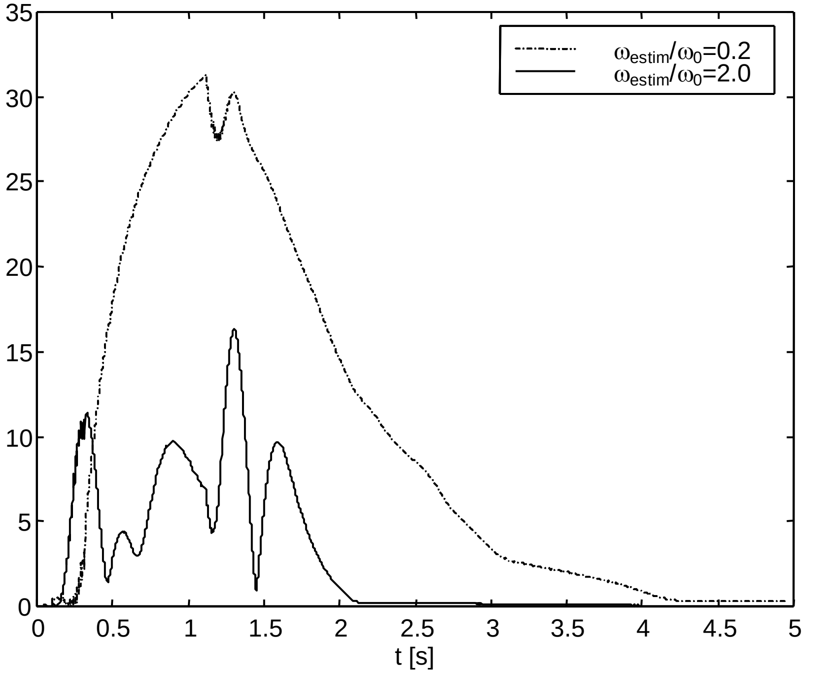




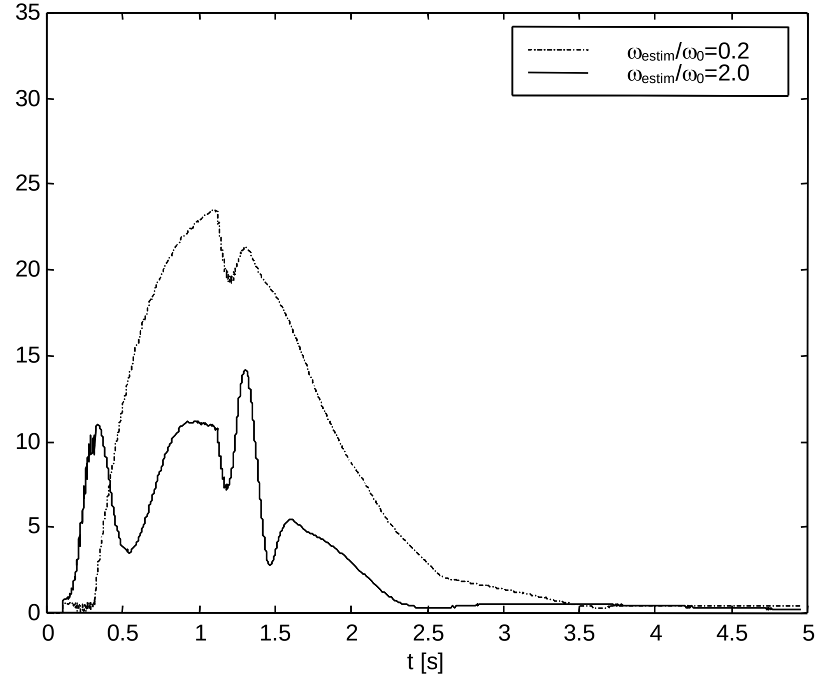


Reference trajectory, joint space

Design of disturbance estimator



Tracking error of robot tip [*mm*] in task space for *ξ=0.65*



Tracking error of robot tip [*mm*] in task space for *ξ=1*

The suitable location of the poles determined by the pole ratio $\omega_{estim}/\omega_n=2.0$ or more, and damping factor being near critical $\xi \cong 1$ result in good performance.

REPORT REQUIRED

Exercise: CT-like controller with PI disturbance estimator

- Draw the controller scheme.
- Design the controller for 2-DOF robot and realize it by simulation in MATLAB/Simulink.
- Test the performance for different parameters to confirm the efficiency of the controller and the results of analysis concerning the parameter's influence.