ROBUST CONTROL
SLIDING MODE CONTROL

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OUTLINE

- Introduction
- Sliding mode control
- Sliding mode robot controller
- Summary
- Exercise: Sliding mode controller

Introduction

- CT control relies on the knowledge of robot dynamics and is sensitive to (large) errors in estimation of its parameters and structure.
- Robust controllers are designed not to be so sensitive to these estimation errors.
- Many different robust controllers exist (based on feedback linearization, nonlinear controllers, variable structure controller). Mostly they are based on Lypunov design.
- We will address **variable structure systems and application of sliding mode control** for a robot motion control.

Introduction

- **Variable structure (VS) systems theory and sliding mode (SM)** control was developed in Soviet Union.
- It was first presented by Emelynov in 1959 but became known in 1976 after a book by Itkis and (in 1977) a survey paper by Utkin were published in English.
- Today VS/SM theory is used in all kinds of control design (model based, estimators, adaptive controllers, soft computing controllers...) and hybrid controllers design. Many publications which deal with a speciall classes of sliding mode control systems are available.

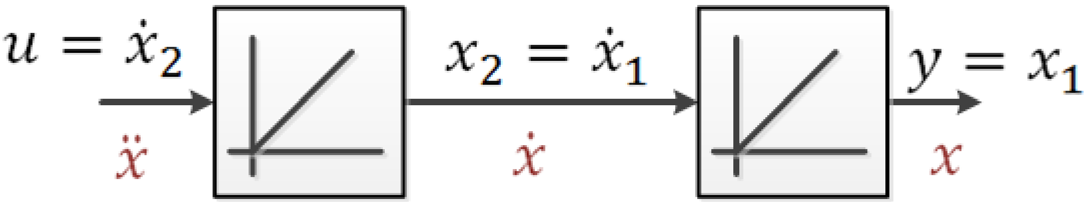
Introduction

Variable structure systems

- VS system has dynamic $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t)$ where $\boldsymbol{f}(\boldsymbol{x}, t)$ has discontinuoties with respect of some of its arguments.
- We have such situations in physics and directly in engineering in electric motors and power electronics converters.
- Control law is naturally discontinous = state dependent switching feedback control (bang-bang control, relay control).

Sliding mode control

SM concept for double integrator



SM condition 1: System motion on sliding manifold is described by equations of reduced order.

SM condition 2: System states, once in SM, should move toward zero.

Proposition: Conditions 1 and 2 are fulfilled by the following choice of sliding manifold (sliding line for 2nd order system): $s = cx + \dot{x} = cx_1 + \dot{x}_1$
However we need to find control law that will drive the system stated to sliding manifold.

Proposition: Following control law (switching function) will force the system's states to the sliding line and keep them there: $u = \begin{cases} u^+, s > 0 \\ u^-, s > 0 \end{cases}$

respectively

$$u = -u_0 sgn(s)$$

Sliding mode control

SM concept for double integrator

$$\ddot{x} = u,$$
$$u = -u_0 \operatorname{sgn}(s), \quad s = cx + \dot{x}$$

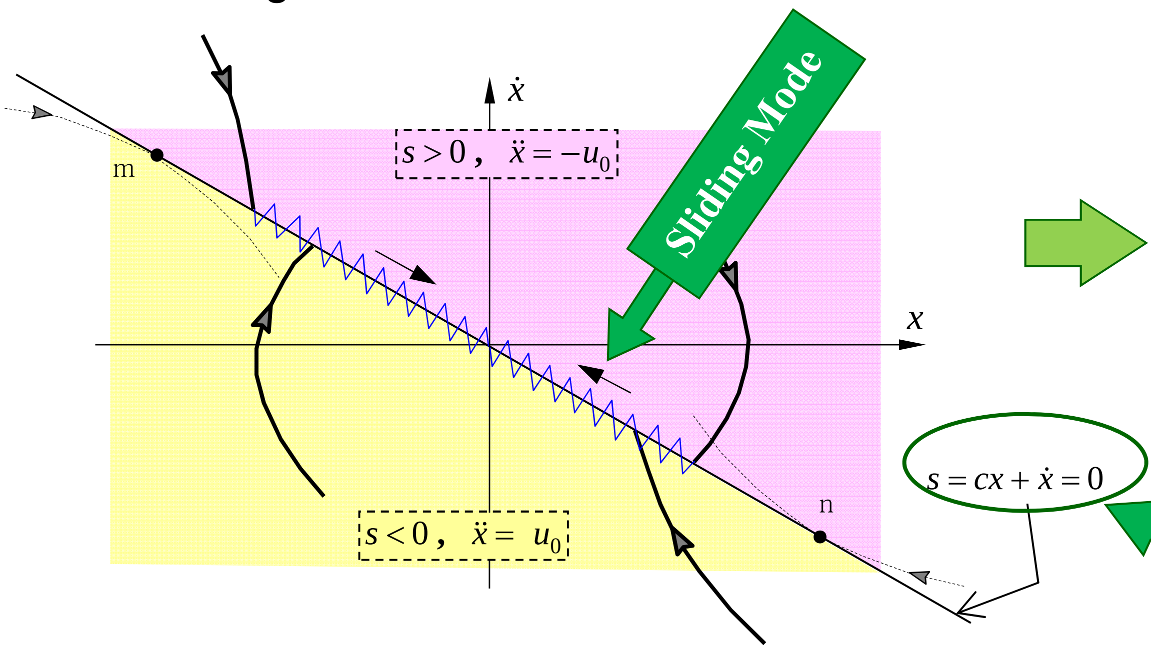
Upper half-plane : $s>0 \rightarrow u=-u_0 \rightarrow \ddot{x}=-u_0$

Lower half-plane : $s<0 \rightarrow u=u_0 \rightarrow \ddot{x}=u_0$

1. State trajectories are towards the sliding line *s=0*

2. State trajectories are on to the sliding line *s=0* SLIDING MODE

3. In SM regime motion is toward zero $s = cx + \dot{x} = 0$ SLIDING MODE EQUATION



In SM:

- o System motion is described by 1st order equation (reduced order).
- o System motion depends only on sliding line equation and not on the system dynamics.

Sliding mode control

- SMC design has two phases:
  - Selection of stable sliding manifold(s) in the state/error space on which the motion of the system in SM regimee should be restricted. Sliding manifolds govern system dynamics in SM. They should be asimptoticaly stable so that system‘s states/errors reach the origin of state/space.
  - Design of switched control that will drive the states/errors to the sliding manifold and keep them there (Lyapunov approach).

Sliding mode control

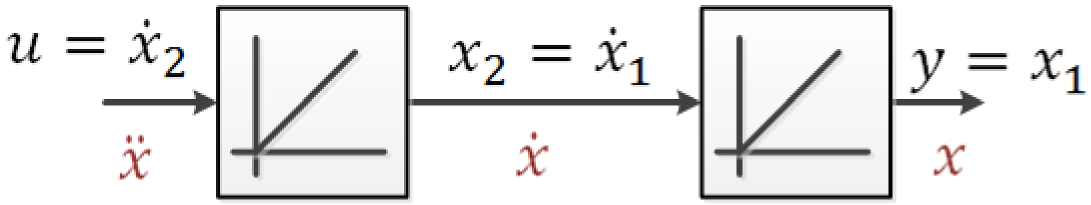
- Motion of the system‘s states/errors, when starting from non-zero conditions has two phases:
  - Reaching phase where the system‘s states/errors are pointed toward sliding manifold(s).
  - Sliding mode, system motion is restricted to sliding manifold(s), stays there and moves toward equilibrium state.

Sliding mode control

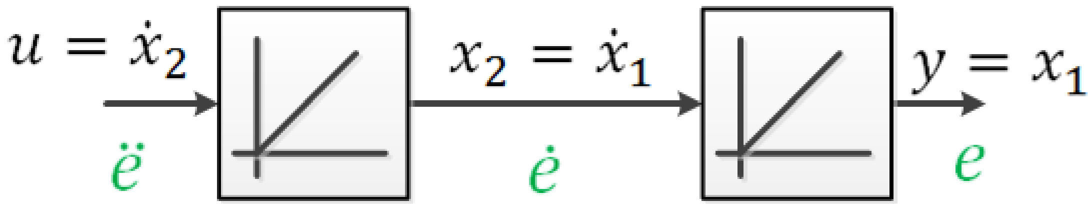
- Most important property of system in SM is invariance and robustness to bounded uncertainties such as modeling errors (unknown parameters and structure) and external disturbances.
- VSC assumes infinitely fast switching between different control structures which results in SM but also in chattering.

Sliding mode control

We had this:



Think about this:



Sliding mode control

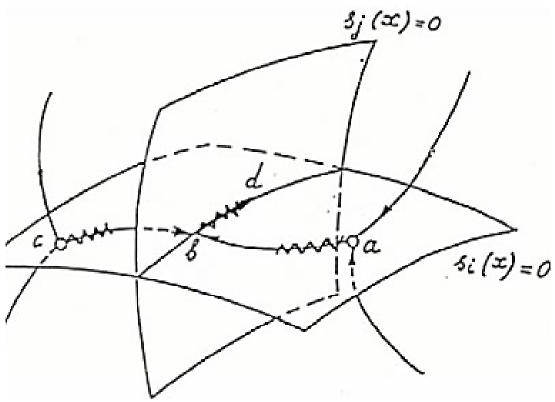
- Nonlinear high-order system ($\boldsymbol{x} \in \Re^n, \boldsymbol{u} \in \Re^m$) with control:

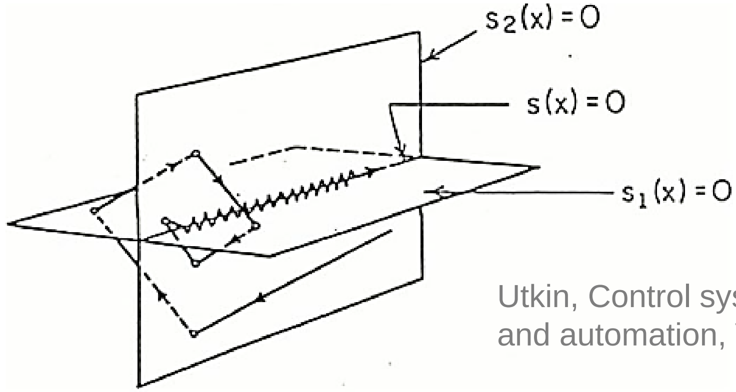
$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t) + B(\boldsymbol{x}, t)\boldsymbol{u}$$

$$u_i = \begin{cases} u^+(\boldsymbol{x}, t), s_i(\boldsymbol{x}) > 0 \\ u^-(\boldsymbol{x}, t), s_i(\boldsymbol{x}) < 0 \end{cases}, i = 1..m$$

- Sliding manifolds $s_i(\boldsymbol{x})$ are continuous functions of $\boldsymbol{x} \in R^n$.

States' velocity vectors point toward sliding manifolds and SM arises along it and/or at intersections.





Utkin, Control systems, robotics and automation, Vol. XIII

General: SM exists on intersection of all manifolds $s_i = 0$ or in the manifold $\boldsymbol{s}(\boldsymbol{x}) = \boldsymbol{0} \in \Re^{n-m}$ where $\boldsymbol{s}^T(\boldsymbol{x}) = [s_1(\boldsymbol{x}), .. s_m(\boldsymbol{x})]$

Sliding mode control

Lyapunov based approach for deriving control law

1. For $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t) + B(\boldsymbol{x}, t)\boldsymbol{u}$ define sliding manifold $\boldsymbol{s}(\boldsymbol{x})^T = [s_1(\boldsymbol{x}), .. s_m(\boldsymbol{x})]$
2. Choose positive definite Lyapunov function:
$$V(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{s}(\boldsymbol{x}) > 0$$
3. Calculate derivative $\dot{V}(\boldsymbol{x}) = \frac{\partial V(\boldsymbol{x})}{\partial s}\dot{\boldsymbol{s}}(\boldsymbol{x}) = \boldsymbol{s}(\boldsymbol{x})^T\dot{\boldsymbol{s}}(\boldsymbol{x})$
$$\dot{\boldsymbol{s}}(\boldsymbol{x}) = \frac{\partial s(\boldsymbol{x})}{\partial x}[\boldsymbol{f}(\boldsymbol{x}, t) + B(\boldsymbol{x}, t)\boldsymbol{u}(t)]$$
$$\dot{V}(\boldsymbol{x}) = \boldsymbol{s}(\boldsymbol{x})^T\frac{\partial s(\boldsymbol{x})}{\partial x}[\boldsymbol{f}(\boldsymbol{x}, t) + B(\boldsymbol{x}, t)\boldsymbol{u}(t)]$$
4. Choose $\boldsymbol{u}(t)$ so that $\dot{V}(\boldsymbol{x})$ is negative definite.

Sliding mode control

Lyapunov based approach for deriving control law

Control law is usually composed of two parts:

$$\boldsymbol{u}(t) = \boldsymbol{u_{eq}}(t) + \boldsymbol{u_c}(t)$$

- Corrective control that makes sure that the system‘s states/error reach sliding manifold. It is usually chosen as $\boldsymbol{u_c}(t) = -U(\boldsymbol{x})sgn(\boldsymbol{s})$. Its amplitude $U(\boldsymbol{x})$ is chosen by estimationg the upper bound on uncertainities.
- Equivalent control $\boldsymbol{u}_{eq}(t)$ makes the system‘s states/errors stay in SM once there. For $\boldsymbol{s}(\boldsymbol{x})^T = [s_1(\boldsymbol{x}), .. s_m(\boldsymbol{x})]$; $\boldsymbol{s}(\boldsymbol{x}) = \boldsymbol{S}\boldsymbol{x}$; $S \in \Re^{mxn}$ it can be shown that suitable equivalent control is:

$$\boldsymbol{u_{eq}}(t) = -(SB)^{-1}SA\boldsymbol{x}$$

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Sliding mode control

SM controller [Slotine 1985]

Theorem:

For SM controller

$$\tau = \hat{M}\ddot{q}_s + \hat{V}_m\dot{q}_s + \hat{G} + Ksgn(s);\ \dot{q}_s = \Lambda e + \dot{q}_d$$

with sliding surface $s = \Lambda e + \dot{e}$ and

$$K = diag[k_1, .., k_n], k_i > 0, \Lambda = diag[\lambda_1, .. \lambda_n], \lambda_i > 0$$

$$sgn(s) = [sgn(s_1), .. sgn(s_n)]^T; \quad sgn(s_i) = \begin{cases} 1, s_i > 0 \\ -1, s_i < 0 \end{cases}$$

error will reach the sliding surface in finite time. Furthermore, once on surface, $q(t)$ will converge to $q_d(t)$ exponentially.

Sliding mode control

SM controller [Slotine 1985]

Proof: Following Lypunov function candidate is considered:

$$V(s) = \frac{1}{2} s^T M(q) s$$

$$s = \Lambda e + \dot{e}$$
$$\dot{s} = \Lambda \dot{e} + \ddot{e}$$
$$\dot{q}_s = \Lambda e + \dot{q}_d$$

Its derivative is:

$$\dot{V}(s) = s^T \left(M(q)\dot{s} + \frac{1}{2}\dot{M}(q)s \right)$$

By substituting $\dot{q}_s$, $\dot{s}$ and using skew-symmetry of $\dot{M}(q) - 2V_m(q, \dot{q})$:

$$\dot{V}(s) = s^T (M(q)\ddot{q}_s + V_m \dot{q}_s + G - \tau)$$

By substituting controller $\tau = \widehat{M}\ddot{q}_s + \hat{V}_m \dot{q}_s + \hat{G} + Ksgn(s)$:

$$\dot{V}(s) = s^T \left(\widetilde{M}(q)\ddot{q}_s + \tilde{V}_m \dot{q}_s + \tilde{G}\right) - \sum_{i=1}^{n} k_i |s_i|$$

Where: $\widetilde{M} = M - \widehat{M}$, $\tilde{V}_m = V_m - \hat{V}_m$, $\tilde{G} = G - \hat{G}$

Sliding mode control

SM controller [Slotine 1985]

$$\dot{V}(s) = s^T\left(\tilde{M}(q)\ddot{q}_s + \tilde{V}_m\dot{q}_s + \tilde{G}\right) - \sum_{i=1}^{n} k_i|s_i|$$

When is this negative definite?

For

$$k_i \geq \left|\left[\tilde{M}(q)\ddot{q}_s + \tilde{V}_m\dot{q}_s + \tilde{G}\right]_i\right| + \eta_i; \quad \eta_i > 0$$

Lypunov function derivative is

$$\dot{V}(s) \leq -\sum_{i=1}^{n} \eta_i|s_i|$$

This means that sliding surface is reached in finite time and error converges exponentially fast to zero.

Sliding mode control

2DOF planar robot

$$\tau = \widehat{M}\ddot{q}_s + \widehat{V}_m\dot{q}_s + \widehat{G} + Ksgn(s);$$

$$\dot{q}_s = \Lambda e + \dot{q}_d$$

Parameters:

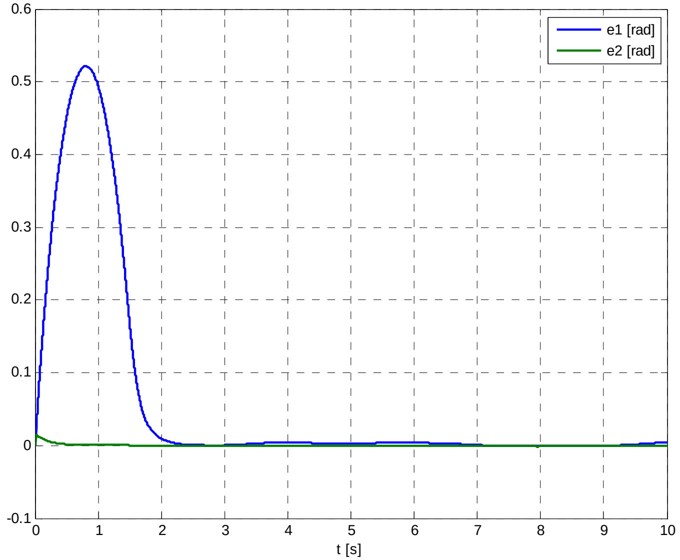
$M_{est} = 0.25M$

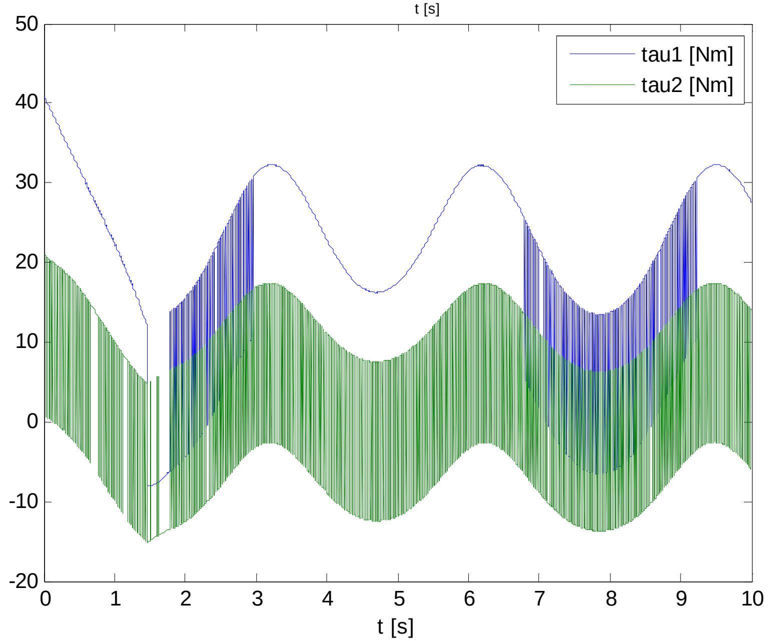
$V_{m\_est} = 0.75V_m$

$G_{est} = 0.75G$

$q_d = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}$ $K =$

$10I, \Lambda = 5I$

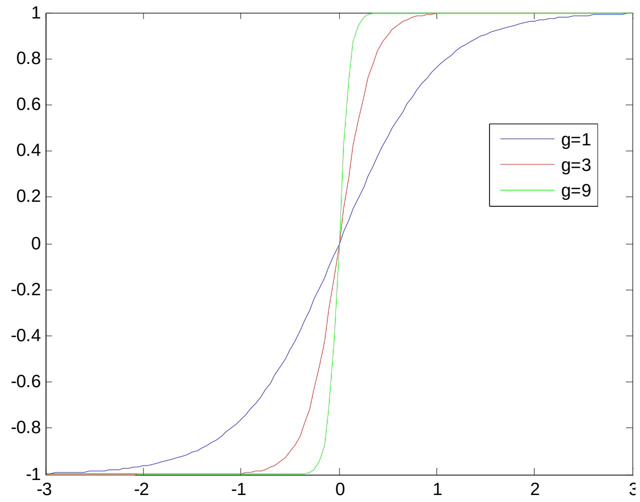




Sliding mode control

Chattering

- In theory the switching frequency is infinite. In practice it is only high.
- High frequency chattering may excite unmodeled high frequency dynamics.
- A common solution is introduction of boundary layer where instead of *sign* function another function is used, like saturation function of $tanh(gs)$.
- By using boundary layer asymptotic stability of control is sacrificed!



Summary

- SM is a useful tool for controlling uncertain dynamical systems. Exact parameters of dynamic model are not required, only bounds on these parameters are enough.
- Sliding modes are a usual behavior in switching systems. In sliding mode trajectory tracking error doesn’t depends on the dynamic’s parameters.

Summary

- Ideal SM is not implemented in practice because infinite frequency switching is required and only an approximate sliding can be achieved.
- Suggested further reading on topics: Shtessel, Y., Edwards, C., Fridman, L., Levant, A., ‘Sliding Mode Control and Observation’, Springer, 2013

REPORT REQUIRED

Exercise: Sliding mode controller

- Summarize sliding mode robot controller.
- By simulation check the performance of SM controller for 2DOF planar robot with masses *1 kg* and links' lengths *1 m*. $q_d = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}$. $M_{est} = 0.25M, V_{m\_est} = 0.75V_m, G_{est} = 0.75G$. Observe responses $(e, \dot{e}, \tau)$ for different parameters and comment the results. Investigate the influence of Λ.
- $M_{est} = 0.25M, V_{m\_est} = 0.75V_m, G_{est} = 0$. Adjust parameters, observe responses $(e, \dot{e}, \tau)$ and comment the results.

REPORT REQUIRED

Exercise: Sliding mode controller

- Repeat the simulations from previous exercise with introduction of the boundary layer in the SM controller.
- Initially use the same controller parameters, then adjust them for better results.
- Comment the results.