Heat exchanger model

A flow model consisting of mass and energy balances

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Model equations and initial condition

- The fuel flows through a cylindrical tube at constant velocity. The position of the left end is z_0 and that of the right end is z_f.
- The coolant flows in the opposite direction in a cylindrical tube covering the inner fuel tube.
- The fuel and coolant are not in direct contact: Heat is only exchanged through the tubing material separating the two fluids.
- We account for the fuel and coolant flows, but we disregard conduction in the tubing as well as losses to the surroundings.
- The inlet fuel and coolant temperatures and concentrations (essentially, identical to the densities) are known.

Model equations

- Fuel and coolant molar fluxes (steady state and homogeneous in space)

$$N^f(t,z) = v^f c^f, \tag{1a}$$
$$N^c(t,z) = v^c c^c \tag{1b}$$

- Fuel, tubing, and coolant energy balances

$$\partial_t u^f = -\partial_z H(T^f, P^f, N^f) + Q^{tf}, \tag{2a}$$
$$\partial_t u^t = Q^{ft} + Q^{ct}, \tag{2b}$$
$$\partial_t u^c = -\partial_z H(T^c, P^c, N^c) + Q^{tc} \tag{2c}$$

- Heat transfers (Newton's law of cooling)

$$Q^{tf} = -k_{tf}(T^f - T^t), \qquad Q^{ft} = -Q^{tf}, \tag{3a}$$
$$Q^{tc} = -k_{tc}(T^c - T^t), \qquad Q^{ct} = -Q^{tc} \tag{3b}$$

- Boundary conditions etc.

$$T^f(t,z_0) = T^f_{in}, \qquad T^c(t,z_f) = T^c_{in}, \qquad \operatorname{sgn} v^f = -\operatorname{sgn} v^c \tag{4}$$

Thermodynamics

- Internal energies (the product of concentration, molar specific heat capacity at constant pressure, and temperature)

$$u^f = c^f c_P^f T^f, \qquad u^t = c^t c_P^t T^t, \qquad u^c = c^c c_P^c T^c \tag{5}$$

- Pure component enthalpies

$$H^f(T,P,n) = n\,h^f(T,P), \quad h^f(T,P) = h^f(T_0,P_0) + c_P^f(T - T_0), \tag{6a}$$
$$H^c(T,P,n) = n\,h^c(T,P), \quad h^c(T,P) = h^c(T_0,P_0) + c_P^c(T - T_0) \tag{6b}$$

- Partial derivative of enthalpy (N is molar flux)

$$\partial_z H(T,P,N) = \underbrace{\partial_z N}_{=0}\, h(T,P) + N\partial_z h(T,P), \quad \partial_z h(T,P) = c_P \partial_z T \tag{7}$$

Bibliography

[1] F. L. Hinton and R. D. Hazeltine, “Theory of plasma transport in toroidal confinement systems,” *Reviews of Modern Physics*, vol. 48, no. 2, pp. 239–308, 1976.