

# Model equations and initial condition

- ▶ The fuel flows through a cylindrical tube at constant velocity. The position of the left end is  $z_0$  and that of the right end is  $z_f$ .
- ▶ The coolant flows in the opposite direction in a cylindrical tube covering the inner fuel tube.
- ▶ The fuel and coolant are not in direct contact: Heat is only exchanged through the tubing material separating the two fluids.
- ▶ We account for the fuel and coolant flows, but we disregard conduction in the tubing as well as losses to the surroundings.
- ▶ The inlet fuel and coolant temperatures and concentrations (essentially, identical to the densities) are known.

# Model equations

- Fuel and coolant molar fluxes (steady state and homogeneous in space)

$$N^f(t, z) = v^f c^f, \quad (1a)$$

$$N^c(t, z) = v^c c^c \quad (1b)$$

- Fuel, tubing, and coolant energy balances

$$\partial_t u^f = -\partial_z H(T^f, P^f, N^f) + Q^{tf}, \quad (2a)$$

$$\partial_t u^t = Q^{ft} + Q^{ct}, \quad (2b)$$

$$\partial_t u^c = -\partial_z H(T^c, P^c, N^c) + Q^{tc} \quad (2c)$$

- Heat transfers (Newton's law of cooling)

$$Q^{tf} = -k_{tf}(T^f - T^t), \quad Q^{ft} = -Q^{tf}, \quad (3a)$$

$$Q^{tc} = -k_{tc}(T^c - T^t), \quad Q^{ct} = -Q^{tc} \quad (3b)$$

- Boundary conditions etc.

$$T^f(t, z_0) = T_{in}^f, \quad T^c(t, z_f) = T_{in}^c, \quad \text{sgn } v^f = -\text{sgn } v^c \quad (4)$$

# Thermodynamics

- Internal energies (the product of concentration, molar specific heat capacity at constant pressure, and temperature)

$$\partial_t u^f = c^f c_P^f \partial_t T^f, \quad \partial_t u^t = c^t c_P^t \partial_t T^t, \quad \partial_t u^c = c^c c_P^c \partial_t T^c \quad (5)$$

- Pure component enthalpies

$$H^f(T, P, n) = n h^f(T, P), \quad h^f(T, P) = h^f(T_0, P_0) + c_P^f(T - T_0), \quad (6a)$$

$$H^c(T, P, n) = n h^c(T, P), \quad h^c(T, P) = h^c(T_0, P_0) + c_P^c(T - T_0) \quad (6b)$$

- Partial derivative of enthalpy ( $N$  is molar flux)

$$\partial_z H(T, P, N) = \underbrace{\partial_z N}_{=0} h(T, P) + N \partial_z h(T, P), \quad \partial_z h(T, P) = c_P \partial_z T \quad (7)$$

# Bibliography