Laboratory 4

Introductory to analytic combinatorics course at Wroclaw University of Science and Technology 2020/2021

Deadline: 21.12.2020

Exercise 1 (2 points)

What is the number of permutations of 30-element set that decompose into cycles of length at most 5?

We can describe any permutation as set of cycles. So permutations that decompose into cycles of length at most 5 is simply a set of cycles of size 1, 2, 3, 4 or 5. The EGF function for such permutation is exp(z/1+ ... + z^r/r).

```
P[r_,n_] := n! * SeriesCoefficient[Exp[Sum[z^i/i,{i,1,r}]],{z,0,n}]
```

In[∘]:= `P[5, 30]`

Out[∘]= 19 054 894 203 999 260 640 645 120 000

Exercise 2 (2 points)

What is the number of permutations of 30-element set that decompose into 5 or less cycles?

Such problem is described by Stirling cycle numbers. Number of permutation of size n that decompose into r cycles is
P^(r)_n = n! / r! [z^n](log 1/(1-z))^r.
So to get permutations decomposing into 5 or less cycles we need to calculate P^(5)_n + P^(4)_n + P^(3)_n + P^(2)_nP^(1)_n.
Let's define function P^(r)_n and then answer stated question.

```
P [r_,n_] := n!/r! *SeriesCoefficient[(Log [1/(1-z)])^r,{z,0,n}]
P[5,30]+P[4,30]+P[3,30]+P[2,30]+P[1,30]
N[P[5,30]+P[4,30]+P[3,30]+P[2,30]+P[1,30]]
```

Out[∘]= 222 444 420 953 341 570 876 847 086 387 200

Out[∘]= 2.22444×10^{32}

Exercise 3 (2 points)