List 2, variant 15 - Tobiasz Wojnar

Digital Signal Processing 2024/25

University of Bielsko-Biala, semestr 1

Task

Generate three sine signals of given f_1, f_2, and f_3 and amplitude $|x[k]|_{max}$ for the sampling frequency fs in the range of $0 \leq k < N$.

Plot:

1. the "normalized" level of the DFT spectra.
2. the window DTFT spectra normalized to their mainlobe maximum. The intervals for f, Ω, and amplitudes should be chosen by yourself for the best interpretation purposes.

Interpret the results of the figures obtained regarding the best and worst case for the different windows. Why do the results for the signals with frequencies f_1 and $_2$ differ?

Data (Variant 15):

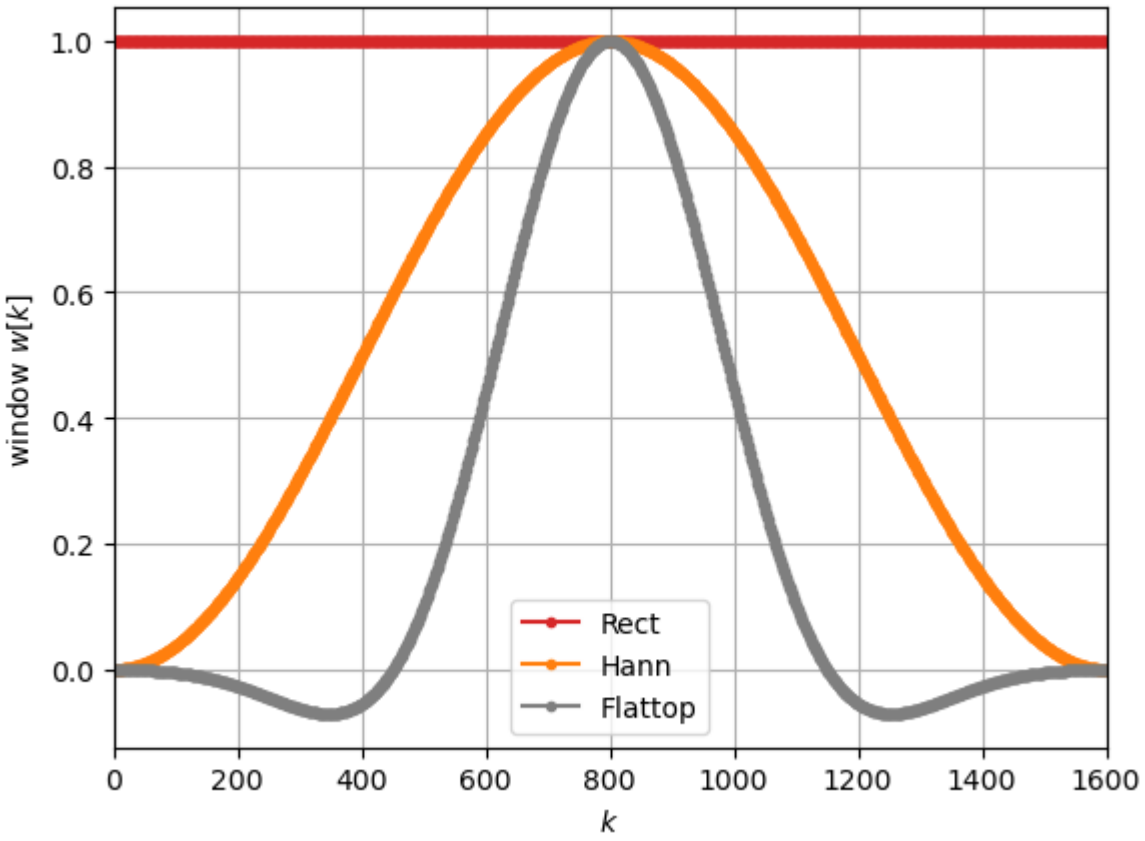
- $f_1 = 500$
- $f_2 = 500.25$
- $f_3 = 499.75$
- $|x[k]|_{max} = 4$
- $f_s = 800$
- $N = 2000$

In [85]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy.fft import fft, ifft, fftshift
from scipy.signal.windows import hann, flattop
```

In [118…
```
f1 = 500    # Hz
f2 = 500.25 # Hz
f3 = 499.75 # Hz
fs = 800 # Hz
N = 1600
k  = np.arange (N)
max_amplitude = 4

x1 = np.sin(2*np.pi * f1 / fs * k )*max_amplitude
x2 = np.sin(2*np.pi * f2 / fs * k )*max_amplitude
x3 = np.sin(2*np.pi * f2 / fs * k )*max_amplitude
```

In [119…
```
#Generate window functions
```

```
w_rect = np.ones(N)
w_hann = hann(N, sym=False)
w_flattop = flattop(N, sym=False)

plt.plot(w_rect, 'C3o-', ms=3, label='Rect')
plt.plot(w_hann, 'C1o-', ms=3, label='Hann')
plt.plot(w_flattop, 'C7o-', ms=3, label='Flattop')
plt.xlabel('$k$')
plt.ylabel('window $w[k]$')
plt.xlim(0,N)
plt.legend()
plt.grid()
```



In [120…

```
x1_w_rect = fft(x1 * w_rect)
x1_w_hann = fft(x1 * w_hann)
x1_w_flattop = fft(x1 * w_flattop)

x2_w_rect = fft(x2 * w_rect)
x2_w_hann = fft(x2 * w_hann)
x2_w_flattop = fft(x2 * w_flattop)

x3_w_rect = fft(x3 * w_rect)
x3_w_hann = fft(x3 * w_hann)
x3_w_flattop = fft(x3 * w_flattop)
```

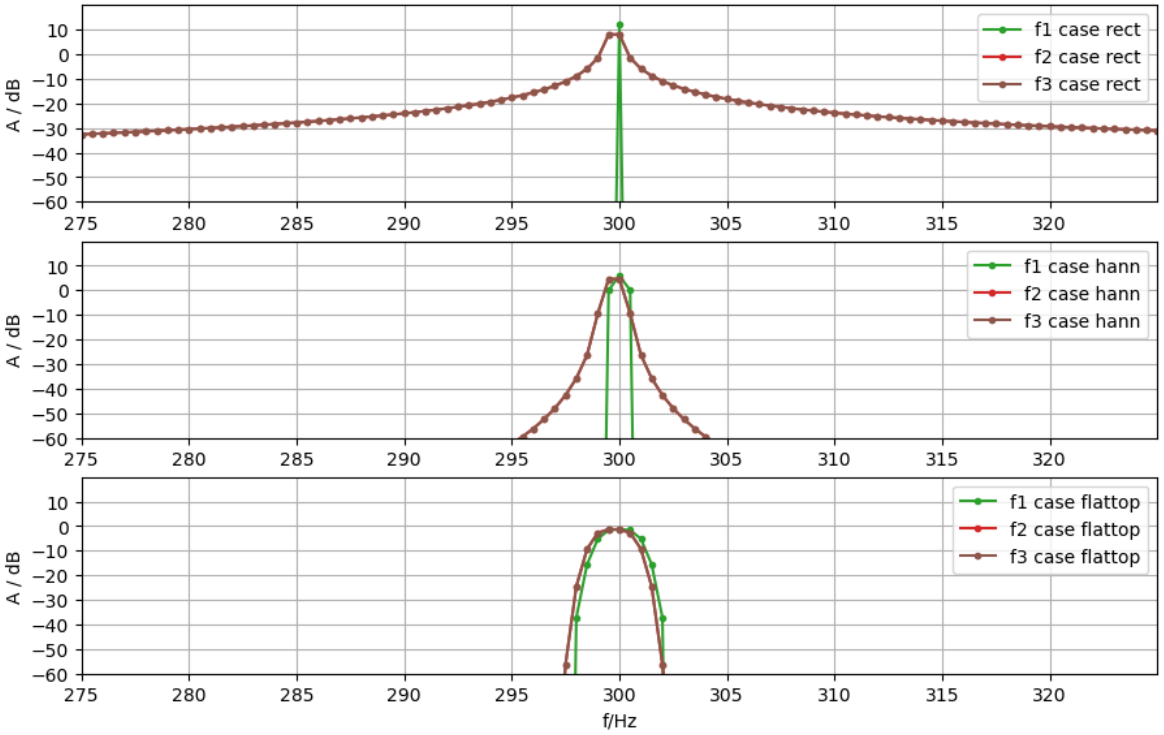
In [121…

```
def fft2db(X):
    N = X.size
    Xtmp = 2/N *X
    Xtmp [ 0 ] *= 1/2
    if N % 2 == 0 :
        Xtmp [N//2] = Xtmp [N//2] /2
    return 20*np.log10(np.abs(Xtmp))
```

In [122…
```
df = fs/N
f = np.arange(N)*df
```

In [137…
```
def plotComparison(x_1, x_2, x_3, window_name,plotNr):
    plt.subplot( 3, 1, plotNr)

    plt.plot(f, fft2db(x_1), 'C2o-', ms=3, label=f'f1 case {window_name}')
    plt.plot(f, fft2db(x_2), 'C3o-', ms=3, label=f'f2 case {window_name}')
    plt.plot(f, fft2db(x_3), 'C5o-', ms=3, label=f'f3 case {window_name}')
    plt.xlim(275,275+50)
    plt.ylim(-50,20)

    plt.xticks(np.arange(275,275+50,5))
    plt.yticks(np.arange(-60,20,10))

    plt.xlabel('f/Hz')
    plt.ylabel('A / dB')

    plt.legend()
    plt.grid()
```

In [138…
```
plt.figure(figsize=(16/1.5, 10/1.5))
plotComparison(x1_w_rect,x2_w_rect,x3_w_rect,'rect',1)
plotComparison(x1_w_hann,x2_w_hann,x3_w_hann,'hann',2)
plotComparison(x1_w_flattop,x2_w_flattop,x3_w_flattop,'flattop',3)

# We can see that f2 and f3 have when transform return the same values
```



In [139…
```
def winDTFTdB(w):
    N  = w.size
    Nz = 100*N
    W  = np.zeros(Nz)

    W[0:N] = w
    W  = np.abs( fftshift (fft(W)) )
    W /= np.max(W) # normalize
```

```
    W = 20*np.log10(W)
    Omega = 2*np.pi/Nz*np. arange(Nz) - np.pi
    return Omega, W
```

In [140…

```
# Check form here


plt.plot([-np.pi,+np.pi], [-3.01,-3.01],'gray')
plt.plot([-np.pi,+np.pi], [-13.3,-13.3],'gray')
plt.plot([-np.pi,+np.pi], [-31.5,-31.5],'gray')
plt.plot([-np.pi,+np.pi], [-93.6,-93.6],'gray')

Omega, W = winDTFTdB(w_rect)
plt.plot(Omega, W, label='rect')
Omega, W = winDTFTdB(w_hann)
plt.plot(Omega, W, label='hann')
Omega, W = winDTFTdB(w_flattop)
plt.plot(Omega, W, label='flattop')

# plt.xlim(-np.pi,np.pi)
plt.xlim(-np.pi/100,np.pi/100) # zoom into mainlobe
plt.ylim(-120,10)

plt.xlabel(r'$\Omega$')
plt.ylabel(r'|W($\Omega$)| / dB$')

plt.legend()
plt.grid(True)
```

```
/tmp/ipykernel_332/226293270.py:10: RuntimeWarning: divide by zero encountered in
log10
  W = 20*np.log10(W)
```