Aufgabe 2

4.

Bayes' Theorem:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Chain Rule of Probability:

$$P(A \wedge B) = P(A) \cdot P(B)$$

Conditional Independence Assumption:

$$P(A \wedge B|C) = P(A|C) \cdot P(B|C) \quad \text{if A and B are conditionally independent given C}$$

Goal: Estimating conditional probability of y

$$P(y = k|x_1, x_2, \ldots, x_n) = \frac{P(x_1, x_2, \ldots, x_n|y = k) \cdot P(y = k)}{P(x_1, x_2, \ldots, x_n)}$$

Expanding the numerator using conditional independence:

$$= \frac{P(x_1|y = k) \cdot P(x_n|y = k)}{P(x_1, x_2, \ldots, x_n)} \cdot P(y = k)$$

Using the product rule for conditional probabilities:

$$= \frac{1}{Z} P(y = k) \prod_{i=1}^{n} P(x_i|y = k)$$

with $Z = P(x) = P(x_1, x_2, \ldots, x_n)$.