Exercise 1.3

October 30, 2024

1 Formulate mathematically a loss function

$$\sum_{i<j} 1(y_i > y_j)$$

Boolean statements will evaluate to either 1 (True) or 0 (False). This loss function checks that each input is higher ranked than the following. Any correct rankings do not contribute to the loss of value, whereas incorrect ones do.

2 Which model is better at ranking? Why is the squared error problematic in this case?

The tuples that meet the condition $i < j$ are $(1, 2)$, $(1, 3)$ and $(2, 3)$, so these will be used in the summation.

For the first model $\hat{y}_1$, the loss function evaluates to the following.

$$1(1 > 3) + 1(1 > 2) + 1(3 > 2)$$
$$= 0 + 0 + 1$$
$$= 0 \quad (1)$$

For the second model $\hat{y}_2$, the loss function evaluates to the following.

$$1(2 > 3) + 1(2 > 7) + 1(3 > 7)$$
$$= 0 + 0 + 0$$
$$= 0 \quad (2)$$

So the second model $\hat{y}_2$ is better. Squared error is problematic because it would penalise $\hat{y}_2$ more for having a larger range despite it having a more correct ranking.