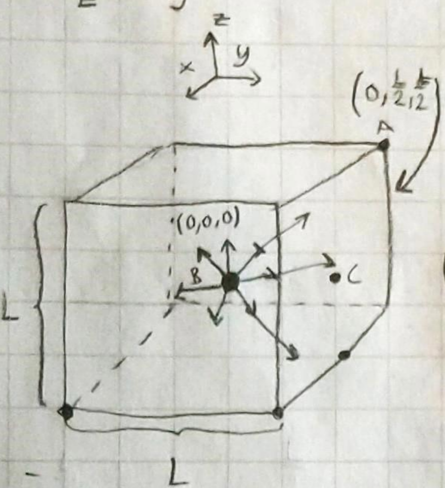
Project work, problem 2 mathematical derivations related to the code

$\frac{q}{\varepsilon_0} = \text{constant} = 1$

$d\bar{A} = \hat{n}\, dA = \hat{n}\, dx dy$

$\Phi_E = \oint E \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0}$, $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$ $L = \text{constant} = 1$

$\Phi_E = \int E \cdot dA$



Flux through this, $A = L \cdot L = L^2 = 1$

When calculating through a plane, one of x, y, z is constant ($-\frac{L}{2}$ or $\frac{L}{2}$), while the other two range from $-\frac{L}{2}$ to $\frac{L}{2}$.

$(0, \frac{L}{2}, 0)$

$(0, \frac{L}{2}, -\frac{L}{2})$

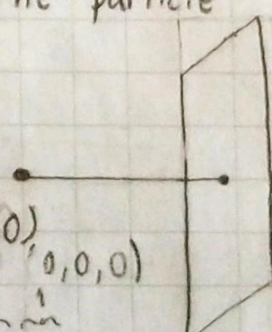
When the point charge is at the middle, the distance to the plane varies from $\frac{L}{2}$ to $\sqrt{(\frac{L}{2})^2 + (\frac{L}{2})^2 + (\frac{L}{2})^2} = \frac{\sqrt{3}}{2}$, $L = 1$

distance from the particle

$\bar{r} = \overline{BA} = -\overline{OB} + \overline{OA}$
$= -(0,0,0) + (0, \frac{L}{2}, \frac{L}{2})$
$= (0, \frac{L}{2}, \frac{L}{2})$

$r = \sqrt{x^2 + y^2 + z^2}$

$r^2 = x^2 + y^2 + z^2$



$\hat{r} = \frac{\bar{r}}{|\bar{r}|} = \frac{1}{\sqrt{0^2 + (\frac{L}{2})^2 + (\frac{L}{2})^2}} (0, \frac{L}{2}, \frac{L}{2})$, $\bar{n} = (1,0,0), (0,1,0)$ or $(0,0,1)$ $(0,0,0)$

$$\Phi_E = \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2 + \frac{L^2}{4} + z^2} \hat{r} \cdot \hat{n}\, dx dz$$, $\hat{r} \cdot \hat{n} = \underbrace{|\hat{r}|}_{1}\underbrace{|\hat{n}|}_{1} \cos(\theta) = \cos\theta$

$\theta \in [0, \frac{\pi}{4}]$

$$= \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{4\pi} \frac{1}{x^2 + \frac{1}{4} + z^2} \cos\theta\, dx dz$$

$$\Rightarrow \Phi_E = \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{4\pi} \frac{1}{x^2 + \frac{L^2}{4} + z^2} \text{dot}(\hat{r}, \hat{n})\, dx dz$$, Coordinate of the point charge and the plane point for the unit vectors