

1. Proyek Pembangunan gedung Perkuliahan dapat diselesaikan dalam waktu x hari dengan menghabiskan biaya proyek $(2x - 100 + \frac{3000}{x})$ ratus ribu rupiah. Biaya minimum proyek pembangunan gedung tersebut adalah?

Jawab

Total biaya

$$B(x) = x \cdot \left(2x - 100 + \frac{3000}{x} \right)$$

$$B(x) = 2x^2 - 100x + 3000$$

Biaya akan minimum saat $B'(x) = 0$

$$B'(x) = 4x - 100$$

$$0 = 4x - 100$$

$$4x = 100$$

$$x = 25$$

Jadi, biaya minimumnya adalah

$$B(25) = 2(25)^2 - 100(25) + 3000$$

$$= 1250 - 2500 + 3000$$

$$= 1750 \text{ ratus ribu rupiah}$$

$$= \text{Rp. } 175.000.000$$

2. $F(x, y) = 7x^3y^4 - 5x^6y^2$ maka $\frac{\partial F(x, y)}{\partial y}$

Jawab

$$F(x, y) = 7x^3y^4 - 5x^6y^2$$

Turunan terhadap y (x dianggap konstanta)

$$\frac{\partial F(x, y)}{\partial y} = 4 \cdot 7x^3y^{4-1} - 2 \cdot 5x^6y^{2-1}$$

$$= 28x^3y^3 - 10x^6y$$

3. ~~$\int \frac{1}{5} x^5 - 7x^9 + e^{2x} - \frac{x^5 \sqrt{x}}{x^3} + 1000 dx$~~ $\int \frac{1}{5} x^5 - 7x^9 + e^{2x} - \frac{x^5 \sqrt{x}}{x^3} + 1000 dx$

Jawab

$$= \int \left(\frac{1}{5} x^5 - 7x^9 + e^{2x} - \frac{x^5 \cdot x^{\frac{1}{2}}}{x^3} + 1000 \right) dx$$

$$= \int \left(\frac{1}{5} x^5 - 7x^9 + e^{2x} - x^{5+\frac{1}{2}-3} + 1000 \right) dx$$

$$= \int \left(\frac{1}{5} x^5 - 7x^9 + e^{2x} - x^{\frac{5}{2}} + 1000 \right) dx$$

$$= \frac{1}{5} \cdot \frac{1}{5+1} x^{5+1} - \frac{7}{4+1} x^{4+1} + \frac{1}{2} e^{2x} - \frac{1}{\frac{5}{2}+1} x^{\frac{5}{2}+1} + 1000x + C$$

$$= \frac{1}{5} \cdot \frac{1}{6} x^6 - \frac{7}{5} x^5 + \frac{1}{2} e^{2x} - \frac{1}{\frac{7}{2}} x^{\frac{7}{2}} + 1000x + C$$

$$= \frac{1}{30} x^6 - \frac{7}{5} x^5 + \frac{1}{2} e^{2x} - \frac{2}{7} x^{\frac{7}{2}} + 1000x + C$$

$$4. \int_0^2 5x^2 + \frac{1}{2}x + 11 \, dx$$

Jawab

$$= \left[\frac{5}{3} x^3 + \frac{1}{2(2)} x^2 + 11x \right]_0^2$$

$$= \left[\frac{5}{3} (2)^3 + \frac{1}{4} (2)^2 + 11(2) \right] - 0$$

$$= \frac{5}{3} (2)^3 + \frac{1}{4} (2)^2 + 11(2) - 0$$

$$= \frac{40}{3} + \frac{1}{4} + 22$$

$$= \frac{40}{3} + 1 + 22$$

$$= \frac{40+3+66}{3}$$

$$= \frac{109}{3}$$

$$5. \int \sin 4x + 2 \cos \frac{1}{3}x - \csc^2 x \, dx$$

Jawab

$$= \int \sin 4x \, dx + 2 \cos \frac{1}{3}x - \csc^2 x \, dx$$

$$= -\frac{1}{4} \cos 4x + 6 \sin \frac{1}{3}x + \cot x$$

$$= -\frac{1}{4} \cos 4x + 6 \sin \frac{1}{3}x + \cot x + C$$

$$6. \int \frac{8}{\sin^2 2x} \, dx$$

Jawab

$$\text{substitusi: } u = 2x$$

$$du = 2 \, dx$$

$$\frac{1}{2} du = dx$$

$$= \int 8 \csc^2 2x \, dx$$

$$= \int 8 \csc^2 u \cdot \frac{1}{2} du$$

$$= \int 4 \csc^2 u \, du$$

$$= 4 \int \csc^2 u \, du = -4 \cot 2x + C$$

$$= 4(-\cot u) + C$$

$$= -4 \cot u + C$$

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Tentukan dengan teknik substitusi sederhana

$$\int 2x^5 (x^6 - 9)^{12} dx$$

Jawab

$$\int 2x^5 (x^6 - 9)^{12} dx = \int 2x^5 (x^6 - 9)^{12} dx$$

$$u = x^6 - 9 \quad = 2 \int x^5 (x^6 - 9)^{12} dx$$

$$du = 6x^5 dx$$

$$= 2 \int \frac{du}{6} (u)^{12}$$

$$x^5 \frac{du}{6} = dx$$

$$= \frac{2}{5} \left(\frac{1}{13} u^{13} \right) + C$$

$$= \frac{2}{65} (x^6 - 9)^{13} + C$$

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~~Tentukan dengan teknik persig~~ Tentukan dengan teknik persig

$$\int 7x \sin 7x dx$$

Jawab

$$u = 7x \quad dv = \sin 7x$$

$$du = 7 \quad v = -\frac{1}{7} \cos 7x$$

$$uv - \int v du$$

$$= 7x \sin 7x - \int -\frac{1}{7} \cos 7x \cdot 7$$

$$= 7x \sin 7x - \left[-\frac{1}{7} \sin 7x \right] + C$$

$$= 7x \sin 7x + \frac{1}{7} \sin 7x + C$$

$$= \sin 7x \left(7x + \frac{1}{7} \right) + C$$

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Tentukan dengan teknik substitusi II

$$\int \sin^7 x \cos^2 x dx$$

Jawab

misal

$$u = \cos x$$

$$du = -\sin x dx$$

$$\frac{1}{-\sin x} du = dx$$

$$-\sin x$$

$$= \int \sin^6 x \cdot \sin x \cos^2 x dx$$

$$= \int (\sin^3 x)^3 \sin x \cos^2 x dx$$

$$= \int (1 - \cos^2 x)^3 \sin x \cos^2 x dx$$

$$= \int (1 - 3\cos^2 x + \cos^4 x) \sin x \cos^2 x dx$$

$$= \int (1 - 3\cos^2 x + \cos^4 x) \cos^2 x \sin x dx$$

$$= \int (\cos^2 x - 3\cos^4 x + \cos^6 x) \sin x dx$$

$$= \int (U^2 - 2U^4 + U^6) \sin x = \frac{1}{-\sin x} du$$

$$= - \int (U^2 - 2U^4 + U^6) du$$

$$= - \left[\frac{1}{3} U^3 - \frac{2}{5} U^5 + \frac{1}{7} U^7 \right] + C$$

$$= -\frac{1}{3} \cos^3 x - \frac{2}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$$

10. Tentukan nilai dari integral lipat berikut

$$\int_{-1}^0 \int_0^1 x^2 y - \frac{1}{3} x^3 y^5 dx dy$$

Jawab

$$(i) \int_0^1 x^2 y - \frac{1}{3} x^3 y^5 dx$$

$$= \left[\frac{1}{3} x^3 y - \frac{1}{4} x^4 y^5 \right]_0^1$$

$$= \left[\frac{1}{3} (1)^3 y - \frac{1}{4} (1)^4 y^5 \right] - 0$$

$$= \frac{1}{3} y - \frac{1}{4} y^5$$

$$(ii) \int_{-1}^0 \frac{1}{3} y - \frac{1}{4} y^5 dy$$

$$= \left[\frac{1}{4} y^2 - \frac{1}{5} y^6 \right]_{-1}^0$$

$$= 0 - \left[\frac{1}{4} (-1)^2 - \frac{1}{5} (-1)^6 \right]$$

$$= 0 - \left[-\frac{1}{4} + \frac{1}{5} \right]$$

$$= 0 - \left[-\frac{1}{4} + \frac{1}{5} \right]$$

$$= \frac{5+4}{20}$$

$$= \frac{9}{20}$$