PERTEMUAN 5

Program Studi Informatika Universitas Indraprasta PGRI

Limit Fungsi Trigonometri

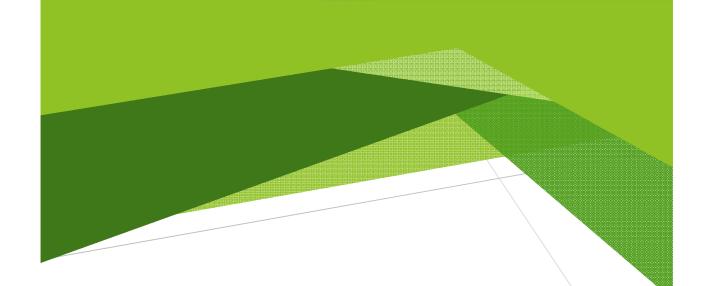
Rumus dasar Limit Trigonometri

(1)
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(3)
$$\underset{x \to 0}{\text{Limit}} \frac{\tan x}{x} = 1$$

(2)
$$\underset{x \to 0}{\text{Limit}} \frac{x}{x \to 0} = 1$$

(4) Limit
$$\frac{x}{x \to 0} = 1$$



 $\frac{a}{b}$

П ax bx

 $\frac{bx}{bx} = \underset{x \to 0}{\text{Limit}} \frac{\sin ax}{ax} \times \frac{bx}{\tan bx} \times \frac{x}{a}$

 $\frac{\text{Limit sin ax}}{x \to 0} = \frac{\text{Limit sin ax}}{\tan bx} \times \frac{ax}{ax} \times \frac{ax}{ax} \times \frac{ax}{ax}$

Limit
$$\frac{\sin ax}{bx} = \frac{\text{Limit}}{x \to 0} \frac{\sin ax}{bx} \times \frac{a}{a} = \frac{\text{Limit}}{x \to 0} \frac{\sin ax}{ax} \times \frac{a}{b} = \frac{a}{b}$$

Limit $\frac{bx}{x \to 0} = \frac{1}{\sin ax} = \frac{1}{x \to 0} \frac{ax}{\sin ax} \times \frac{a}{a} = \frac{1}{x \to 0} \frac{b}{\sin ax} \times \frac{b}{a} = \frac{b}{a}$

Limit $\frac{\tan ax}{x \to 0} = \frac{1}{bx} = \frac{1}{bx} \times \frac{a}{a} = \frac{1}{x \to 0} \frac{ax}{\sin ax} \times \frac{b}{b} = \frac{b}{b}$

Limit $\frac{bx}{x \to 0} = \frac{bx}{bx} = \frac{1}{x \to 0} \frac{ax}{bx} \times \frac{a}{a} = \frac{1}{x \to 0} \frac{ax}{ax} \times \frac{b}{b} = \frac{b}{b}$

Limit $\frac{bx}{x \to 0} = \frac{bx}{ax \to 0} = \frac{1}{ax} \times \frac{a}{a} = \frac{1}{a} = \frac{a}{a}$

$$\underset{x \to 0}{\text{Limit}} \ \frac{\sin ax}{bx} = \frac{a}{b} \quad \text{dan} \quad \underset{x \to 0}{\text{Limit}} \ \frac{bx}{x \to 0} = \frac{b}{a}$$

(1) Limit
$$\frac{\text{sun } ax}{\text{bx}} = \frac{a}{b}$$

(2) Limit $\frac{\tan ax}{\text{con } bx} = \frac{a}{b}$

(2) Limit
$$\frac{\tan ax}{x \to 0} = \frac{a}{bx}$$

(3) Limit $\frac{\sin ax}{x \to 0} = \frac{a}{a}$

Limit
$$\frac{bx}{x \to 0} = \frac{1}{\tan ax}$$

Limit $\frac{an bx}{a \to 0} = \frac{b}{\sin ax}$

dan

П

Limit
$$\frac{bx}{x \to 0} = \frac{a}{\sin ax}$$

Limit $\frac{bx}{\tan ax} = \frac{b}{a}$

Limit $\frac{\tan bx}{x \to 0} = \frac{b}{\sin ax}$

dan

Contoh Soal

(a) Limit
$$\left[\frac{\sin 4x}{x \to 0} + \frac{\tan 2x}{3x} + \frac{8x}{\sin 6x} + \frac{8x}{\tan 2x}\right] = \frac{4}{3} - \frac{2}{6} + \frac{8}{2} = \frac{4}{3} - \frac{1}{3} + 4 = 5$$

(b) Limit
$$\frac{\sin^2 6x}{x \to 0} = \text{Limit} \frac{\sin 6x}{x \to 0} \frac{\sin 6x}{3x \cdot \tan 4x} = \frac{\sin 6x}{x \to 0} \frac{\sin 6x}{3x} \cdot \frac{\sin 6x}{\tan 4x} = \frac{6}{3} \left(\frac{6}{4}\right) = \frac{36}{12} = 3$$

(c) Limit
$$\frac{2\tan^2 3x \cdot \sin 2x}{4x^2 \cdot \sin 6x} = \frac{\text{Limit }}{x \to 0} 2 \left(\frac{\tan 3x}{4x}\right) \left(\frac{\tan 3x}{x}\right) \left(\frac{\sin 2x}{\sin 6x}\right)$$

$$= 2\left(\frac{3}{4}\right)\left(\frac{3}{1}\right)\left(\frac{2}{6}\right)$$
$$= 3/2$$

(d) Limit
$$\frac{6.\sin^3 2x}{x \to 0}$$
 = $\lim_{x \to 0} \frac{\sin 2x}{\sin 4x}$ $\sin 4x$ $\sin 3x$ $\sin 3x$ $\sin 3x$

$$= 6\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\sin(20)$$

$$= \left(\frac{24}{12}\right)^{\mathsf{G}}$$

(e) Limit
$$\begin{bmatrix} \sin 2x + 4x \\ x \to 0 \end{bmatrix} = \text{Limit} \begin{cases} \sin 2x + 4x \\ x \to 0 \end{cases}$$

$$x \to 0$$

$$\begin{cases} \sin 2x + 4x \\ x \to 0 \end{cases}$$

$$= \begin{bmatrix} \frac{2}{1} + \frac{4}{1} \\ \frac{3}{1} + \frac{1}{1} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2+4}{3+1} \\ \frac{3+1}{3+1} \end{bmatrix}$$

$$= 3/2$$

(e) Limit
$$\begin{bmatrix} 6x^2 + \sin^2 3x \\ \tan^2 2x - x^2 \end{bmatrix}$$
 = Limit $\begin{bmatrix} 6x^2 + \sin^2 3x \\ x \to 0 \end{bmatrix}$ $\frac{x^2 + \sin^2 3x}{\tan^2 2x - x^2}$

$$= \left(\frac{6+9}{4-1}\right)$$

Contoh Soal

(a) Limit
$$\frac{3\sin(x-2)}{x\to 2}$$
 = Limit $\frac{3\sin(x-2)}{4(x-2)}$ = $3\left(\frac{1}{4}\right)$ = $\frac{3}{4}$

(b) Limit
$$\frac{6\tan^2(2x-6)}{(3x-9)^2} = \frac{x-2}{x \to 3} \left(\frac{\tan(2x-6)}{3x-9}\right)^2$$

$$= \underset{\mathbf{x} \to 3}{\text{Limit}} \ 6 \left(\frac{\tan 2(x-3)}{3(x-3)} \right)^2$$

$$= 6\left(\frac{2}{3}\right)^2$$

$$= 8/3$$

$$\sin(2x - 8)$$

(c) Limit
$$\left[\frac{\sin(2x-8)}{x \to 4} \left[\frac{\sin 2(x-4)}{\tan(x-4) + (3x-12)} \right] = \frac{\text{Limit}}{x \to 4} \left[\frac{\sin 2(x-4)}{\tan(x-4) + 3(x-4)} \right] = \frac{\text{Limit}}{x \to 4} \left[\frac{\sin 2(x-4)}{(x-4)} \right]$$

$$= \left(\frac{2}{1+3}\right)$$



(d) Limit
$$\frac{\tan(2x^2 - 6x + 4)}{3x^2 - 9x + 6} = \frac{\text{Limit}}{x \to 1} \frac{\tan 2(x^2 - 3x + 2)}{3(x^2 - 3x + 2)}$$

$$= \frac{\text{Limit}}{x \to 1} \frac{\tan 2(x + 3)(x - 1)}{3(x + 3)(x - 1)}$$

$$= \frac{2}{x}$$

(e) Limit
$$\frac{\sin(3x-6)}{x^2 + 2x - 8} = \frac{\text{Limit}}{x \to 2} \frac{\sin 3(x-2)}{(x+4)(x-2)}$$

$$= \frac{\text{Limit}}{x \to 2} \frac{1}{(x+4)} \cdot \frac{\sin 3(x-2)}{(x-2)}$$

$$= \left(\frac{1}{2+4}\right) \left(\frac{3}{1}\right)$$

$$= \frac{3}{6}$$

$$= \frac{3}{6}$$

Rumus tambahan untuk penyelesaian Limit Trigonometri

(1)
$$1 - \cos 2\alpha = 2 \sin^2 \alpha$$

$$(3) 1 - \cos^2 \alpha = \sin^2 \alpha$$

(2)
$$\cos 2\alpha - 1 = -2 \sin^2 \alpha$$

(4)
$$\operatorname{ctg} \alpha = \frac{1}{\tan \alpha}$$

(5)
$$\cos A - \cos B = -2\sin^{\frac{1}{2}}(A + B).\sin^{\frac{1}{2}}(A - B)$$

Contoh Soal

(a) Limit
$$\frac{1-\cos 6x}{x \rightarrow 0}$$
 = Limit $\frac{1-\cos 2(3x)}{x \rightarrow 0}$

$$= \underset{x \to 0}{\text{Limit}} \frac{2.\sin^2 3x}{-2.\sin^2 4x}$$

$$= \underset{x \to 0}{\text{Limit}} - \left(\frac{.\sin 3x}{.\sin 4x} \right)^2$$

$$= -\left(\frac{.3}{.4}\right)^2$$

(b)
$$\lim_{x\to 0} \frac{3\cos 4x - 3}{2.\sin^2 3x} = \lim_{x\to 0} \frac{3(\cos 2(2x) - 1)}{2.\sin^2 3x}$$

$$= \underset{x \to 0}{\text{Limit}} \frac{3(\cos 2(2x) - \cos 2(2x))}{2 \cdot \sin^2 3x}$$

=
$$\lim_{x\to 0} \frac{-3.\sin^2 2x}{2.\sin^2 3x}$$

= $\lim_{x\to 0} \frac{3}{2} \frac{(\sin 2x)^2}{(\sin 3x)^2}$

$$= -\frac{3}{2} \left(\frac{2}{3} \right)^2$$

(c) Limit
$$\frac{4-4\cos 2}{x \to 0}$$

(c) Limit
$$\frac{4-4\cos 2x}{x\to 0}$$
 = Limit $\frac{4(1-\cos 2x)}{1-\cos^2 3x}$

$$\begin{array}{c}
\text{Limit} \\
x \to 0 \\
\hline
1 - \cos 2 3
\end{array}$$

$$= \underset{x \to 0}{\text{Limit}} \frac{4.(2\sin^2 x)}{\sin^2 3x}$$

$$= \underset{x \to 0}{\text{Limit}} 8 \left(\frac{.\sin x}{.\sin 3x} \right)^2$$
$$= 8 \left(\frac{1}{.3} \right)^2$$

$$= 8\left(\frac{1}{3}\right)^2$$

(d) Limit
$$\frac{2-2\cos^2 6x}{x\to 0}$$
 $\frac{2-2\cos^2 6x}{3\cos^2 2x - 3}$

$$\frac{2 - 2\cos 6x}{\cos^2 2x - 3} = L$$

= Limit
$$\frac{2(1-\cos^2 6)}{x \to 0}$$
 $3(\cos^2 2x - \frac{1}{2})$

(d) Limit
$$\frac{2-2\cos^2 6x}{x \to 0}$$
 = Limit $\frac{2(1-\cos^2 6x)}{x \to 0}$
= Limit $\frac{2(1-\cos^2 6x)}{3(\cos^2 2x - 1)}$
= Limit $\frac{4.(\sin^2 6x)}{3.(-\sin^2 2x)}$

$$\frac{\text{Limit}}{\text{x} \to 0} - \frac{4}{3} \left(\frac{\sin 6x}{\sin 2x} \right)^2$$

$$- \frac{4}{3} \left(\frac{6}{2} \right)^2$$

$$- \frac{4}{3} (9)$$

$$- 12$$

$$=-\frac{4}{3}\left(\frac{6}{3}\right)^2$$

$$=\frac{4}{3}(9)$$



(e)
$$\lim_{x \to 0} \frac{\cos 5x - \cos 3x}{1 - \cos 8x} = \lim_{x \to 0} \frac{-2\sin \frac{1}{2}(5x + 3x).\sin \frac{1}{2}(5x - 3x)}{1 - \cos 2(4x)}$$

$$= \lim_{x \to 0} \frac{-2\sin 4x.\sin x}{2\sin^2 4x}$$

$$= \lim_{x \to 0} \frac{-\sin x}{\sin 4x}$$

II

(f)
$$\lim_{x\to 0} \frac{3\cos 6x - 3\cos 2x}{1 - \cos^2 2x} = \lim_{x\to 0} \frac{3(\cos 6x - \cos 2x)}{1 - \cos^2 2x}$$

$$= \lim_{x\to 0} \frac{-6\sin\frac{1}{2}(6x + 2x).\sin\frac{1}{2}(6x - 2x)}{\sin^2(2x)}$$

$$= \lim_{x\to 0} \frac{-\cos\frac{1}{2}(6x + 2x).\sin\frac{1}{2}(6x - 2x)}{\sin^2(2x)}$$

$$= \underset{x \to 0}{\text{Limit}} \frac{-6\sin 4x}{\sin 2x}$$
$$= -6\left(\frac{4}{2}\right)$$
$$= -12$$

Contoh Soal Lainya

(a)
$$x \rightarrow \frac{\pi}{2}$$
 $\frac{3.\sin 4x}{\cos x}$ Jawab

$$\frac{\pi}{2}$$
 3. $\sin 4y$

(b) Limit $\sin 6x + \sin 2x$

 $x \rightarrow \frac{\pi}{4}$ $\cos 5x + \cos x$

(a)
$$\lim_{x \to \frac{\pi}{2}} \frac{3.\sin 4x}{\cos x} =$$

$$\frac{\ln 4x}{\cos x} = \frac{\text{Limit}}{x \to \pi} \frac{3.(2\sin 2x.\cos 2x)}{\cos x}$$

$$= \underset{2}{\text{Limit}} 12.\sin x \cdot \cos 2x$$

=
$$12 \sin \frac{\pi}{2} \cdot \cos 2\left(\frac{\pi}{2}\right)$$

= $12 \sin \frac{\pi}{2} \cdot \cos \pi$
= $12 (1)(0)$
= 0

= 12
$$\sin \frac{\pi}{2}$$
.cos n

(b)
$$x \to \frac{\pi}{4} \frac{\sin 6x + \sin 2x}{\cos 5x + \cos x} = \frac{\text{Limit}}{4} \frac{2 \cdot \sin 3x \cdot \cos 2x}{2 \cdot \cos 4x \cdot \cos 2x}$$

$$= \frac{\text{Limit}}{x \to \frac{\pi}{4}} \frac{\sin 3x}{\cos 4x}$$

$$= \frac{\sin 3\frac{\pi}{4}}{\cos 4}$$

$$= \frac{\sin 3\frac{\pi}{4}}{\cos 5}$$

$$= \frac{\sin 3\pi}{\cos 5}$$

$$= \frac{\sin 3\pi}{\sin 5}$$

$$= \frac{$$

$$= \lim_{x \to \pi} \frac{\sin 3x}{\cos 4x}$$

$$= \frac{\sin 3\left(\frac{\pi}{4}\right)}{\cos 4\left(\frac{\pi}{4}\right)}$$

$$= \frac{\sin \frac{3\pi}{4}}{\cos \pi}$$

$$=\frac{\frac{1}{2}\sqrt{3}}{-1}$$

$$= -\frac{1}{2}\sqrt{3}$$

Latihan Soal

$$\lim_{x \to 2} \frac{1 - \cos^2(x-2)}{3x^2 - 12x + 12} = \dots$$

 $\lim_{x \to 0} \frac{\cos 4x - 1}{x \tan 2x} = \dots$

$$\lim_{x \to 2} \frac{\frac{1 - \cos^2(x - 2)}{3x^2 - 12x + 12}}{3x^2 - 12x + 12}$$

$$\lim_{x \to \pi} \frac{x^2 \sin 2x}{x - \pi} = \dots$$

 $\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \dots$

$$Nilai \lim_{x \to 0} \frac{1 - \cos 2x}{4x^2} = \dots$$

 $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \dots$

$$\lim_{x \to 0} \frac{\sin \frac{1}{2} x \tan 2\sqrt{x}}{x\sqrt{x}} = \dots$$

 $\lim_{x \to 2} \frac{(t^2 - 5t + 6)\sin(t - 2)}{(t^2 - t - 2)^2} = \dots$

$$\lim_{x \to 0} \frac{\sin 3x - \sin 3x \cos 2x}{2x}$$

 $\lim_{x \to 0} \frac{\sin 6x}{\sin 2x} = \dots$

$$\lim_{x \to 4} \frac{\sin(4-2\sqrt{x})}{4-x}$$

 $\lim_{x \to 0} \frac{\tan x}{x^2 + 2x} = \dots$

$$\lim_{x \to 0} \frac{2x \sin 3x}{1 - \cos 6x} = \dots$$

 $\lim_{x \to 0} \frac{\cos x - \cos 2x}{x^2} = \dots$

$$\lim_{x \to \mathcal{I}} \frac{1 - 2\sin^2 x}{\cos x - \sin x} = \dots$$

$$\lim_{x \to \mathcal{I}} \frac{1 - \sin 2x}{\cos^2 2x} = \dots$$

$$x \to \frac{\pi}{4} \cos^2 2x$$

$$\lim_{x \to 1} \left(\frac{x^3 - 1}{x^{-1}} + \frac{\cos(\frac{\pi}{2} - x + 1)}{x^{-1}} \right) = 1$$

$$\lim_{x \to 1} \left(\frac{x^3 - 1}{x - 1} + \frac{\cos(\frac{\pi}{2} - x + 1)}{x - 1} \right) = \dots$$

$$\lim_{x \to a} \frac{x - a}{3x - 3a + \tan(x - a)} = \dots$$

$$\lim_{h \to 0} \frac{\sin(\frac{1}{3}\pi + h) - \sin\frac{1}{3}\pi}{h} = \dots$$

$$\lim_{x \to \infty} x^2 \sin \frac{1}{x} \tan \frac{1}{x} = \dots$$

$$\lim_{x \to 0} \frac{(x^2 - 1)\sin 6x}{x^3 + 3x^2 + 2x} = \dots$$