Simple Linear Regression

Lecture 07 - CPSC392

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    - The 2 variables are a dependent variable (denoted by y) and an independent variable (denoted by x)
- Linear regression is in fact a comparison of 2 models

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- Let’s plot the data

Model 1

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- Our best estimate, in this case, is the mean
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“ with only one variable, and no other information, the best prediction for the next measurement is the mean of the sample”

Goodness of Fit

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- This is very similar to the concept of standard deviation

Residuals/Errors

- Distance between the best fit line to the observed values

Sum of Squared Errors (SSE)

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- Measure of discrepancy between the data and the estimated model

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- Measure of discrepancy between the data and the estimated model
- Calculated by squaring all errors and summing them up

Goals

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- If the regression model is “significant”, it will take away a large chunk of the SSE
- The model should “fit” the data better and minimize the residuals once we introduce an independent variable

The SSE of the model with just test scores is 20. Let’s introduce a new independent variable, total hours of study, and see if we can create a linear regression model using this attribute

Lines

Lines

- **y = mx + b**

- **m** = slope (rise/run)
- **b** = y-intercept (point where x = 0)

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- $y = mx + b$
- $y = \beta_0+\beta_1 x+\varepsilon$

- β_1 = slope parameter
- β_0 = y-intercept parameter
- ε = error term (unexplained variation in y)

Lines

- $y = mx + b$
- **$y = \beta_0+\beta_1 x+\varepsilon$ (for population data)**

- β_1 = slope parameter
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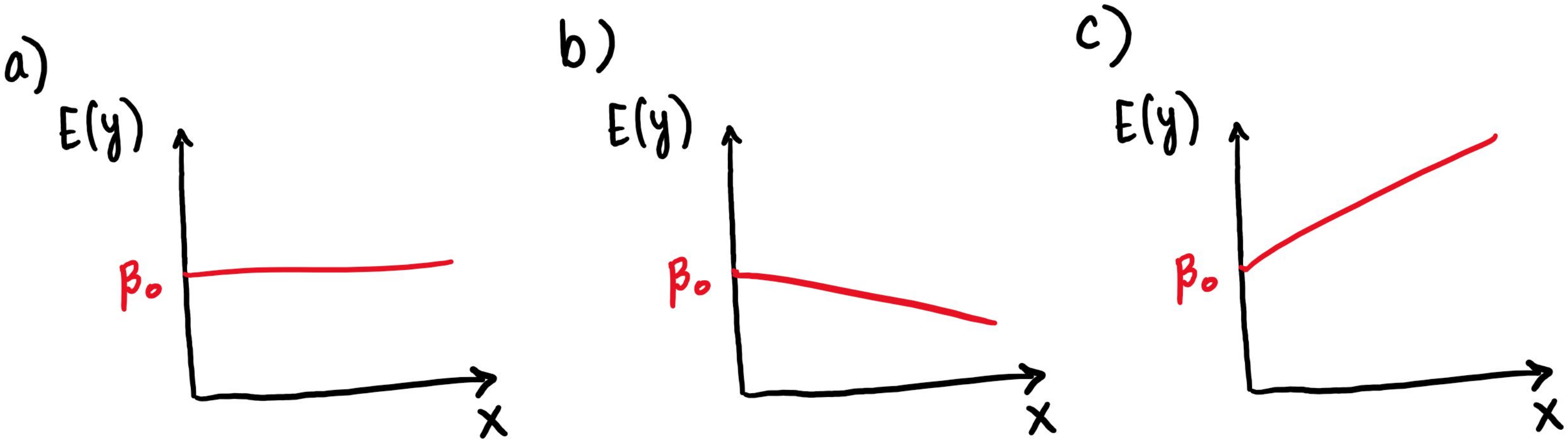
- ε = error term (unexplained variation in y)

Lines (Simple Linear Regression)

- **$E(y) = \beta_0+\beta_1x$**

- **β_1** = slope parameter
- **β_0** = y-intercept parameter
- **E(y)** = mean or expected value of y, given some x

What are the equations for these lines?



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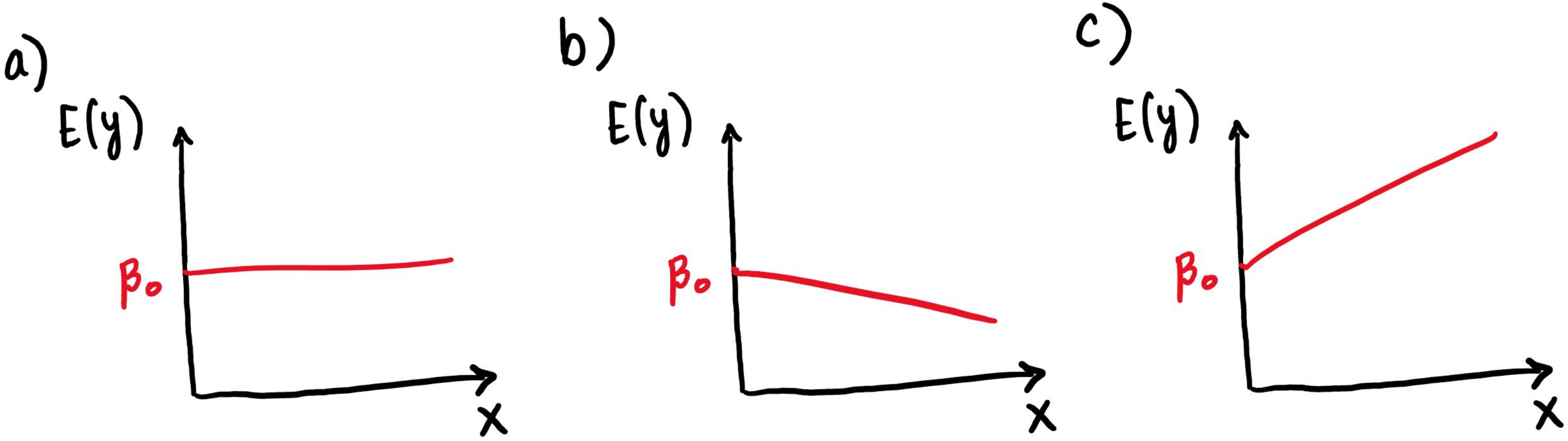

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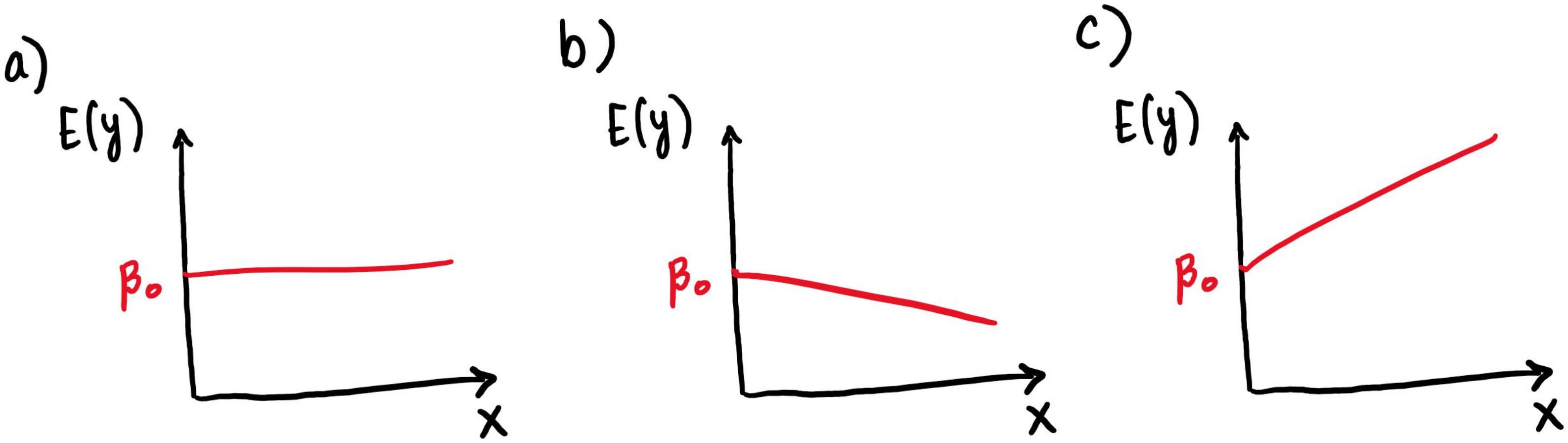

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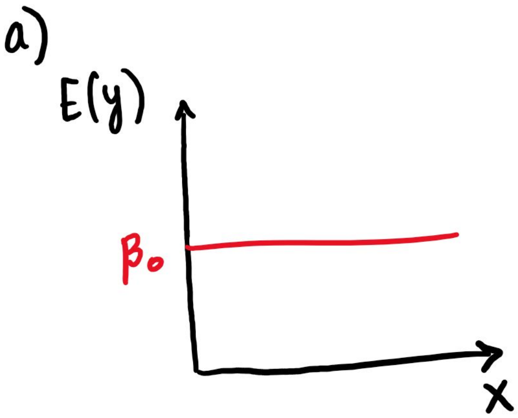

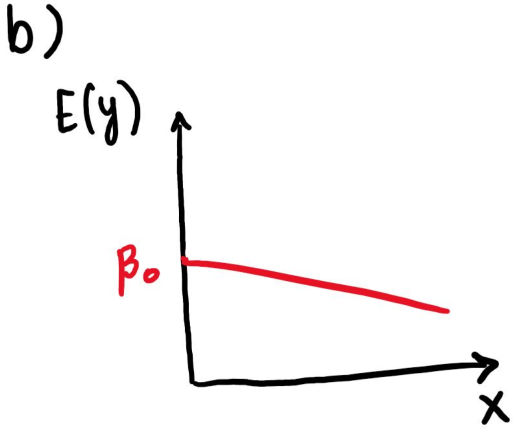

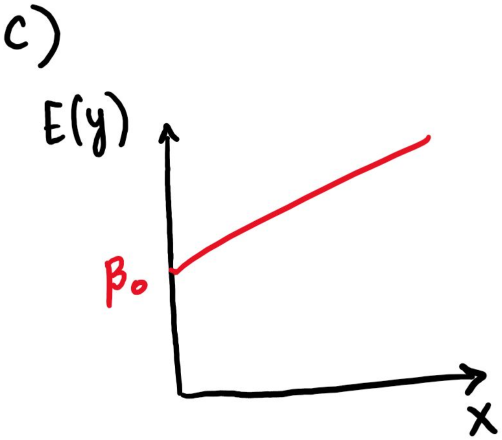

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Linear Regression for a Sample

- $E(y) = \beta_0+\beta_1 x$
- **$\hat{y} = b_0+b_1 x$**

- **$\hat{y}$ (y-hat)** = estimator of $E(y)$

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- If yes, then we can fit a linear regression line to predict future scores
- If no, then the linear regression model might be useless

$\hat{y} = b_0 + b_1 x$

$$\hat{y}_i = b_0 + b_1 x_i$$

$$b_1 = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2} \quad , \; b_0 = \hat{y}_i - b_1 x_i$$

$\bar{x}$ = mean of independent variable
x_i = value of independent variable
$\bar{y}$ = mean of dependent variable
y_i = value of dependent variable.

Best-fit Line

$$\hat{y}_i = 3.2 + 0.95x_i$$

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- For every 1 hour increase in study time, you expect to get an increase in score by 0.95 points
- If you don't study, x = 0, then you will end up with a score of 3.2 (practical?)

Least Square Criterion

$$\min \Sigma (y_i - \hat{y}_i)^2$$

y_i = observed value of test score

$\hat{y}_i$ = predicted value of test score

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- Goal is to minimize the sum of the squared differences between the actual value of dependent variable and the estimated (predicted) value
- We can find this sum and compare with the SSE of Model 1 to see how much linear regression minimizes the distance

SSR & SST

- SSE = sum of squared errors
- SST = sum of squared total
  - Equals to SSE when no independent variable is used in model
- SSR = sum of squared regression
  - SSR = SST - SSE

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- **GOOD FIT!**