­­Toby Chappell

CPSC 390

**Homework #3 Proposition Logic**

**I. (5 points) Models and satisfiability**

1. How many models does (A∧B) ⇒ (B∧C) satisfy?

The statement satisfies 7 models (the only model not satisfied is A=true, B=true, and C=false).

2. Tell if each of the following sentences is valid (that is, *True* in all models), unsatisfiable, or neither (Explain your answers):

SMOKE ⇒ FIRE

Neither, if smoke is true and fire is false implies model is false (therefore sentence is not valid) and all other models are true (therefore model is satisfiable).

(SMOKE ⇒ FIRE) ⇒ (¬FIRE ⇒ ¬SMOKE)

Valid, all models are true.

(SMOKE ∧ (SMOKE ⇒ FIRE)) v ¬FIRE

Neither, if smoke is false and fire is true implies model is false (therefore sentence is not valid) and all other models are true (therefore model is satisfiable).

**II. (5 points) Clause form and resolution refutation**

Consider the following sentences in proposition logic:

1. (1)  Battery-OK ∧ Bulbs-OK ⇒ Headlights-Work
2. (2)  Battery-OK ∧ Starter-OK ∧ ¬Empty-Gas-Tank ⇒ Engine-Starts
3. (3)  Engine-Starts ∧ ¬Flat-Tire ⇒ Car-OK
4. (4)  Radio-Works ⇒ Battery-OK
5. (5)  ¬Headlights-Work
6. (6)  ¬Empty-Gas-Tank
7. (7)  Radio-Works
8. (8)  Starter-OK
9. (9)  ¬Flat-Tire
10. Transform these sentences into clauses.
    1. ¬Battery-OK v Headlights-Work (1)
    2. ¬Bulbs-OK v Headlights-Work (1)
    3. ¬Battery-OK v Engine-Starts (2)
    4. ¬Starter-OK v Engine-Starts (2)
    5. Empty-Gas-Tank v Engine-Starts (2)
    6. ¬Engine-Starts v Car-OK (3)
    7. Flat-Tire v Car-OK (3)
    8. ¬Radio-Works v Battery-OK (4)
    9. ¬Headlights-Work (5)
    10. ¬Empty-Gas-Tank (6)
    11. Radio-Works (7)
    12. Starter-OK (8)
    13. ¬Flat-Tire (9)
11. Use refutation resolution with set-of-support strategy to prove Car-OK. Present your proof with one clause per line. Label each clause by an integer (use consecutive integers) and indicate after each derived clause the numbers of their parents.
    1. ¬Battery-OK v Headlights-Work
    2. ¬Bulbs-OK v Headlights-Work
    3. ¬Battery-OK v Engine-Starts
    4. ¬Starter-OK v Engine-Starts
    5. Empty-Gas-Tank v Engine-Starts
    6. ¬Engine-Starts v Car-OK
    7. Flat-Tire v Car-OK
    8. ¬Radio-Works v Battery-OK
    9. ¬Headlights-Work
    10. ¬Empty-Gas-Tank
    11. Radio-Works
    12. Starter-OK
    13. ¬Flat-Tire
    14. ¬Car-OK
    15. ¬Engine-Starts (6)
    16. ¬Battery-OK (3)
    17. ¬Radio-Works (8)
    18. *False* (11)

**III. (5 points) Formulating a problem in logic**

State the following:

“If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mammal. All mammals are mortal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.”

in a collection of sentences in propositional logic. [Either use a single proposition to represent the facts of being mortal and being immortal, or use two distinct propositions and add sentences in proposition logic stating that if one proposition is *True*, then the other is *False*.]

1. Mythical ⇒ ¬Mortal
2. ¬Mythical ⇒ Mammal
3. Mammal ⇒ Mortal
4. ¬Mortal v Mammal ⇒ Horned
5. Horned ⇒ Magical

Can you construct a proof (application of successive sound inference rules) showing that (in each case, if your answer is yes, give the proof as part of your answer):

1. The unicorn is mythical?

No

1. The unicorn is magical?

Yes

* 1. ¬Mythical v ¬Mortal
  2. Mythical v Mammal
  3. ¬Mammal v Mortal
  4. Mortal v Horned
  5. ¬Mammal v Horned
  6. ¬Horned v Magical
  7. ¬Magical
  8. ¬Horned (6)
  9. Mortal (4)
  10. ¬Mythical (1)
  11. Mammal (2)
  12. Horned (5)
  13. *False* (8)

**IV. (5 points) Soundness and completeness of resolution refutation**

1. Tell me what it means for an inference rule to be sound. Prove that full resolution is sound.

An inference is sound if it generates only entailed sentences.

Let m be a model of:

L1 v … v Lp and M1 v … v Mq

Where Li and Mj are complementary literals

Either Li or Mj must be False in m, hence L1 v … v L(i-1) v L(i+1) v … v Lp v M1 v … v M(j-1) v M(j+1) v … v Mq must be True.

1. Tell me what it means for resolution refutation to be complete.

Resolution refutation is complete if every entailed sentence can be obtained by applying some finite succession of the resolution refutation rule.