a)

$$\log\left(\frac{P(Y=k_1|X=x)}{P(Y=k_2|X=x)}\right) = \log\left(\frac{\dfrac{\pi_{k_1}\frac{1}{\sigma_{k_1}\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{k_1}^2}(x-\mu_{k_1})^2}}{\sum_{i=1}^{n}\pi_i\frac{1}{\sigma_i\sqrt{2\pi}}e^{-\frac{1}{2\sigma_i^2}(x-\mu_i)^2}}}{\dfrac{\pi_{k_2}\frac{1}{\sigma_{k_2}\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{k_2}^2}(x-\mu_{k_2})^2}}{\sum_{i=1}^{n}\pi_i\frac{1}{\sigma_i\sqrt{2\pi}}e^{-\frac{1}{2\sigma_i^2}(x-\mu_i)^2}}}\right)$$

$$= \log\left(\frac{\pi_{k_1}\frac{1}{\sigma_{k_1}\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{k_1}^2}(x-\mu_{k_1})^2}}{\pi_{k_2}\frac{1}{\sigma_{k_2}\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{k_2}^2}(x-\mu_{k_2})^2}}\right)$$

Assume $\sigma_{k_1} = \sigma_{k_2} = \sigma$

$$= \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right) + \log\left(e^{-\frac{1}{2\sigma^2}(x-\mu_{k_1})^2 + \frac{1}{2\sigma^2}(x-\mu_{k_2})^2}\right)$$

$$= \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right) + \frac{(x-\mu_{k_2})^2 - (x-\mu_{k_1})^2}{2\sigma^2}$$

$$= \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right) + \frac{x^2 - 2x\mu_{k_2} + \mu_{k_2}^2 - x^2 + 2x\mu_{k_1} - \mu_{k_1}^2}{2\sigma^2}$$

$$= \frac{\mu_{k_1} - \mu_{k_2}}{\sigma^2}x - \frac{\mu_{k_1}^2 - \mu_{k_2}^2}{2\sigma^2} + \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right)$$

b)

$$\log\left(\frac{P(Y=k_1 \mid X=x)}{P(Y=k_2 \mid X=x)}\right)$$

$$= \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right) + \log\left(e^{-\frac{1}{2\sigma_{k_1}^2}(x-\mu_{k_1})^2 + \frac{1}{2\sigma_{k_2}^2}(x-\mu_{k_2})^2}\right)$$

$$= \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right) - \frac{(x-\mu_{k_1})^2}{2\sigma_{k_1}^2} + \frac{(x-\mu_{k_2})^2}{2\sigma_{k_2}^2}$$

$$= \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right) - \frac{x^2 - 2x\mu_{k_1} + \mu_{k_1}^2}{2\sigma_{k_1}^2} + \frac{x^2 - 2x\mu_{k_2} + \mu_{k_2}^2}{2\sigma_{k_2}^2}$$

$$= \left(-\frac{1}{2\sigma_{k_1}^2} + \frac{1}{2\sigma_{k_2}^2}\right)x^2 + \left(\frac{\mu_{k_1}}{\sigma_{k_1}^2} - \frac{\mu_{k_2}}{\sigma_{k_2}^2}\right)x - \frac{\mu_{k_1}^2}{2\sigma_{k_1}^2} + \frac{\mu_{k_2}^2}{2\sigma_{k_2}^2} + \log\left(\frac{\pi_{k_1}}{\pi_{k_2}}\right)$$