Image Processing and Pattern Recognition—Homework 4

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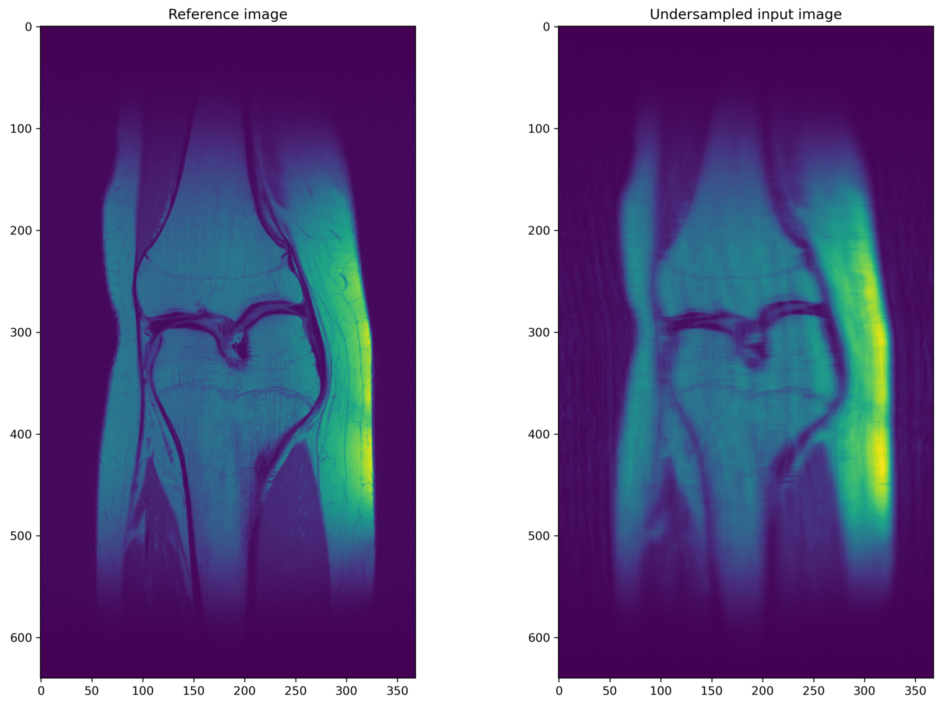
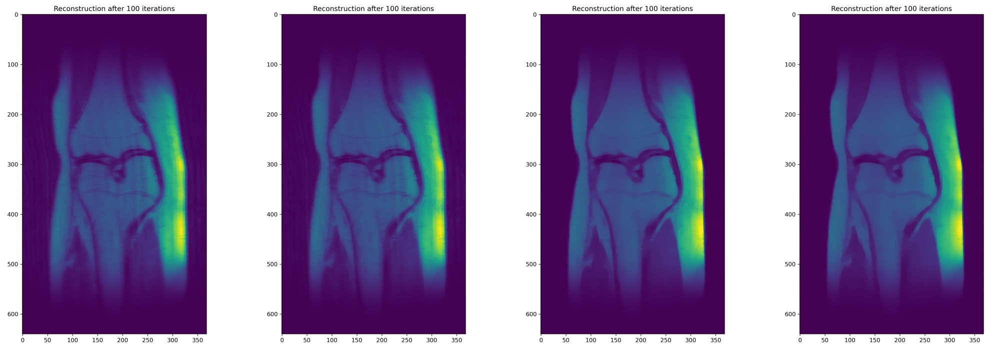


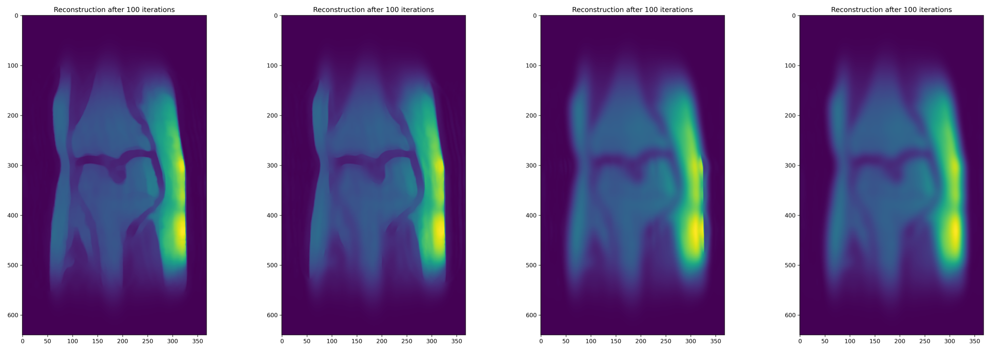
(a) Densely sampled reconstruction (b) Undersampled reconstruction

Figure 1: There is a significant amount of artifacts in the original undersampled reconstruction.

We have implemented the total variation algorithm as described on the assignment sheet in order to apply it on undersampled MRI data visualized in Figure 1. Any parameters except for λ remain unchanged and all reconstructions were run for 100 iterations.



(a) $\lambda = 5 \cdot 10^{-3}$ (b) $\lambda = 1 \cdot 10^{-2}$ (c) $\lambda = 5 \cdot 10^{-2}$ (d) $\lambda = 1 \cdot 10^{-1}$



(e) $\lambda = 5 \cdot 10^{-1}$ (f) $\lambda = 1$ (g) $\lambda = 5$ (h) $\lambda = 50$

Figure 2: If λ is too small, there is no visible improvement; if it becomes too large, more and more detail is lost and the reconstructed image becomes blurry.

Figure 2 shows the result of the TV reconstruction for a wide range of the parameter λ—the best results can indeed be obtained with $\lambda = 5 \cdot 10^{-2}$ (Figure 2c), as there are still artifacts left in Figure 2b, while details start to disappear starting with Figure 2d. We can see that it is vital to find a good trade-off between these two phenomena in tuning λ.

This behavior can be explained by looking at the variational problem we need to solve:

$$\min_{\{u_c\}_{c=1}^{C}} \lambda \sum_{c=1}^{C} \|\nabla u_c\|_{2,1} + \frac{1}{2} \sum_{c=1}^{C} \|M \odot \mathcal{F} u_c - f_c\|^2; \tag{1}$$

we can see that λ is a weighting factor that controls how much emphasis is put on data fidelity versus the TV norm. If the parameter is small, the data fidelity term is weighted more heavily, which is why the reconstruction is closer to the original undersampled reconstruction. When it becomes larger, however, more emphasis is put on the TV norm, the reconstructed image is thus allowed to be more different and, if it becomes too large, the reconstruction is smoothed out more and more.

The total variation algorithm yields results that should be good enough for many diagnostic scenarios, in so far we would say that it is a good prior. However, there are a few issues. One of the major drawbacks of total variation in image reconstruction is that it favors a piecewise constant solution—this leads to a staircasing effect, particularily evident in Figure 2c and Figure 2d in the bright parts on the right. Furthermore, another drawback in comparison to other MRI reconstruction algorithms is the fact that λ needs to be tuned well and that slightly smaller or larger values already have a negative effect, in some other algorithms this tuning does not need to be as precise.

Some of these issues can be remedied by slightly restating the variational problem: using the so-called Huber function instead of the norm reduces staircasing, for example; for MRI reconstruction the total generalized model has also been explored as a possible improvement. Moreover, since the gradient operator does not capture the complicated statistics of natural images, using more complex operators Φ, such as shearlets can also improve the performance of TV.