Consumer Fairness Concerns and Dynamic Pricing in a Channel

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Abstract

The extant literature has shown that when a firm increases its price due to increased demand or consumer valuation, some consumers may have fairness concerns and experience psychological disutility when buying the firm's product. This paper provides a two-period model to study the effects of consumers' fairness concerns on firms' dynamic pricing strategies and profits in a channel. Our analysis reveals a strategic link between the two periods—the retailer has a cost-reduction incentive of lowering its first-period price to induce the manufacturer to reduce the wholesale price in the second period. When the retailer's cost-reduction incentive prevails, in equilibrium, the retail price stays unchanged while the wholesale price decreases over time. Hence, our results provide an alternative explanation for the empirical observation that retail prices typically do not decrease when wholesale prices do (Anderson et al. 2015). Further, we find that a higher demand increase in the second period can lead to a *decrease* in both wholesale and retail prices. Importantly, we show that consumer fairness concerns can result in a win-win outcome for the manufacturer and the retailer, which suggests that firms may prefer not using tactics such as price framing to alleviate fairness concerns.

Key words: fairness, behavioral economics, dynamic pricing, channel

1. Introduction

A new product's demand is typically low initially, even when the product may have high expected quality. Over time, product demand may shift upward due to consumer learning through positive word-of-mouth (e.g., online reviews) or increased brand awareness among consumers. Similarly, when a new marketing campaign is launched for an existing product, or when the product receives endorsements from celebrities or media outlets, consumers' willingness to pay for the product and market demand are also likely to increase over time. Naturally, when consumers' willingness to pay or the product demand increases, the firm can have an incentive to raise its price to reap the economic surplus that its product generates. However, the consumer-behavior literature shows that consumers often perceive price increases as unfair (e.g., Kahneman et al. 1986, Dickson and Karapurakal 1994), which reduces consumers' purchase intention and willingness to pay for the product (Campbell 1999) and may even induce some consumers to boycott the product (Friedman 2002). The extant literature has used both lab and field experiments to demonstrate that a price increase may be perceived as unfair in product categories such as food (Tripathi 2017), toys (Campbell 1999), apparel (Bolton, Warlop and Alba 2003), drugs (Bolton and Alba 2006; Stanley 2016) and health insurance and other services (Maxwell et al. 2013). Campbell (1999) shows that consumers tend to consider increased demand as a negative motive for a firm's price increase; in such situations, a price increase may aggravate consumers' fairness concerns, reducing their willingness to purchase the firm's product. For example, SPAR, a multinational retailer, faced backlash from consumers when it increased the price of hand soap in one of its stores; many consumers felt that the retailer was exploiting the increased demand for soap, calling it "shameful" and "greedy" on social media (Irving 2020). Consulting agencies such as McKinsey warn managers about the importance of consumers' psychological perceptions of price increases (Bondi et al. 2021). Managers tend to be cognizant of the negative psychological effects that a price increase may invoke among consumers (Selove 2019). Blinder et al. (1998) find, from more than 200 interviews with managers, that most managers cited the fear of "antagonizing" customers as the top reason to avoid a price increase. The desire to avoid antagonizing consumers with price increases can also create a conflict between channel members. For example, when Unilever, the manufacturer of famous brands such as Marmite and Ben & Jerry's, announced a 10% increase in its wholesale prices, it created a backlash among many consumers, some of whom vouched to boycott Unilever products; Tesco, the biggest retailer in the UK, refused to pass the price increase on to consumers and threatened to stop selling some Unilever products (Linning et al. 2016). After the companies reached a compromise about the magnitude of the price increase, Tesco still avoided increasing its retail prices for several months (Hayward and Rodger 2017).

The conflict between a firm's desire to raise its price to extract more surplus and the risk of invoking some consumers' negative perceptions (such as fairness concerns) raises several interesting managerial questions. Should firms charge a high initial price to avoid the need for a future price increase or reduce the relative amount of the future price increase to dampen the potential backlash from consumers' fairness concerns? Would firms necessarily benefit from reducing consumer fairness concerns, e.g., by framing low initial prices as one-time introductory offers to reduce the consumers' negative perception of the firms' future price increase? How do consumers' fairness concerns affect the dynamics of a decentralized channel, e.g., the manufacturer's and retailer's optimal dynamic pricing decisions and profits as well as the channel coordination?

One might intuit that consumers' fairness concerns will prevent or mitigate price hikes, making consumers better off and firms worse off. Our analysis shows that this intuition holds in a centralized channel; this implies that a manufacturer selling its product directly to consumers will have an incentive to alleviate consumers' fairness concerns, e.g., by framing its low initial prices as an introductory promotion. However, we show that in a decentralized channel, the effects of consumers' fairness concerns on firms can be qualitatively different; for example, even if the firms can alleviate fairness concerns (e.g., by framing their price changes differently), they may not want to do so. We construct an analytical framework with an upstream manufacturer selling its product through a retailer in two periods in a market where the demand in the second period may increase, e.g., due to consumer learning or positive reviews that increase the consumer's willingness to pay. There is a segment of consumers in the market who have fairness concerns about price increases; more specifically, if the retailer raises its second-period price, there will be a demand drop due to consumers' fairness concerns. Our analysis reveals several interesting findings.

First, consumer fairness concerns will, in equilibrium, induce the manufacturer to charge a higher wholesale price in the first period and a lower wholesale price in the second period than when consumers do not have fairness concerns. Our analysis reveals an important strategic link across the two periods when the market has a segment of consumers with fairness concerns—the lower the retailer's *first-period* retail price, the lower the manufacturer's best-response *second-period* wholesale price. This suggests that the retailer can have an incentive to lower its first-period retail price to induce the manufacturer to decrease its second-period wholesale price, which reduces the retailer's marginal cost in the second period. We refer to this incentive as the retailer's *cost-reduction incentive*. Anticipating this incentive, the manufacturer will increase its first-period wholesale price to exploit its first-period profit margin, knowing that the retailer will not increase its first-period retail price too much in response to the increased first-period wholesale price. What about the manufacturer's second-period wholesale price? Note that consumer fairness concerns make product demand more sensitive to the retail price, which induces the retailer to decrease its second-period price to alleviate or avoid triggering consumers' fairness concerns. The increased price sensitivity of demand and the retailer's incentive to reduce its second-period price allow the manufacturer to obtain a higher increase in unit sales with a marginal decrease in its second-period wholesale price. The manufacturer's incentive to reduce its second-period wholesale price is further strengthened when the retailer strategically reduces its first-period price, lowering the reference price for the second period; doing so allows the retailer to obtain a lower wholesale price in the second period because if the manufacturer does not decrease its wholesale price, then the retail price will be high, leading to a significant profit loss because of consumers' fairness concerns. For these reasons, consumers' fairness concerns will reduce the second-period equilibrium wholesale price.

Second, counterintuitively, consumer fairness concerns can, in equilibrium, reduce both the first-period and second-period retail prices. One might intuit that consumer fairness concerns will induce the retailer to increase its first-period retail price and reduce its second-period price to mitigate the potential demand drop due to fairness concerns in the second period. This intuition holds for a centralized channel. However, we find that in a decentralized channel, with a segment of consumers having fairness concerns, the retailer has to weigh two *opposing* incentives when choosing its first-period price. On the one hand, the retailer has an incentive to raise its first-period price to mitigate or eliminate the second-period demand drop due to fairness concerns. We refer to this as the retailer's *fairness-mitigation incentive*. On the other hand, the retailer has a *cost-reduction incentive*—reducing its first-period retail price to induce the manufacturer to lower the second-period wholesale price, which directly reduces the retailer's marginal cost in the second period. Our analysis shows that as long as the demand intercept does not increase too much in the second period, the retailer's cost-reduction incentive will dominate its fairness-mitigation incentive, making it optimal for the retailer to charge a lower first-period price than in the case without fairness concern.

Third, we find that a higher increase in the second-period demand can induce both the manufacturer and the retailer to *decrease* their prices in *both* periods. This is counterintuitive since one would intuit that a higher increase in the second-period demand intercept would allow firms to raise their prices. We show that this intuition typically holds, except when the increase in the second-period demand transitions from below a critical threshold (the low-growth region) to above the threshold (the high-growth region), in which case there can be a paradigm shift in the manufacturer's pricing strategy. Intuitively, in the low-growth region, the retailer does not have a strong incentive to increase its second-period price, and hence, the retailer's fairness-mitigation incentive is weak and is dominated by its cost-reduction incentive, i.e., the retailer wants to keep its first-period retail price low in order to obtain a low wholesale price in the second period. Realizing this, the manufacturer optimally sets a high wholesale price to exploit the retailer's cost-reduction incentive, knowing that the retailer will not add a high markup on top of the wholesale price. By contrast, in the high-growth region, the large increase in the second-period demand gives a natural tendency for a high second-period retail price, thus creating a strong fairness-mitigation incentive for the retailer in the first period, i.e., raising its first-period retail price so that consumers' fairness concerns in the second period will not be too strong. In this situation, the manufacturer finds it optimal to substantially reduce its first-period wholesale price because otherwise the retailer's first-period price would be too high, yielding small first-period sales and profits for the manufacturer. The drop in the manufacturer's first-period wholesale price leads to a discrete drop in subsequent retail and wholesale prices.

Fourth, even though consumer fairness concerns have a negative direct effect on the second-period demand, the second-period *equilibrium* unit sales will be higher than when consumers do not have fairness concerns. This is because consumer fairness concerns give the retailer a stronger incentive to reduce its second-period price, which also allows the manufacturer to increase sales more efficiently by decreasing its wholesale price. In other words, fairness concerns help alleviate the double-marginalization problem, allowing the channel to achieve higher sales and profits in the second period. This insight is crucial for understanding the profit implications of fairness concerns, which we discuss below.

Fifth, we find that consumers' fairness concerns can make the manufacturer better off in both periods. Namely, in the first period, the manufacturer exploits the retailer's cost-reduction incentive of keeping its first-period retail price low to obtain a lower wholesale price in the second period. That is, knowing that the retailer will not increase its first-period price too much, the manufacturer can charge a high wholesale price in the first period, capturing a large share of the channel profits. The manufacturer also gains more profit in the second period due to alleviated double marginalization; even though consumers' fairness concerns reduce the equilibrium wholesale price in the second period, the manufacturer's gain from increased sales will dominate the drop in its profit margin, making the manufacturer better off than when consumers do not have fairness concerns.

Lastly, we show that the manufacturer's benefit from consumer fairness concerns does not have to come at the retailer's expense, i.e., the retailer can also benefit from the consumers' fairness concerns. The retailer's profit can increase mainly because fairness concerns allow it to reduce its second-period cost by decreasing its first-period retail price to induce the manufacturer to lower its second-period wholesale price. Essentially, consumer fairness concerns enable the retailer to use a low first-period mark-up (over the first-period wholesale price) to "commit" to a low second-period retail mark-up, sacrificing some first-period profits to reduce its marginal cost (i.e., the wholesale price) in the second period. Through these inter-period pricing dynamics, fairness concerns can make the channel more coordinated, reducing the double-marginalization problem and improving the channel pricing efficiency (especially in the second period) by a high enough amount to allow both firms to benefit.

In short, in markets where some consumers exhibit fairness concerns against price increases, the retailer strategically chooses its first-period price by weighing its cost-reduction and fairness-mitigation incentives. The strategic choice of the first-period price allows the retailer to improve channel efficiency in the second period, leading to increased sales and higher channel profits, which can make both the manufacturer and retailer better off. First-period channel efficiency can also improve due to the retailer's incentive to keep its first-period price low (cost-reduction incentive), which increases the manufacturer's first-period profits.

Our analytical results are consistent with some empirical evidence about wholesale and retail price dynamics and may offer an alternative perspective for explaining the data. Specifically, several studies have shown that retail prices in many product categories often do not decrease when wholesale prices go down (e.g., Peltzman 2000, Anderson et al. 2015). This may seem puzzling in a static setting, where a reduced wholesale price would give the retailer incentives to decrease its retail price. To explain this phenomenon, the past literature has mainly relied on the existence of menu costs for the retailer that make it costly to adjust the prices (e.g., Levy et al. 1997). However, Anderson et al. (2015) show that menu costs cannot explain why retail prices do not decrease when wholesale prices do, and they suggest that reductions in wholesale prices may be associated with a different paradigm than increases in wholesale prices. In our dynamic setting with consumers' fairness concerns, the lack of retail price adjustment after a wholesale price reduction is consistent with our equilibrium results for products that do not experience significant changes in demand conditions over time. Namely, we find that in equilibrium, the retail price will stay unchanged over time even when the wholesale price drops in the future. This is essentially the retailer's cost-reduction mechanism that our analysis reveals—the retailer strategically charges a low initial markup knowing that this will induce the manufacturer to drop its wholesale price in the future in order to prevent the retailer from increasing the retail price. We acknowledge that other factors may be at play to explain such empirical observations. However, our analysis reveals another plausible contributing factor in a practically relevant setting. We hope that our research motivates more empirical research to test our theoretical results.

2. Literature Review

One significant stream of literature in consumer behavior and psychology has studied consumers' perception of price fairness and its impact on the consumers' purchase decisions. Kahneman et al. (1986), Sinha and Batra (1999), and Maxwell (2002) demonstrate that consumers who perceive a firm as unfair will become less willing to purchase that firm's product. Kahneman et al. (1986) document that most consumers consider it unfair for a firm to raise prices to exploit higher product demand. Campbell (1999) shows that consumers tend to consider increased demand as a negative motive for a firm's price increase and that such a price increase will be perceived as unfair, reducing consumers' willingness to purchase the firm's product. Furthermore, some studies have found that consumers often take past prices as a reference point and thus tend to perceive a firm's price increase as a loss, making them less likely to purchase from the firm. This paper develops a dynamic, analytical model, taking it as given that consumers may perceive a firm's price increase as unfair and thus be less likely to purchase the firm's product due to fairness concerns. We investigate how such fairness concerns will affect the manufacturer's and the retailer's dynamic pricing decisions and profits.

A stream of literature has studied the effects of managers' and consumers' behavioral preferences on market outcomes. For example, the extant literature has analyzed reciprocity among players (Charness and Rabin 2002), consumers' concerns for their payoff standing relative to other players (Bolton and Ockenfels 2000), context-dependent preferences (Narasimhan and Turut 2013), outcome-dependent behavioral preferences (Jiang et al. 2017; Zou et al. 2020), the existence of reference groups (Amaldoss and Jain 2008), reference products (Amaldoss and He 2018), and behavioral preferences by managers or firms (Jiang and Liu 2019; Jiang et al. 2014). Our work is more related to studies that have explored the strategic implications of consumers' fairness concerns. One stream of this literature has studied distributional fairness based on how the surplus is divided between different parties involved in the economic transaction (Fehr and Schmidt 1999). For example, Guo (2015) explores fairness concerns in a buyer-seller context when the buyer faces uncertainty about the seller's cost; Guo and Jiang (2016) investigate the effect of fairness concerns on the firm's pricing and product quality. Another stream of this literature has focused on peer-induced fairness concerns when a consumer gets a disutility if her payoff is less than others'. Peer-induced fairness concerns have been studied in several contexts, such as bargaining (Ho and Su 2009) and product-line pricing (Chen and Cui 2013). Li and Jain (2015) study peer-induced fairness concerns that arise when a firm charges its existing customers and new customers different prices. In contrast to this line of research, we explore a dynamic setting where firms can change their prices over time, but they cannot price discriminate against consumers at a given point in time. Our dynamic framework allows us to focus on another well-documented source of consumer fairness concerns based on consumers' comparison of current and past prices. Selove (2019) also explores a dynamic setting with consumers' fairness concerns where the firm may not be able to meet the full demand due to limited capacity. He finds that the interaction of fairness concerns with travel costs can result in a firm setting stable prices, which will lead to shortages during high demand. By contrast, we study the manufacturer's and the retailer's dynamic pricing decisions in a distribution channel and explore the effect of fairness concerns on channel coordination and profitability.

Note that, from a conceptual point of view, fairness concerns in the aforementioned literature can be seen as a type of loss aversion (or, more generally, reference-dependent consumer preferences) since fairness concerns can be framed as consumers being averse to paying a higher price than the reference price. In this paper, we will use the "fairness" terminology since, in many behavioral studies, fairness is a central theme when it comes to the survey participants' perceptions regarding product price increases. Some existing literature has examined firms' pricing decisions when consumers perceive the firm's price as a gain or a loss relative to a reference price.[1] Greenleaf (1995) studies a firm selling directly to consumers and analyzes the effect of an exogenously given reference-price on the firm's optimal price promotions. Kopalle et al. (1996) extend Greenleaf (1995) by examining a multiproduct firm to show that reference prices can lead to constant prices over time if consumers are very sensitive to losses relative to the reference prices. Kuksov and Wang (2014) show that when consumers' search cost is sufficiently small, the existence of loss-averse consumers can alleviate price competition, making firms better off. Popescu and Wu (2007)

[1] For a review of the reference-price literature, see Mazumdar et al. (2005).

show that if consumers are loss-averse, a firm's optimal prices will converge to a constant steady-state price. These studies on reference-dependent consumer preferences have focused on firms that sell directly to consumers; in contrast, this paper examines how reference-dependent preferences like fairness concerns affect the upstream and downstream firms' dynamic pricing strategies in a *decentralized* channel.

Several papers in the literature have studied reference-dependent preferences in distribution channels. Ho and Zhang (2008) show that when the retailer's utility is reference-dependent, the manufacturer's use of two-part tariffs may reduce channel efficiency. Cui et al. (2007) show that when the retailer cares about distributional fairness of channel profits, the manufacturer can achieve full channel coordination with a simple wholesale price. Zhang et al. (2014) use a continuous-time, infinite-horizon model to study reference-price effects by assuming that the consumer's reference price evolves following a differential equation with respect to time; they show that both centralized and decentralized channels always prefer consumers to have a higher initial exogenous reference price and stronger reference-price sensitivity. Yi et al. (2018) show that if consumers have very strong fairness concerns, the manufacturer may want to sell its product through an intermediary (i.e., an agent) rather than directly to consumers because selling through an intermediary will prevent consumers from feeling unfair about the manufacturer's high profit margin. We complement this stream of literature by explicitly analyzing the dynamic wholesale and retail pricing decisions in a market with some consumers having fairness concerns about price increases. In our dynamic setting, the retailer's optimal pricing decisions are driven by its inter-period *fairness-mitigation* and *cost-reduction* incentives, which, in turn, affect a strategic manufacturer's dynamic wholesale pricing decisions. We identify an inter-period strategic link between the first-period retail price and the second-period wholesale price, which plays an important role in influencing the dynamic outcomes of the channel.

3. Model

Consider a manufacturer selling a nondurable product through a retailer in two time periods $t \in \{1,2\}$ to meet market demand. In each period t, the manufacturer sets its wholesale price (w_t) and subsequently, the retailer chooses its retail price (p_t). The manufacturer has a constant marginal cost, which is, without loss of generality, normalized to zero. The retailer's marginal cost is the wholesale price paid to the manufacturer. Formally, the game proceeds as follows. First, the manufacturer chooses its first-period wholesale price (w_1). Second, the retailer chooses its first-period retail price (p_1) and subsequently, the first-period demand and sales are realized. Third, the manufacturer chooses its second-period wholesale price (w_2). Lastly, the retailer chooses its second-period retail price (p_2) and the second-period demand and sales are then realized. The manufacturer and the retailer will choose their pricing strategies to maximize their respective overall profits from both periods.

To clearly identify and explain the effects of fairness concerns, let us start with a benchmark in which no consumers have fairness concerns. We use a superscript "NF" on the demand and equilibrium variables to indicate this "no fairness concern" benchmark. In the first period, the market demand is given by $D_1^{NF} = 1 - \beta p_1$, where β represents how elastic the market demand is. As we discussed in the Introduction, product demand may shift over time due to a variety of factors. Thus, in our model, the market demand in the second period is expressed as $D_2^{NF} = 1 + \delta - \beta p_2$, where δ represents the change in demand.[2] In the main paper, we focus on the situation where $\delta \geq 0$, i.e., the market demand either stays the same or increases in the second period.[3] In section 5, we will discuss the case of diminishing demand and demand uncertainty in the second period. Such reduced-form demand formulations are commonly used in the literature (e.g., McGuire and Staelin 1983, Moorthy 1988). Note that our two-period, reduced-form demand framework can be rationalized by consumer utility-based models. For example, suppose that consumers' expected valuations for the product in the first period are uniformly distributed. Past research has shown that consumers' satisfaction with a product or service can increase their future willingness to pay for it (Homburg et al. 2005). Increased willingness to pay can also be driven by positive word-of-mouth or

[2] In the discussion section, we will consider an alternative demand model where the second-period demand (without fairness concerns) is $D_2^{NF} = (1 + \delta)(1 - \beta p_2)$ rather than $D_2^{NF} = 1 + \delta - \beta p_2$.

[3] In practice, it may take time for a firm to build up consumer awareness, communicate product information and create brand value. Hence, as time passes, the firm's product demand may increase. In Part I of the Online Appendix, we provide a model with a micro-foundation, showing how the revelation of an ex-ante uncertain positive quality about some product attributes can lead to an increase in the firm's future product demand.

reviews about the product, improved brand reputation and image, celebrity or influencer endorsements, etc. Hence, in the second period, consumers' valuations for the product increase by δ. This setting would lead to similar first-period and second-period linear demand functions as those described above. Please see the Online Appendix for more details of two different models with a micro-foundation that can lead to a similar linear demand framework as in our core model. Note that we use the linear demand framework in the main paper to make our key analysis and insights more transparent and easier to follow without imposing extra layers of complexity on the already complex dynamic framework.

One can solve the equilibrium outcome using backward induction. All proofs in this paper have been relegated to the Online Appendix. In the second period, for a given second-period wholesale price w_2, the retailer chooses p_2 to maximize its second-period profit $\pi_{R2}^{NF} = (p_2 - w_2)\, D_2^{NF}$, where $D_2^{NF} = 1 + \delta - \beta p_2$. The retailer's optimal second-period price satisfies the first-order condition

$$(p_2 - w_2)\frac{dD_2^{NF}}{dp_2} + D_2^{NF} = 0. \tag{1}$$

Solving (1), we find the retailer's optimal price, $p_2^{*NF}(w_2) = \frac{1+\delta+\beta w_2}{2\beta}$. The manufacturer anticipates the retailer's response and chooses w_2 to maximize its second-period profit, $\pi_{M2}^{NF} = w_2 \cdot D_2^{NF}(p_2^{*NF})$, where $D_2^{NF}(p_2^{*NF}) = 1 + \delta - \beta p_2^{*NF} = \frac{1+\delta-\beta w_2}{2}$. The corresponding first-order condition is as follows.

$$w_2 \frac{dD_2^{NF}}{dp_2}\frac{dp_2^{*NF}}{dw_2} + D_2^{NF}(p_2^{*NF}) = 0. \tag{2}$$

Solving (2), we obtain the manufacturer's second-period equilibrium price: $w_2^{*NF} = \frac{1+\delta}{2\beta}$. Upon plugging w_2^{*NF} into $p_2^{*NF}(w_2)$, we find the second-period equilibrium retail price: $p_2^{*NF} = \frac{3(1+\delta)}{4\beta}$. Next, using the equilibrium wholesale and retail prices, we obtain the retailer's and the manufacturer's second-period profits. $\pi_{R2}^{*NF} = \frac{(1+\delta)^2}{16\beta}$ and $\pi_{M2}^{*NF} = \frac{(1+\delta)^2}{8\beta}$.

In the first period, for a given first-period wholesale price w_1, the retailer chooses p_1 to maximize its total profit $\pi_R^{NF} = (p_1 - w_1)\, D_1^{NF} + \pi_{R2}^{*NF}$, where $D_1^{NF} = 1 - \beta p_1$. Since $\frac{d\pi_{R2}^{*NF}}{dp_1} = 0$, the first-order condition characterizing the retailer's optimal price is as follows.

$$(p_1 - w_1)\frac{dD_1^{NF}}{dp_1} + D_1^{NF} = 0. \tag{3}$$

One can readily show that the solution to (3) is $p_1^{*NF}(w_1) = \frac{1+\beta w_1}{2\beta}$. Then the manufacturer chooses w_1 to maximize its total profit $\pi_M^{NF} = w_1\, D_1^{NF}(p_1^{*NF}) + \pi_{M2}^{*NF}$, where $D_1^{NF}(p_1^{*NF}) = 1 - \beta p_1^{*NF} = \frac{1-\beta w_1}{2}$. The manufacturer's optimal first-period wholesale price satisfies the following first-order condition:

$$w_1 \frac{dD_1^{NF}}{dp_1}\frac{dp_1^{*NF}}{dw_1} + D_1^{NF}(p_1^{*NF}) = 0. \tag{4}$$

Solving (4), we find the manufacturer's optimal wholesale price $w_1^{*NF} = \frac{1}{2\beta}$. Plugging w_1^{*NF} into $p_1^{*NF}(w_1)$, we find the corresponding first-period retail price: $p_1^{*NF} = \frac{3}{4\beta}$. Finally, using the equilibrium prices, we find the manufacturer's and retailer's first-period equilibrium profits: $\pi_{M1}^{*NF} = \frac{1}{8\beta}$ and $\pi_{R1}^{*NF} = \frac{1}{16\beta}$. Their profits over the two periods are given by $\pi_M^{*NF} = \frac{1+(1+\delta)^2}{8\beta}$ and $\pi_R^{*NF} = \frac{1+(1+\delta)^2}{16\beta}$.

Lemma 1 shows that as one would expect, in the absence of consumer fairness concerns, the increase in demand will lead to higher wholesale and retail prices in the second period.

LEMMA 1. *In the absence of consumer fairness concerns, an increase* ($\delta > 0$) *in the second-period demand leads to higher wholesale and retail prices in the second period than in the first period*; mathematically, $w_2^{*NF} > w_1^{*NF}$ and $p_2^{*NF} > p_1^{*NF}$.

Next, we explain how we model consumer fairness concerns. As documented by Kahneman et al. (1986), most consumers consider it unfair for a firm to raise prices to exploit higher product demand. Campbell (1999) also shows that consumers tend to consider increased demand to be a negative motive for a firm's price increase, which may be perceived as unfair and reduce consumers' willingness to purchase the firm's product. Moreover, many studies (e.g., Rinne 1981, Gurumurthy and Little 1988, Kalwani et al. 1990, Hardie et al. 1993) have documented significant reference-price effects on retail demand when consumers take past prices as a reference point and perceive a firm's price increase as a loss, making them less likely to buy the firm's product. As mentioned earlier, consumer fairness concerns are conceptually a type of loss-aversion or context-dependent preferences by the consumer. Past behavioral studies (e.g., Guo and Jiang 2016) also typically show that consumers may have different degrees of fairness concerns or loss aversion—some consumers have strong fairness concerns, whereas others may have no such concerns. To capture the consumer heterogeneity in fairness concerns, we assume that consumers can be of two types—one with fairness concerns and one without fairness concerns. Let $\lambda \in (0,1)$ denote the fraction of consumers with fairness concerns and $1-\lambda$ be the fraction of consumers without fairness concerns. Note that a price increase may not trigger fairness concerns in consumers who do not recall or are not aware of lower past prices; our model has essentially included these consumers in the segment of consumers without fairness concerns.

In the first period, the market demand is given by $D_1 = 1 - \beta p_1$, where p_1 is the retail price of the product in the first period. In the second period, if the retailer raises its price (p_2), the demand will decrease not only because of the price increase's conventional direct effect on demand but also its negative effect due to consumers' fairness concerns. We can express the retailer's second-period demand as the weighted average of the demand from the two segments of consumers with or without fairness concerns: $D_2 = \lambda(1 + \delta - \beta p_2 - \gamma \max\{p_2 - p_1, 0\}) + (1-\lambda)(1 + \delta - \beta p_2)$, where the first term represents the demand from consumers with fairness concerns and the second term represents the demand from consumers without fairness concerns. The parameter γ represents the strength of the consumers' fairness concerns, and $\gamma \max\{p_2 - p_1, 0\}$ captures the potential negative effect of fairness concerns on demand.[4] One can see that if the retailer does not increase its second-period price (i.e., $p_2 \leq p_1$), its second-period demand will simply be $D_2 = 1 + \delta - \beta p_2$. In essence, the first-period retail price (p_1) serves as the consumer's reference price in the second period. If the second-period retail price p_2 is higher than p_1, then some of the consumers with fairness concerns may decide not to buy the product, resulting in a demand drop in the second period proportional to $\gamma \max\{p_2 - p_1, 0\}$.

[4] Note that if γ is too high, firms will not serve the consumers with such strong fairness concerns, and it is as if there are no such consumers in the market. Thus, for non-trivial analysis, we assume that the consumers' fairness is not overwhelmingly strong (in particular, $\gamma \leq \frac{\beta}{3}$) such that in equilibrium, firms serve some of these customers.

3.1 Benchmark: Effects of Fairness Concerns in Centralized Channel

Before analyzing the core model, let us consider a centralized channel as a benchmark and study how the existence of a positive fraction of consumers having fairness concerns affects the market outcome when the manufacturer sells its product *directly* to consumers. Other aspects of the model are the same as those in the core model. We will use a tilde on equilibrium variables to indicate the case of a centralized channel. Note that the first-period and second-period demand functions are the same as those specified in the core model: $D_1 = 1 - \beta p_1$ and $D_2 = \lambda\,(1 + \delta - \beta p_2 - \gamma \max\{p_2 - p_1, 0\}) + (1 - \lambda)(1 + \delta - \beta p_2)$. Also note that the second-period demand can be expressed as follows: $D_2 = D_2^{NF} - \lambda\gamma \max\{p_2 - p_1, 0\}$, where $D_2^{NF} = 1 + \delta - \beta p_2$ is demand component without fairness concerns and $-\lambda\gamma \max\{p_2 - p_1, 0\}$ captures the effect of fairness concerns on demand. The centralized manufacturer's total profit can be expressed as $\pi = D_1\, p_1 + D_2\, p_2$. We solve the game by backward induction. First, , given p_1, the manufacturer chooses its second-period price p_2 to maximize its second-period profit, $\pi_2 = D_2\, p_2$.

In the Appendix, we demonstrate that if the first-period price is very high ($p_1 > \frac{1+\delta}{2\beta}$), then the second-period optimal price, $\tilde{p}_2$, will be lower than the first-period price ($\tilde{p}_2 < p_1$) and will satisfy the following first-order condition:

$$p_2 \frac{dD_2^{NF}}{dp_2} + D_2^{NF} = 0. \tag{5}$$

Solving (5), one can readily show that $\tilde{p}_2 = \frac{1+\delta}{2\beta}$. However, if the first-period price is low ($p_1 < \frac{1+\delta}{2\beta+\gamma\lambda}$), then the second-period price will satisfy $\tilde{p}_2 > p_1$, and the relevant first-order condition will be

$$p_2 \frac{dD_2}{dp_2} + D_2 = 0, \tag{6}$$

where $D_2 = D_2^{NF} - \lambda\gamma(p_2 - p_1)$. Using the expression for D_2 and rearranging terms, (6) becomes

$$p_2\left(\frac{dD_2^{NF}}{dp_2} - \lambda\gamma\right) + D_2^{NF} - \lambda\gamma(p_2 - p_1) = 0. \tag{7}$$

Solving (7), we find that $\tilde{p}_2 = \frac{1+\delta+\lambda\gamma p_1}{2(\beta+\gamma)}$. Comparing equations (5) and (7), we see that the term $-\lambda\gamma p_2$ captures the downward pressure on second-period price from fairness-induced increased price-sensitivity of demand. The term $-\lambda\gamma(p_2 - p_1)$ captures the downward pressure on price from decreased demand due to fairness concerns. Hence, as we would expect, the second-period retail price tends to be lower with consumers' fairness concerns.

Finally, when p_1 is in an intermediate range ($\frac{1+\delta}{2\beta+\gamma\lambda} \le p_1 \le \frac{1+\delta}{2\beta}$), we find that the retailer's optimal price is at a corner solution: $\tilde{p}_2 = p_1$.

In the Appendix, we show that the manufacturer's optimal second-period price is

$$\tilde{p}_2 = \begin{cases} \frac{1+\delta+\lambda\gamma p_1}{2(\beta+\gamma)} & \text{if } 0 < p_1 < \frac{1+\delta}{2\beta+\gamma\lambda} \\ p_1 & \text{if } \frac{1+\delta}{2\beta+\gamma\lambda} \le p_1 \le \frac{1+\delta}{2\beta}. \\ \frac{1+\delta}{2\beta} & \text{if } p_1 > \frac{1+\delta}{2\beta} \end{cases} \tag{8}$$

Using $\tilde{p}_2$, we can obtain the manufacturer's second-period subgame equilibrium profits:

$$\tilde{\pi}_2 = \begin{cases} \frac{(1+\delta+p_1\gamma\lambda)^2}{4(\beta+\gamma\lambda)} & \text{if } 0 < p_1 < \frac{1+\delta}{2\beta+\gamma\lambda} \\ p_1(1+\delta-\beta p_1) & \text{if } \frac{1+\delta}{2\beta+\gamma\lambda} \le p_1 \le \frac{1+\delta}{2\beta}. \\ \frac{(1+\delta)^2}{4\beta} & \text{if } p_1 > \frac{1+\delta}{2\beta} \end{cases} \tag{9}$$

In the first period, the manufacturer chooses p_1 to maximize its overall profit $\pi = D_1 p_1 + \tilde{\pi}_2$. The first-order condition corresponding to an interior solution is given by

$$p_1 \frac{dD_1}{dp_1} + D_1 + \frac{d\tilde{\pi}_2}{dp_1} = 0. \tag{10}$$

Notice that $\frac{d\tilde{\pi}_2}{dp_1} \ge 0$, with strict inequality when $p_1 < \frac{1+\delta}{2\beta}$. In words, the manufacturer gains an incentive to charge a higher first-period price to alleviate consumers' fairness concerns and increase its second-period profit. Lemma 2 provides the manufacturer's equilibrium first- and second-period prices.

LEMMA 2. *In a market with consumer fairness concerns, the centralized manufacturer's equilibrium first-period and second-period prices are given by*

$$\tilde{p}_1 = \begin{cases} \frac{2+\delta}{4\beta} & \text{if } 0 \leq \delta < \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \\ \frac{2\beta+\gamma\lambda(3+\delta)}{4\beta^2+4\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \leq \delta < \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}\text{, and} \\ \frac{1+\delta}{2\beta} & \text{if } \delta \geq \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \end{cases} \tag{11}$$

$$\tilde{p}_2 = \begin{cases} \frac{2+\delta}{4\beta} & \text{if } 0 \leq \delta < \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \\ \frac{2\beta(1+\delta)+\gamma\lambda}{4\beta^2+4\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \leq \delta < \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}\text{, respectively.} \\ \frac{1+\delta}{2\beta} & \text{if } \delta \geq \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \end{cases} \tag{12}$$

Note that if no consumers have fairness concerns (i.e., $\lambda = 0$), the centralized manufacturer's optimal first-period price, $\tilde{p}_1^{NF} = \frac{1}{2\beta}$, does *not* depend on the second-period demand. But if $\lambda > 0$ fraction of consumers have fairness concerns, the manufacturer's optimal decision for its first-period price will have to weigh the potential impact of the first-period price on the *second-period* demand. Specifically, as one can verify from Lemma 2, if δ is not too large ($\delta < \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}$), then consumers' fairness concerns induce the centralized manufacturer to increase its first-period price (i.e., $\tilde{p}_1 > \frac{1}{2\beta} = \tilde{p}_1^{NF}$) and decrease its second-period price (i.e., $\tilde{p}_2 < \frac{1+\delta}{2\beta} = \tilde{p}_2^{NF}$) to mitigate the negative effect of consumers' fairness concerns. Note that if there is a very large demand increase in the second period (i.e., if $\delta \geq \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}$), the manufacturer's optimal second-period price when facing $\lambda > 0$ fraction of consumers with fairness concerns will be the same as that when no consumers have fairness concerns ($\lambda = 0$). Intuitively, this is because, with a very large δ, the manufacturer will essentially give up the relatively low first-period demand by setting a high reference price ($\tilde{p}_1$) so that it can reap the benefit of the high demand in the second period without triggering consumer fairness concerns.

From Lemma 2, one can easily show that $\frac{d\tilde{p}_1}{d\delta} > 0$ and $\frac{d\tilde{p}_2}{d\delta} > 0$, i.e., with a fraction of consumers having fairness concerns in the market, a larger demand increase (δ) in the second period will lead to higher equilibrium retail prices in *both* periods. This is intuitive. As δ increases, the centralized manufacturer gains incentives to raise its second-period price, which will, in turn, increase the manufacturer's incentive to raise the first-period price $\tilde{p}_1$ to lessen the second-period demand drop (of $\gamma(\tilde{p}_2 - \tilde{p}_1)$) due to fairness concerns.

Using the equilibrium prices in Lemma 2, we can obtain the centralized manufacturer's profit:

$$\tilde{\pi} = \begin{cases} \frac{(2+\delta)^2}{8\beta} & \text{if } 0 \leq \delta < \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \\ \frac{\beta(1+(1+\delta)^2+\gamma\lambda(2+\delta)}{4\beta^2+4\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \leq \delta < \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}. \\ \frac{(1+\delta)^2}{4\beta} & \text{if } \delta \geq \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \end{cases} \tag{13}$$

As one would expect, the existence of a segment of consumers with fairness concerns tends to reduce the centralized manufacturer's profit, i.e., $\tilde{\pi} < \frac{1+(1+\delta)^2}{4\beta} = \tilde{\pi}^{NF}$, where $\tilde{\pi}^{NF}$ is the centralized manufacturer's profit if no consumers have fairness concerns. This suggests that if a centralized manufacturer expects to increase the price after its product gains traction in the market, it will have an incentive to frame the initial low price as an introductory promotion price to alleviate consumer fairness concerns when it raises its price in the future. In the next section, we will show that the effects of fairness concerns in a *decentralized* channel will be qualitatively different from those in a centralized channel. We will show that firms in a decentralized channel can *benefit* from the consumers' fairness concerns; this suggests that firms may *not* want to alleviate fairness concerns even if they can do so without extra cost (e.g., by framing initial low prices as introductory promotion).

4. Effects of Fairness Concerns on Dynamics and Market Outcome of Decentralized Channel

We now consider our core model of a decentralized channel, where the manufacturer sells its product through an independent retailer in a market that has a fraction $\lambda > 0$ of consumers having fairness concerns. In this section, we assume δ to be below a certain threshold (denoted by δ^{**}) so that, in equilibrium, the retailer will serve a positive fraction of consumers in the first period. We will discuss the case of $\delta \geq \delta^{**}$ in Section 5.

The manufacturer's and the retailer's total profits can be written as $\pi_M = \pi_{M1} + \pi_{M2}$ and $\pi_R = \pi_{R1} + \pi_{R2}$, where π_{Mt} and π_{Rt} denote the manufacturer's and the retailer's profits in period $t \in \{1, 2\}$, respectively. We solve the game by backward induction, starting with the second period. All technical details of the analysis are provided in the Online Appendix; this section outlines the main analytical steps. Given the first-period wholesale and retail prices, the retailer chooses its optimal second-period price p_2 conditional on the manufacturer's second-period wholesale price w_2. If the retailer's second-period price is below its first-period price ($p_2 < p_1$), then consumers' fairness concerns will not be evoked and $D_2 = D_2^{NF}$. In this subgame situation, the analysis is similar to that for our benchmark without fairness concerns, and one can show that the retailer's best response price is $p_2^* = \frac{1+\delta+\beta w_2}{2\beta}$. Let us consider the more interesting scenario in which the retailer's second-period price exceeds its first-period price ($p_2 > p_1$). The first-order condition characterizing the retailer's best-response p_2 is as follows:

$$(p_2 - w_2)\frac{dD_2^{NF}}{dp_2} + D_2^{NF} - \lambda\gamma p_2 - \lambda\gamma(p_2 - p_1) = 0. \tag{14}$$

Akin to equation (7) in the benchmark with a centralized channel, the terms $-\lambda\gamma p_2$ and $-\lambda\gamma(p_2 - p_1)$ in equation (14) capture the downward pressure on the second-period retail price due to consumers' fairness concerns. Solving (14), we find $p_2^* = \frac{1+\delta+(\beta+\gamma\lambda)w_2+\gamma\lambda\, p_1}{2(\beta+\gamma\lambda)}$.

In the Online Appendix, we provide a full characterization of the retailer's best-response second-period price. Our analysis shows that

$$p_2^*(w_2) = \begin{cases} \frac{1+\delta+\beta w_2}{2\beta} & \text{if } 0 \le w_2 < \omega_a \\ p_1 & \text{if } \omega_a \le w_2 < \omega_b \\ \frac{1+\delta+(\beta+\gamma\lambda)w_2+\gamma\lambda\, p_1}{2(\beta+\gamma\lambda)} & \text{if } \omega_b \le w_2 \le \omega_c \\ \frac{1+\delta+\beta w_2}{2\beta} & \text{if } \omega_c < w_2 \le \frac{1+\delta}{\beta} \end{cases}, \tag{15}$$

where $\omega_a = \max\{0, \frac{2p_1\beta-1-\delta}{\beta}\}$, $\omega_b = \max\{0, \frac{2p_1\beta+p_1\gamma\lambda-1-\delta}{\beta+\gamma\lambda}\}$, and $\omega_c = \frac{1+\delta+\gamma p_1}{\beta+\gamma} - \frac{\gamma(1+\delta-\beta p_1)}{\beta+\gamma}\sqrt{\frac{1-\lambda}{\beta(\beta+\gamma\lambda)}}$.

We see that when $0 \le w_2 < \omega_a$, the retailer's best-response second-period price is strictly below its first-period price, thus triggering no fairness concerns from consumers. When $\omega_a \le w_2 < \omega_b$, the retailer will set its second-period price to be equal to its first-period price, hence also triggering no fairness concerns. For higher wholesale prices $\omega_b \le w_2 \le \omega_c$, the retailer's best-response second-period price will be higher than its first-period price, provoking fairness concerns from some consumers. Finally, when the wholesale price is even higher ($\omega_c < w_2 \leq \frac{1+\delta}{\beta}$), the retailer's second-period price will be very high, and sales will be positive only in the consumer segment without fairness concerns.

Anticipating the retailer's best response $p_2^*(w_2)$, the manufacturer chooses w_2 to maximize its second-period profit, $\pi_{M2} = w_2 \cdot D_2(p_2^*(w_2))$. We find that the manufacturer's optimal wholesale price is

$$w_2^*(p_1) = \begin{cases} \frac{1+\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 \leq p_1 \leq \rho_a \\ \frac{2p_1\beta-1-\delta+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \rho_a \leq p_1 \leq \rho_b \\ \frac{1+\delta}{2\beta} & \text{if } p_1 > \rho_b \end{cases}, \tag{16}$$

where $\rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$ and $\rho_b = \frac{(1+\delta)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. Detailed technical derivations of $w_2^*(p_1)$ are provided in the Online Appendix. Below we outline the main steps for finding $w_2^*(p_1)$.

First, we find that when $0 \leq p_1 < \rho_a$, the manufacturer's best-response second-period price $w_2^*(p_1)$ belongs to the interval (ω_b, ω_c), and from (15), we know that the retailer's best-response price satisfies $p_2^*(w_2) = \frac{1+\delta+(\beta+\gamma\lambda)w_2+\gamma\lambda\, p_1}{2(\beta+\gamma\lambda)} > p_1$ for a given $w_2 \in (\omega_b, \omega_c)$. In this situation, we have $D_2(p_2^*(w_2)) = D_2^{NF}(p_2^*(w_2)) - \lambda\gamma[p_2^*(w_2) - p_1]$; using the chain rule, we obtain the first-order condition that characterizes $w_2^*(p_1)$:

$$w_2 \frac{dD_2}{dp_2}\frac{dp_2^*}{dw_2} + D_2(p_2^*(w_2)) = 0. \tag{17}$$

Note that $\frac{dp_2^*}{dw_2} = \frac{dp_2^{*NF}}{dw_2} = \frac{1}{2}$ and $\frac{dD_2}{dp_2} = \frac{dD_2^{NF}}{dp_2} - \lambda\gamma < \frac{dD_2^{NF}}{dp_2}$. Hence, consumers' fairness concerns make demand more price-sensitive, which tends to induce the manufacturer to decrease w_2. As w_2 decreases, the manufacturer's profit margin decreases, but its sales, $D_2(p_2^*(w_2))$, will increase; the equilibrium is reached when the manufacturer's gain and loss from a marginal decrease in wholesale price exactly balance each other out. Solving (17), we get $w_2^*(p_1) = \frac{1+\delta+p_1\gamma\lambda}{2\beta+2\gamma\lambda}$.

Second, we find that when $\rho_a \leq p_1 \leq \rho_b$, the manufacturer's profit-maximization problem has a corner solution: $w_2^*(p_1) = \frac{2p_1\beta+p_1\gamma\lambda-1-\delta}{\beta+\gamma\lambda}$. Intuitively, when p_1 belongs to this intermediate region, the manufacturer optimally chooses a wholesale price w_2 that will induce the retailer to charge $p_2^*(w_2) = p_1$. From equation (15), we can see that $w_2^* = \frac{2p_1\beta + p_1\gamma\lambda - 1 - \delta}{\beta + \gamma\lambda}$ is the highest w_2 such that $p_2^*(w_2) = p_1$.

Finally, when $p_1 > \rho_b$, w_2^* belongs to the interval $[0, \omega_a)$ and in this situation, $p_2^*(w_2) < p_1$, i.e., consumers' fairness concerns will not arise and $D_2(p_2^*) = D_2^{NF}(p_2^*)$. The analysis is similar to the one in our benchmark without fairness concerns; we find that the manufacturer's optimal price is $w_2^* = \frac{1+\delta}{2\beta}$.

As we will later show, in equilibrium, the retailer's first-period price will not exceed ρ_b, i.e., the full equilibrium path entails $p_1 < \rho_b$. Proposition 1 shows that when $p_1 < \rho_b$, the higher the retailer's first-period price p_1, the higher the manufacturer's best-response wholesale price $w_2^*(p_1)$ in the second period.

PROPOSITION 1. *When the market has a segment of consumers with fairness concerns, the higher (lower) the retailer's first-period price, the higher (lower) the manufacturer's best-response wholesale price in the second period. Mathematically,* $\frac{\partial w_2^*(p_1)}{\partial p_1} \geq 0$*, with strict inequality for* $p_1 < \rho_b$.

Note that the second-period wholesale price is the retailer's marginal cost in the second period. Thus, Proposition 1 reveals a mechanism with which the retailer can lower its first-period price to reduce its second-period marginal cost. Let us examine the intuition of this result. First, when $p_1 < \rho_a$, in equilibrium, $p_2^*(p_1, w_2) > p_1$ and a decrease in p_1 increases $p_2^* - p_1$, exacerbating consumers' fairness concerns and reducing the manufacturer's sales. In this situation, the manufacturer has an incentive to reduce w_2 to incentivize the retailer to decrease p_2 to mitigate the demand reduction from fairness concerns. Second, when $\rho_a \leq p_1 < \rho_b$, in equilibrium, $p_2^* = p_1$. In this situation, if p_1 decreases, the manufacturer needs to decrease w_2^* to induce the retailer to keep charging a lower price of $p_2^* = p_1$; without a reduction in w_2, the retailer will have an incentive to charge a price above p_1, which will trigger consumers' fairness concerns and hurt the manufacturer's sales.

As alluded to in the above discussion, the retailer's first-period price has an important effect on its second-period profit. When deciding its first-period price p_1, the retailer will have to consider the effects of p_1 on both its first-period and second-period profits. We now determine the retailer's optimal first-period price in anticipation of the subgame equilibrium outcomes. Given the first-period wholesale price w_1, the retailer chooses p_1 to maximize its total profits from both periods, $\pi_R = (p_1 - w_1)D_1 + \pi_{R2}^* = (p_1 - w_1)\, D_1 + (p_2^* - w_2^*)\, D_2^*$, where $D_2^* = D_2(p_2^*, p_1)$. Note that with abuse of notation, we have used p_2^* to denote $p_2^*(w_2^*)$ to reduce the notational clutter.

We provide the full characterization of the retailer's best-response first-period price, $p_1^*(w_1)$, in the Online Appendix. Since the expressions are rather lengthy and algebraically cumbersome, we do not replicate them in the main paper. Nonetheless, below we provide the first-order condition that characterizes an interior solution to the retailer's profit-maximization problem. Our discussion of the terms in the first-order condition outlines the key forces and tradeoffs that guide the retailer's pricing decision in the first period. Specifically, the first-order condition is

$$(p_1 - w_1)\frac{dD_1}{dp_1} + D_1 + \frac{d\pi_{R2}^*}{dp_1} = 0. \tag{18}$$

If $p_1 > \rho_b$, then $\frac{d\pi_{R2}^*}{dp_1} = 0$, and fairness concerns will be moot and will not affect the firms' equilibrium decisions in the second period. However, for $p_1 < \rho_b$, we can use the expression for π_{R2}^* to rewrite (18) as

$$(p_1 - w_1)\frac{dD_1}{dp_1} + D_1 + (p_2^* - w_2^*)\frac{dD_2^*}{dp_1} + \frac{dp_2^*}{dp_1}D_2^* - \frac{dw_2^*}{dp_1}D_2^* = 0. \tag{19}$$

Comparing (19) with (3), we can determine how consumers' fairness concerns influence the retailer's optimal first-period price relative to the benchmark without fairness concerns. First, the term $(p_1 - w_1)\frac{dD_1}{dp_1} + D_1$ in (19) is the same as in (3). Second, one can show $(p_2^* - w_2^*)\frac{dD_2^*}{dp_1} + \frac{dp_2^*}{dp_1}D_2^* > 0$, i.e., the retailer has an incentive to increase p_1 to be able to charge a higher retail price ($\frac{dp_2^*}{dp_1} > 0$) and also obtain higher sales ($\frac{dD_2^*}{dp_1} > 0$ when $p_1 < \frac{3(1+\delta)}{4\beta+\gamma\lambda}$) in the second period. Thus, the term $(p_2^* - w_2^*)\frac{dD_2^*}{dp_1} + \frac{dp_2^*}{dp_1}D_2^* > 0$ captures the retailer's *fairness-mitigation incentive*, i.e., the retailer's use of a higher p_1 to mitigate the negative direct effect of consumer fairness concerns to achieve higher demand and higher price in the second period. Third, recall from Proposition 1 that $\frac{dw_2^*}{dp_1} > 0$; thus, in (19), the term $-\frac{dw_2^*}{dp_1}D_2^* < 0$ captures the retailer's incentive to decrease p_1 to obtain a lower wholesale price in the second period (i.e., reducing its marginal cost in the second period)—we refer to this incentive as the retailer's *cost-reduction incentive*. Note that the retailer's cost-reduction incentive exists even when δ is zero.

Using backward induction, we can now determine the manufacturer's best-response first-period wholesale price in anticipation of all subgame equilibrium outcomes to maximize its total profits over the two periods, $\pi_M = w_1 D_1(p_1^*(w_1)) + \pi_{M2}^* = w_1 D_1^* + w_2^* D_2^*$, where $D_1^* = D_1(p_1^*(w_1))$ and $D_2^* = D_2(p_2^*(w_2^*, p_1^*), p_1^*)$.

We find that the equilibrium w_1^* will be such that, on the equilibrium path, the retailer's prices will satisfy $p_1^* \leq p_2^*$ with strict inequality when δ is above a threshold. The first-order condition characterizing an interior solution to the manufacturer's profit-maximization problem is as follows.

$$w_1 \frac{dD_1}{dp_1} \frac{dp_1^*}{dw_1} + D_1^* + \frac{d\pi_{M2}^*}{dp_1} \frac{dp_1^*}{dw_1} = 0. \tag{20}$$

The effect of consumers' fairness concerns on the manufacturer's incentives becomes clear when we compare the first-order conditions in (20) and (4). First, notice from (20) that the manufacturer's optimal first-period wholesale price will depend on how sensitive the retailer's p_1^* is with respect to w_1. If the retailer's fairness-mitigation incentive prevails, then the retailer has a strong incentive to alleviate consumers' fairness concerns by raising p_1—in this situation, we have $\frac{dp_1^*}{dw_1} > \frac{dp_1^{*NF}}{dw_1} > 0$. By contrast, when the retailer's cost-reduction incentive dominates, the retailer has a strong incentive to keep its first-period price low in order to obtain a lower wholesale price in the second period than when consumers do not have fairness concerns—in this situation, we have $0 \leq \frac{dp_1^*}{dw_1} < \frac{dp_1^{*NF}}{dw_1}$. Depending on which effect dominates, the impact of $\frac{dp_1^*}{dw_1}$ on w_1 can be positive or negative. Second, due to its cost-reduction incentive, the retailer may charge a lower first-period price than when consumers do not have fairness concerns, i.e., $p_1^* < p_1^{*NF}$. In this situation, $D_1^* > D_1^{*NF}$, giving the manufacturer an incentive to charge a higher w_1. By contrast, when the retailer's fairness-mitigation incentive prevails, we have $p_1^* > p_1^{*NF}$ and hence, $D_1^* < D_1^{*NF}$, giving the manufacturer an incentive to charge a lower w_1. Finally, the last term on the right-hand-side of (20) shows how w_1 is affected by the manufacturer's intertemporal profit considerations. As we discussed earlier,

$\frac{d\pi_{M2}^*}{dp_1} > 0$ as long as p_1 is not too high. Since $\frac{dp_1^*}{dw_1} \geq 0$, it follows that $\frac{d\pi_{M2}^*}{dp_1}\frac{dp_1^*}{dw_1} \geq 0$, i.e., the manufacturer benefits from increasing w_1 because doing so will induce the retailer to charge a higher p_1^*, alleviating consumers' fairness concerns in the second period. However, when p_1^* becomes too high, we can have $\frac{d\pi_{M2}^*}{dp_1} < 0$ because high p_1^* allows the retailer to charge high p_2^*, reducing the manufacturer's second-period sales. In this situation, the manufacturer will want to decrease w_1 to increase its second-period profit.

The above discussion provides a general direction of the effect of fairness concerns on the manufacturer's first-period price under different conditions. We provide the technical details of the analysis in the Online Appendix; Lemma 3 fully characterizes the equilibrium wholesale and retail prices in each period. For convenience, in the proof of Lemma 3, we have implicitly defined a critical threshold δ^* for δ, above which there exists a *paradigm shift* for the manufacturer's and the retailer's pricing strategies, and also another critical threshold δ^{**}, above which in equilibrium there will be no sales in the first period.

LEMMA 3. *With a segment of consumers having fairness concerns in the market, the equilibrium wholesale and retail prices are given by*

$$w_1^* = \begin{cases} \hat{w} & \text{if } 0 \leq \delta < \delta^* \\ \frac{128\beta^4+16\beta^3\gamma\lambda(17+\delta)+8\beta^2\gamma^2(17+2\delta)\lambda^2+\beta\gamma^3(\delta-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \delta^* \leq \delta < \delta^{**}, \end{cases}$$

$$p_1^* = \begin{cases} \frac{3(1+\delta)}{4\beta+\gamma\lambda} & \text{if } 0 \leq \delta < \delta^* \\ \frac{24\beta^2+3\beta\gamma(9+\delta)\lambda-\gamma^2\lambda^2}{4\beta(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} & \text{if } \delta^* \leq \delta < \delta^{**}, \end{cases}$$

$$w_2^* = \begin{cases} \frac{2(1+\delta)}{4\beta+\gamma\lambda} & \text{if } 0 \leq \delta < \delta^* \\ \frac{32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)-\beta\gamma^2\lambda^2(\delta-23)-\gamma^3\lambda^3}{8\beta(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \delta^* \leq \delta < \delta^{**}, \end{cases}$$

$$p_2^* = \begin{cases} \frac{3(1+\delta)}{4\beta+\gamma\lambda} & \text{if } 0 \leq \delta < \delta^* \\ \frac{96\beta^3(1+\delta)+24\beta^2\gamma\lambda(7+4\delta)-3\beta\gamma^2\lambda^2(\delta-23)-3\gamma^3\lambda^3}{16\beta(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \delta^* \leq \delta < \delta^{**}, \end{cases}$$

where $\hat{w}$ *is given in the proof of Lemma 3.*

By comparing the equilibrium wholesale prices in Lemma 3 with those in the benchmark without fairness concerns (in Lemma 1), Proposition 2 shows that consumers' fairness concerns will, in equilibrium, increase the manufacturer's first-period wholesale price and decrease its second-period wholesale price.

PROPOSITION 2. *The existence of a fraction of consumers with fairness concerns will, in equilibrium, induce the manufacturer to raise its first-period wholesale price and reduce its second-period wholesale price, i.e.,* $w_1^* > w_1^{*NF}$ *and* $w_2^* < w_2^{*NF}$.

Let us examine the intuitive rationale for the results in Proposition 2. Recall from Proposition 1 that when facing a segment of consumers with fairness concerns, the retailer has an incentive to reduce its p_1 to induce a lower second-period wholesale price (w_2). Anticipating the retailer's cost-reduction incentive, the manufacturer will increase w_1 to exploit its first-period profit margin, knowing that the retailer will not increase p_1 too much in response to increased w_1. Thus, $w_1^* > w_1^{*NF}$. One may intuit that the manufacturer may also want to charge a higher *second-period* wholesale price without worrying about inducing the retailer to increase its second-period retail price too much (since that would give the retailer a large demand drop due to fairness concerns). However, as we discussed earlier, consumers' fairness concerns make the second-period demand more sensitive to price, giving the manufacturer an incentive to lower its w_2 to increase its sales. The manufacturer's incentive to reduce w_2 is further strengthened by the retailer's cost-reduction incentive, with which the retailer strategically charges a low p_1^* to induce the manufacturer to decrease w_2 more (Proposition 1). Thus, in the full equilibrium, consumer fairness concerns will induce the manufacturer to lower its second-period wholesale price, i.e., $w_2^* < w_2^{*NF}$.

One might expect that consumer fairness concerns will induce the retailer to increase its first-period retail price (p_1) and reduce its second-period price (p_2) to mitigate or eliminate the potential demand drop due to fairness concerns. This intuition has been shown to hold for an integrated manufacturer (centralized channel) that sells directly to consumers (see the discussion following Lemma 2). Proposition 3 shows that the effect of fairness concerns on retail prices in a *decentralized* channel can be qualitatively different.

PROPOSITION 3. *If* $\delta < \frac{\gamma\lambda}{4\beta}$, *then* $p_1^* < p_1^{*NF}$; *moreover,* $p_2^* < p_2^{*NF}$. *That is, when the demand increase is relatively small, the existence of a segment of consumers with fairness concerns will, in equilibrium, reduce both the first-period and second-period retail prices.*

Recall that, with a segment of consumers having fairness concerns, the retailer has to weigh two opposing strategic incentives when selecting its first-period price: fairness mitigation and cost reduction. The retailer's fairness-mitigation incentive induces the retailer to increase p_1 to alleviate consumers' fairness concerns in the second period so as to be able to charge a higher p_2. By contrast, the retailer's cost-reduction incentive induces the retailer to decrease p_1 to obtain a lower wholesale price w_2 in the second period, reducing the retailer's marginal cost. Note that, intuitively speaking, the smaller δ is, the less the retailer will need to raise its second-period price and the smaller the retailer's potential profit loss due to consumer fairness concerns. When δ is small enough ($\delta < \frac{\gamma\lambda}{4\beta}$), the retailer's fairness-mitigation incentive will be dominated by its cost-reduction incentive, making it optimal for the retailer to choose a low first-period price, i.e., $p_1^* < p_1^{*NF}$, even though the manufacturer charges a higher wholesale price in the first period ($w_1^* > w_1^{*NF}$). If $\delta > \frac{\gamma\lambda}{4\beta}$, the retailer's fairness-mitigation incentive will dominate its cost-reduction incentive, leading to $p_1^* > p_1^{*NF}$. As for the second-period retail price p_2, note that, regardless of δ, the retailer gains an incentive to charge a lower p_2 due to increased elasticity of demand and decreased wholesale price ($w_2^* < w_2^{*NF}$); thus, in equilibrium, $p_2^* < p_2^{*NF}$.

Next, we investigate how the magnitude of the demand increase (δ) in the second period affects the equilibrium prices in the channel with consumer fairness concerns. Intuitively, a higher demand intercept (i.e., a higher δ) will tend to induce the firms to raise prices. This intuition holds for the benchmark case with a centralized channel (i.e., a manufacturer selling directly to consumers). Proposition 4 shows that this intuition may not hold in a *decentralized* channel; more specifically, an increase in δ may induce both the manufacturer and the retailer to *decrease* their prices.

PROPOSITION 4. *In a decentralized channel with a segment of consumers having fairness concerns, an increase in δ can lead to a decrease in retail and wholesale prices in both periods.*

The result in Proposition 4 is illustrated in Figures 1 and 2. Note that an increase in δ within the lower region ($\delta < \delta^*$) or within the upper region ($\delta > \delta^*$) will lead to an increase in the wholesale prices and, consequently, an increase in the corresponding retail prices—an intuitive result. However, when δ rises from below δ^* to above it, the manufacturer significantly reduces its first-period wholesale price, which leads to a discrete drop in subsequent retail and wholesale prices.

Figure 1 Equilibrium Wholesale Prices[5]

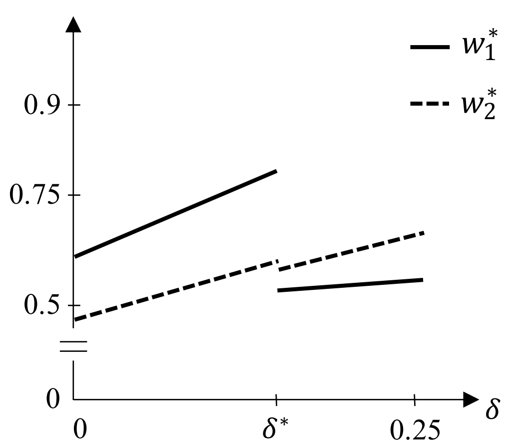
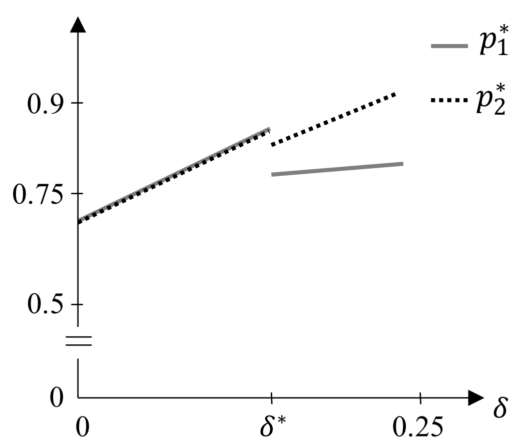


Figure 2 Equilibrium Retail Prices



To see the intuition, let us first consider $\delta < \delta^*$, i.e., the second-period demand does not increase too much. In this parameter range, the retailer will naturally have a weak fairness-mitigation incentive since the demand intercept does not increase much from the first period to the second period. Hence, the retailer's cost-reduction incentive will dominate, i.e., the retailer will want to keep its first-period retail price low in order to obtain a low wholesale price in the second period. Realizing this, the manufacturer optimally sets a relatively high wholesale price to exploit the retailer's cost-reduction incentive, knowing that the retailer will not add a high markup on top of the wholesale price. We refer to this behavior by the manufacturer as the exploitation strategy. With this strategy, the manufacturer pushes its first-period wholesale price high,

[5] Figures 1 and 2 are illustrated using $\beta = 1$, $\gamma = \frac{1}{3}$ and $\lambda = \frac{1}{4}$. Under these values, $\delta^* \approx 0.16$ and $\delta^{**} \approx 2.39$.

extracting significant profits in the first period, while the retailer adds only a small markup in order to induce the manufacturer to reduce its second-period wholesale price. In the second period, the manufacturer will indeed reduce its wholesale price because otherwise the retailer would increase its second-period price, leading to significant losses in sales due to consumers' fairness concerns. Hence, the manufacturer decreases its second-period wholesale price just enough to avoid triggering consumers' fairness concerns (i.e., $p_2^* = p_1^*$) when $\delta < \delta^*$. Next, when $\delta > \delta^*$, the second-period demand is very high, giving a natural tendency for a high second-period price and thus creating a strong fairness-mitigation incentive for the retailer (i.e., raising its first-period retail price). In this situation, if the manufacturer still adopts the exploitation strategy to charge a high first-period wholesale price, the retailer's correspondingly high first-period retail price will lead to very low first-period sales and profits for the manufacturer. Our analysis shows that the manufacturer is better off avoiding such low sales and profits in the first period by switching to a strategy of facilitating first-period sales through the reduction of its first-period wholesale price to induce the retailer to lower its first-period retail price. From the retailer's perspective, this retail price drop not only increases the retailer's first-period sales but also helps it obtain a lower second-period wholesale price than without the price drop. In the second period, given the large increase in demand ($\delta > \delta^*$) and the drop in the first-period retail price, fairness concerns will be triggered, and as expected, both firms will find it beneficial to alleviate fairness concerns by lowering their second-period prices.

In summary, when δ switches from below δ^* to above δ^*, the manufacturer will switch from the exploitation strategy to the facilitation strategy for its first-period wholesale pricing decision; we call this a strategic *paradigm shift*. Table 1 summarizes how retail and wholesale prices change over time in the low-growth ($\delta < \delta^*$) and high-growth ($\delta > \delta^*$) regions.

Table 1 Summary of Price Dynamics

	$\boldsymbol{\delta < \delta^*}$	$\boldsymbol{\delta > \delta^*}$
Equilibrium retail prices	$p_2^* = p_1^*$	$p_2^* > p_1^*$
Equilibrium wholesale prices	$w_2^* < w_1^*$	$w_2^* > w_1^*$

So far, we have analyzed the manufacturer's and the retailer's pricing strategies when facing consumers with fairness concerns. Now, we examine how consumer fairness concerns affect the firms' profits; to better illustrate the intuition, we start by discussing the effect of fairness concerns on the equilibrium sales.

LEMMA 4. *The existence of consumers with fairness concerns will result in an increase in the second-period subgame equilibrium sales in the channel.*

Although consumers' fairness concerns have a negative direct effect on second-period demand, Lemma 4 shows that, surprisingly, the second-period subgame equilibrium sales can *increase* relative to when consumers have no fairness concerns. Let us examine the logic behind this result. First, recall from Lemma 3 that when $\delta < \delta^*$, in equilibrium, we have $p_1^* = p_2^*$ without raising any fairness concerns. Further, as Proposition 3 showed, $p_2^* < p_2^{*NF}$. Hence, when $\delta < \delta^*$, the second-period retail price is lower than that in the absence of fairness concerns and does not trigger consumers' fairness concerns, which leads to higher equilibrium sales. Next, when $\delta^* < \delta < \delta^{**}$, the retailer's second-period equilibrium price exceeds its first-period price (i.e., $p_1^* < p_2^*$), evoking consumers' fairness concerns. Fairness concerns affect the retail demand by raising the intercept by $\gamma\lambda p_1^*$ and also increasing the slope from β to $\beta + \gamma\lambda$; the overall direct effect on demand is negative. However, the increased slope of the demand function gives the retailer a strong incentive to reduce p_2. Furthermore, to the manufacturer, the second-period demand, $D_2(p_2^*(w_2))$, also becomes more sensitive to w_2, inducing the manufacturer to lower w_2^* relative to when consumers do not have fairness concerns. Thus, due to lowered wholesale and retail prices, the second-period subgame equilibrium sales can increase relative to when consumers do not have fairness concerns, i.e., $D_2(p_2^*(w_2^*)) > D_2(p_2^{*NF}(w_2^{*NF}))$.

Lemma 4 helps shed light on the effect of fairness concerns on the double-marginalization and channel-coordination problem. Namely, in the absence of fairness concerns, the manufacturer may not be able to achieve high sales in the second period even if it charges the same (low) wholesale price w_2^*. The reason is that without fairness concerns, the retailer will add a high markup on top of the wholesale price, reducing the manufacturer's sales. By contrast, the existence of consumers with fairness concerns gives the retailer a strong incentive to lower its price, which can lead to higher sales than when consumers do not have fairness concerns. Hence, fairness concerns can help mitigate the traditional double-marginalization problem, allowing the channel to achieve more sales and higher channel profits.

Let us examine the effect of consumer fairness concerns on the manufacturer's profit. Recall from section 3.1 that in a centralized channel, consumers' fairness concerns reduces the manufacturer's profit. However, Proposition 5 reveals that in a decentralized channel, consumers' fairness concerns can actually increase the manufacturer's profit.

PROPOSITION 5. *There exists $\bar{\delta} > 0$ such that if $\delta \in [0, \bar{\delta})$, then $\pi_{M1}^{*} > \pi_{M1}^{*NF}$ and $\pi_{M2}^{*} > \pi_{M2}^{*NF}$. That is, the existence of consumers with fairness concerns increases the manufacturer's profits in both periods. Furthermore, there exists $\tilde{\delta} > \bar{\delta}$ such that $\pi_{M}^{*} > \pi_{M}^{*NF}$ if and only if $\delta < \tilde{\delta}$.*

To see the intuition behind this surprising result, let us start with the manufacturer's second-period profit. As Lemma 4 has shown, consumers' fairness concerns can lead to higher second-period sales due to the retailer's strong incentive to charge a lower price in the second period, which helps alleviate the double-marginalization problem in the channel. Hence, even though the manufacturer's second-period wholesale price is lower than when consumers do not have fairness concerns ($w_2^{*} < w_2^{*NF}$), its profit will increase (i.e., $\pi_{M2}^{*} > \pi_{M2}^{*NF}$) because of more significantly increased sales. Next, as Proposition 2 has shown, consumers' fairness concerns allow the manufacturer to charge a higher wholesale price in the first period ($w_1^{*} > w_1^{*NF}$). However, due to its strong cost-reduction incentive when δ is not too high, the retailer does not increase p_1^{*} too much in response to high w_1^{*} and may even charge a lower p_1^{*} than when consumers do not have fairness concerns (i.e., $p_1^{*} < p_1^{*NF}$ as Proposition 3 has shown). This situation clearly benefits the manufacturer, increasing its profit in the first period relative to when consumers do not have fairness concerns, i.e., $\pi_{M1}^{*} > \pi_{M1}^{*NF}$. However, note that when δ is very large, the retailer will find it optimal to set a fairly high first-period price to mitigate the fairness concerns in the second period, even taking into account its cost-reduction incentive; the high first-period retail price will significantly reduce the first-period sales, thus reducing the manufacturer's first-period profit (though its second-period profit increases).

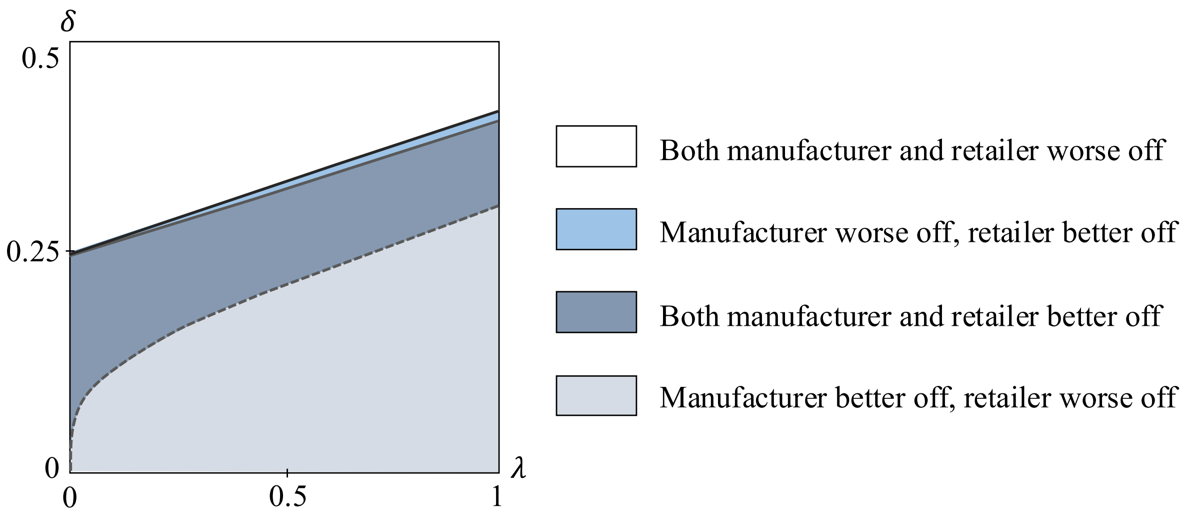
In this situation, when δ is too high, the manufacturer's first-period profit loss can dominate its second-period profit gain from consumer fairness concerns, making the manufacturer worse off than when consumers do not have fairness concerns.

One might wonder whether the manufacturer's gain from consumers' fairness concerns will come at the expense of the retailer. Our analysis of the retailer's profit reveals that consumer fairness concerns can also benefit the retailer. Thus, as shown in Proposition 6, under some conditions, consumer fairness concerns can lead to a win-win outcome for the manufacturer and the retailer.

PROPOSITION 6. *There exists $\tilde{\delta} > \delta^*$ such that if $\delta^* < \delta < \tilde{\delta}$, then consumer fairness concerns can increase both the manufacturer's and the retailer's profits.*

As we discussed earlier, consumers' fairness concerns help improve channel coordination in the second period, allowing the retailer to obtain a lower wholesale price and achieve higher sales. However, consumers' fairness concerns negatively affect the retailer's first-period profit. First, recall that when δ is very low ($\delta < \delta^*$), the manufacturer charges a very high wholesale price in the first period to induce the retailer to charge the same price in the first and second periods ($p_1^* = p_2^*$). In essence, when $\delta < \delta^*$, the manufacturer exploits the retailer's cost-reduction incentive to extract most of the channel profits in the first period. In this situation, the retailer's second-period profit gain is not enough to compensate for its first-period profit loss, making the retailer worse off. Second, when δ increases above δ^*, the manufacturer significantly decreases its first-period wholesale price due to the paradigm shift in the manufacturer's strategy as we discussed earlier (Proposition 4). This reduction in the first-period wholesale price mitigates the retailer's first-period profit loss and can lead to consumer fairness concerns having a net positive effect on the retailer's total profits from both periods. More specifically, if δ is not too high, then the retailer's second-period profit gain exceeds its first-period profit loss, making the retailer better off. Recall from Proposition 5 that when $\delta < \tilde{\delta}$, the manufacturer also benefits from consumers' fairness concerns. Hence, when $\delta^* < \delta < \tilde{\delta}$, consumers' fairness concerns lead to a win-win situation for both the manufacturer and the retailer. However, note that if δ is very high, the retailer wants to charge a much higher price in the second period, and hence, the retailer has to substantially increase its first-period price to alleviate consumers' potential fairness concerns in the second period. In this situation, consumer fairness concerns will lead to a large reduction in the retailer's first-period profit, making the retailer's overall profit lower than when consumers do not have fairness concerns. Figure 3 illustrates the parameter region where both the manufacturer and the retailer benefit from consumers' fairness concerns.

Figure 3 Profit Implications of Consumer Fairness Concerns [6]



Our analysis of the manufacturer's and the retailer's profits shows that even if firms can use marketing communications to alleviate the consumer's fairness concerns about price increases, they may not necessarily want to do so. Note that different factors can lead to win-win outcomes in channel settings, e.g., Harutyunyan and Jiang (2019) show that, in a single-period model, a competing manufacturer's entry can be a win-win for the incumbent manufacturer and retailer because the competitor's entry can change the demand curve in a way to alleviate the double-marginalization problem in the incumbent's channel. By contrast, in our multi-period model, consumer fairness concerns about price increases can result in a win-win outcome because of the inter-period pricing dynamics in the channel driven by the retailer's

[6] Figure 3 is illustrated using the following parameter values: $\beta = 1, \gamma = \frac{1}{3}$. Under these parameter conditions, $\delta^{**} = \frac{3+\lambda}{9}\frac{8\sqrt{2}(12+\lambda)\sqrt{(6+\lambda)(18+\lambda)(24+\lambda)(72-(-24+\lambda)\lambda)}-\sqrt{\lambda}(12-\lambda)(2016+\lambda(540+\lambda))}{\sqrt{\lambda}(13824+\lambda(7056+(696-43\lambda)\lambda))} > 0.5$.

consideration of fairness-mitigation and cost-reduction incentives when deciding its first-period retail price.[7]

5. Discussions

This section provides further discussions about our model and results. First, recall that in our model, δ represents a change in the second-period demand intercept and that our core analysis focused on the case of $\delta \in [0, \delta^{**}]$, where the retailer has a positive equilibrium market share in both periods. One can show that when a segment of consumers have fairness concerns, if δ is above δ^{**} (which is defined in the Online Appendix), the retailer's first-period equilibrium price satisfies $p_1^* > \frac{1}{\beta}$, which will lead to no sales in the first period. In that case, our model essentially becomes equivalent to a model in which firms sell the product only in the second period, but the retailer can choose the reference price (p_1) before selling starts in the second period. Our analysis reveals that, in that case, even though the retailer can costlessly choose any p_1 (e.g., infinity) to be the consumer's reference price, it would prefer a relatively low reference price. This is because, in a decentralized channel, the retailer's optimal choice of the reference price (p_1) will have to balance its fairness-mitigation incentive and its cost-reduction incentive. This result directly contrasts the finding of Zhang et al. (2014) that firms will always prefer consumers' having higher initial reference prices.

Second, in the Online Appendix, we analyze an alternative *multiplicative* demand growth model where the second-period demand (without fairness concerns) is $D_2^{NF} = (1 + \delta)(1 - \beta p_2)$, rather than $D_2^{NF} = 1 + \delta - \beta p_2$. Accordingly, in the presence of consumer fairness concerns, the second-period demand function is given by: $D_2 = \lambda(1 + \delta)(1 - \beta p_2 - \gamma \max\{p_2 - p_1, 0\}) + (1 - \lambda)(1 + \delta)(1 - \beta p_2)$. Other aspects of the model are the same as those in our core model. We show that consumer fairness concerns can still give

[7] One might wonder whether the results from our model of fairness concerns can arise from a simple increase in the demand elasticity due to an increase in the slope of the demand curve in a model without fairness concerns. We can show that in a channel setting (e.g., in our model without fairness concerns), an increase in the slope of the demand curve (i.e., an increase in β, which makes the demand more elastic) will make both the manufacturer and the retailer worse off. This shows that, even though one may intuit that fairness concerns make consumers more sensitive to a price increase, the results and insights from our fairness model cannot be obtained from a simple increase in consumers' price sensitivity in a model without fairness concerns.

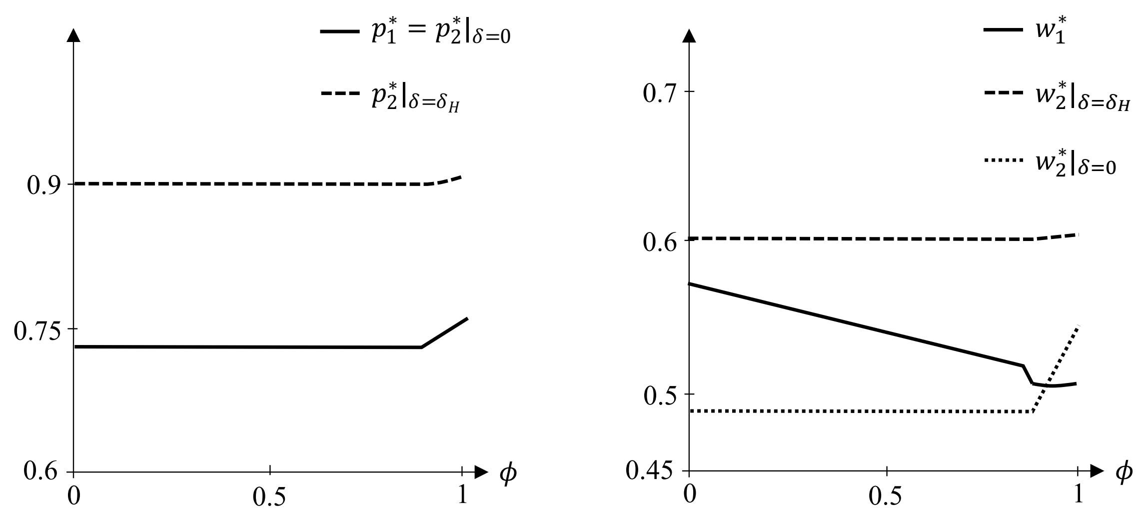
rise to the same two opposing incentives as those in our core model (namely, the fairness-mitigation and the cost-reduction incentives), albeit the fairness-mitigation incentive is weakened; thus, most of our main results can hold across these two types of demand models. Note that, for demand-increasing factors such as positive word-of-mouth, celebrity endorsements, or improved brand image, the consumers' valuations for the product tend to increase, and hence our original demand growth model of $D_2^{NF} = 1 + \delta - \beta p_2$ is more reasonable than the alternative multiplicative demand model that we present in the Online Appendix.

Third, our core model has assumed that the second-period demand is non-decreasing from the first period (i.e., $\delta \geq 0$). If the second-period demand intercept drops (i.e., if δ is negative), the retailer will have a natural tendency to lower its price in the second period. This tends to eliminate or reduce consumers' fairness concerns, especially in a centralized channel. In a decentralized channel, if the decrease in the second-period demand is very large, i.e., δ is very negative, then consumer fairness concerns about price increases will become a moot point since the equilibrium retail price will not increase. But if the decrease in the second-period demand is relatively small, consumer fairness concerns can still play a role in affecting the intertemporal pricing decisions in the channel, and the results will be similar to the special case of $\delta = 0$ in our core model. Namely, when the demand decrease is relatively small, the existence of a segment of consumers with fairness concerns will, in equilibrium, reduce both the first-period and second-period retail prices. Intuitively, our core model fits a setting with a new product that is being positively received in the marketplace, e.g., with favorable reviews or positive word-of-mouth, rather than a product that is at the end of its life cycle with diminished demand and lowering valuations (e.g., due to negative reviews or ratings).

Fourth, while we often observe products or brands that experience increasing demand over time (i.e., $\delta \geq 0$, as assumed in our core model), firms may not always have good estimates of future demand growth. For example, firms' marketing campaign is likely to have a positive effect on future demand, but the magnitude of the positive effect may not be known ex ante, i.e., firms will face uncertainty about future demand. Also, for a new product introduced to the market, there can be uncertainty about whether it will succeed or fail in satisfying customers' needs well; if it fails, then demand will actually decrease. To explore the uncertain demand situations, we extend our model by analyzing uncertainty in δ. To this end, we investigate two scenarios.

In the first scenario, we consider the situation where the second-period demand is expected to rise (e.g., due to celebrity endorsements or positive word-of-mouth), but there is uncertainty about whether the increase will be high or low. To model this situation, we assume that $\delta \in \{\delta_L, \delta_H\}$, where $0 \leq \delta_L < \delta_H$. The probability of $\delta = \delta_H$ is $\phi \in [0,1]$, and the probability of $\delta = \delta_L$ is $1 - \phi$. To simplify the analysis, without loss of generality, we normalize $\delta_L = 0$. In the first period, the manufacturer and the retailer know only the probability distribution of δ, but they will learn or observe the value of δ only at the beginning of the second period. We provide the analysis of the model in the Online Appendix, where we demonstrate that our main qualitative results continue to hold even in the presence of uncertainty about δ. The existence of uncertainty as captured by ϕ moderates our results. Specifically, Figure 4 illustrates the effect of ϕ on equilibrium wholesale and retail prices.

Figure 4 Effect of Uncertainty on Wholesale and Retail Prices [8]

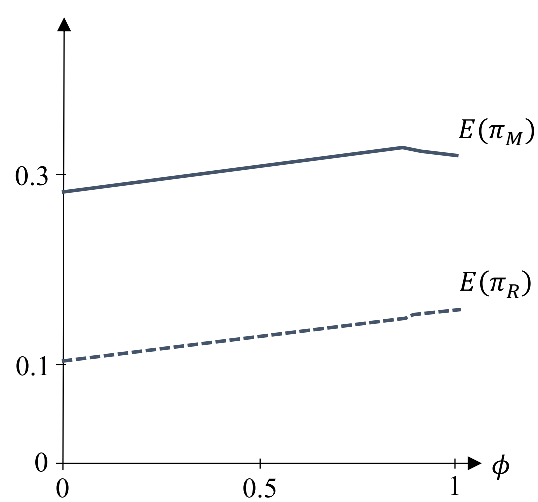


a. Effect of ϕ on retail prices **b**. Effect of ϕ on wholesale prices

[8] Figures 4 is illustrated using $\beta = 1$, $\gamma = \frac{1}{5}$, $\lambda = \frac{1}{2}$ and $\delta_H = \frac{1}{4}$.

Notice from Figure 4a that the retailer's first-period price is the same as its second-period price when there is no demand improvement in the second period (i.e., $p_1^* = p_2^*|_{\delta=0}$), but it is lower than the second-period price when demand improves (i.e., $p_1^* < p_2^*|_{\delta=\delta_H}$). As ϕ increases, the second-period demand improvement ($\delta = \delta_H$) becomes more likely, giving the retailer an incentive to increase its first-period retail price to mitigate consumers' potential fairness concerns in the second period so as to be able to charge a higher second-period price. However, since an increase in p_1 will lead to higher p_2, the manufacturer's expected sales in the second period will decline, reducing its second-period profit. To prevent this, we can see from Figure 4b that the manufacturer will decrease its w_1^* as ϕ increases, inducing the retailer not to increase p_1. However, when ϕ becomes very large, the retailer's (fairness-mitigation) incentive to increase p_1 becomes very strong; to prevent it, the manufacturer would have to reduce w_1 very steeply. Such a reduction in w_1 is no longer profitable for the manufacturer, and the manufacturer's strategy switches from preventing an increase in p_1 to moderating the increase. Hence, we observe that when ϕ is in the very high range, w_1^* will no longer decline as sharply and may even increase as ϕ increases, leading to an increase in the retailer's first-period price, p_1^*.

Figure 5 Effect of Uncertainty on Manufacturer's and Retailer's Expected Profits



One would expect that both the manufacturer and the retailer will benefit as a second-period demand improvement becomes more likely (that is, as ϕ increases). As Figure 5 shows, this is indeed the case with the retailer's ex-ante expected profit, $E(\pi_R)$. However, the manufacturer's expected profit can actually decrease as ϕ increases. Intuitively, when ϕ becomes very high, the retailer's fairness-mitigation incentive becomes very strong, inducing the retailer to raise its first-period price to mitigate the potential negative effect from consumers' fairness concerns. Increased p_1^* negatively affects not only the manufacturer's first-period profit but also the manufacturer's second-period profit in the demand state of $\delta = 0$. The latter effect arises because $p_2^*|_{\delta=0} = p_1^*$ in equilibrium. Hence, when p_1^* increases due to an increase in ϕ, $p_2^*|_{\delta=0}$ also increases, decreasing the manufacturer's second-period sales and profit in the demand state of $\delta = 0$. For these reasons, when ϕ is already very high, a further increase can make the manufacturer worse off.

In the second scenario, we explore the situation where the second-period demand will increase by $\delta > 0$ with probability ψ and decrease by δ with probability $1 - \psi$.[9] Other aspects of the model are the same as in the first scenario described above. The analysis of this model turned out to be algebraically very cumbersome, but a solution to the model can still be obtained numerically. As one would expect, we find that the manufacturer's and retailer's prices and profits increase in ψ, making the firms better off as the future demand increase becomes more likely. We also find that the effect of fairness concerns becomes stronger as ψ increases. Note that if the second-period demand drops (i.e., δ is negative), then the retailer is likely to charge a lower second-period price than in the first period, in which case the effect of fairness concerns will become moot. Hence, we find that the effect of fairness concerns on the manufacturer's and the retailer's expected profits is weak when ψ is low, but it tends to increase as ψ increases (i.e., $\frac{dE(\pi_M^* - \pi_M^{*NF})}{d\psi} > 0$ and $\frac{dE(\pi_R^* - \pi_R^{*NF})}{d\psi} > 0$). One may wonder whether both firms can still benefit from the existence of consumers' fairness concerns. Intuitively, if ψ is very high (say, close to 1), the result will surely hold since the model would approach our core model in the limit. In the Online Appendix, we demonstrate that the win-win result can hold for much smaller values of ψ (e.g., when ψ is 0.5 or even

[9] In this model, we assume $\delta < 1$ so that if the marked demand in the second period is in the low state, the demand intercept is still positive in the second period.

lower). That is, the results and insights from our core model remain quite robust even if there is significant uncertainty about the future demand.

6. Conclusion

Extant research in consumer behavior has shown that some consumers consider a price increase due to a change in demand as unfair and may reduce their purchase intentions. This paper has provided an analytical framework to study the effects of such fairness concerns in a dynamic channel setting. We have examined a decentralized channel, where a manufacturer sells through a retailer in two time periods in a market with a segment of consumers having fairness concerns regarding price increases. Our analysis of such a marketplace reveals several findings in stark contrast with those of static or centralized-channel models.

One might intuit that consumer fairness concerns about price increases will induce the retailer to increase its first-period retail price and reduce its second-period price to mitigate or eliminate the potential demand drop due to fairness concerns. We show that this intuition holds in a centralized channel, but in a decentralized channel with consumer fairness concerns, the retailer has to weigh two *opposing* strategic incentives when choosing its first-period price. On the one hand, the retailer has a *fairness-mitigation* incentive—raising its first-period price to mitigate or eliminate the second-period demand drop due to fairness concerns. On the other hand, the retailer has a *cost-reduction incentive*—reducing its first-period retail price to induce the manufacturer to lower the second-period wholesale price, which reduces the retailer's marginal cost in the second period. The retailer's cost-reduction incentive creates a strategic link between the two periods, which has not been identified in the prior literature. We find that if the second-period demand intercept does not increase much, the retailer's cost-reduction incentive will dominate its fairness-mitigation incentive, making it optimal for the retailer to reduce its first-period price. Anticipating the retailer's incentives when facing consumer fairness concerns, the manufacturer tends to *raise* its first-period wholesale price and *reduce* its second-period wholesale price. Moreover, we find that, interestingly, a larger increase in the second-period demand intercept can induce both the manufacturer and the retailer to *decrease* their prices in *both* periods.

Our results also shed some light on the effect of consumer fairness concerns on firms' profits in a dynamic, multi-period, channel setting. Although consumer fairness concerns have a potential negative direct effect of reducing the second-period demand, the strategic, dynamic effect of fairness concerns can help the manufacturer increase its profits in *both* periods. In the second period, consumer fairness concerns make demand more sensitive to the retail price, inducing the retailer to charge a lower price to mitigate consumers' fairness concerns or avoid triggering them. The increased price sensitivity of demand and the retailer's incentive to reduce its second-period price allow the manufacturer to obtain a higher increase in unit sales with a marginal decrease in its second-period wholesale price—this improves channel coordination, leading to higher equilibrium sales and increased channel profits. In the first period, anticipating the retailer's cost-reduction incentive, the manufacturer can increase its first-period wholesale price without inducing much increase in the first-period retail price, capturing a larger share of channel profits. Finally, we show that the manufacturer's benefit from consumer fairness concerns does not have to come at the retailer's expense. In particular, under some conditions, both the manufacturer and the retailer can benefit from the existence of fairness concerns. In essence, consumer fairness concerns allow the retailer to use a low first-period retail price to "commit" to a relatively low future second-period retail mark-up, sacrificing some first-period profits to reduce its second-period marginal cost. The retailer's benefit from the second period can more than offset its loss in its first-period profit, making the retailer better off. In sum, through the inter-period pricing dynamics, fairness concerns can make the channel more coordinated, sufficiently improving the channel pricing efficiency to allow both firms to benefit.

Below we discuss several managerial insights that our analysis offers.

Should a retailer set a high initial price to reduce or eliminate the need for future price increases to avoid potential fairness concerns? While this can be an optimal strategy in an integrated channel, we find that in a decentralized channel, consumers' fairness concerns may actually give the retailer an incentive to charge a lower initial price instead of a higher price. Doing so can help improve future channel efficiency and coordination, allowing the retailer to obtain a lower wholesale price from the manufacturer. Managers need to weigh the costs and benefits of the direct and strategic effects of consumer fairness concerns.

Should the initial low price be framed as introductory or special temporary offers to mitigate consumers' fairness concerns about the future price increase? We show that in a decentralized channel, the manufacturer and the retailer may not want to mitigate consumers' fairness concerns. This is because consumers' fairness concerns can help reassure the manufacturer that the retailer will not charge a high margin. Consumer fairness concerns can benefit the manufacturer because it can achieve higher sales, and the retailer can also benefit because it can obtain a lower wholesale price than when consumers do not have fairness concerns.

Should the retailer prioritize fairness mitigation or cost reduction? Our results underline the importance of estimating the product's future demand potential. If the retailer anticipates that the product will achieve much higher popularity and willingness to pay among consumers, then the retailer may want to prioritize fairness mitigation by charging a high initial price to avoid very drastic price increases in the future. By contrast, if the future demand improvement is unlikely to be very high, then the retailer should prioritize cost reduction to increase channel efficiency in the future.

We conclude the paper by outlining several directions for future research. First, our model has assumed that the number of consumers with fairness concerns is an exogenous fraction of the market. In practice, it might be possible that a higher price increase can induce more consumers to feel unfair, i.e., λ in our model will tend to increase as $p_2 - p_1$ increases. Such a model will be analytically intractable in our framework. However, we expect that our main results should actually be *strengthened* if the number of consumers having fairness concerns increases with $p_2 - p_1$. This is because a higher number of consumers with fairness concerns will make the second-period demand even more price-sensitive, giving the retailer less incentive to raise its second-period price, more significantly alleviating the double-marginalization problem in the second period and further strengthening the inter-period strategic link between the first-period retail price and the second-period wholesale price. With that said, it might be interesting to explore in more detail the non-linear effect of fairness concerns in future research. Second, our model assumes that the potential change in the second-period demand is exogenous and driven by other non-price elements in the marketing mix that firms have already decided. Future research may want to explore how consumer fairness concerns will affect the firms' incentives to create market demand or increase product valuation by investing in, for example, advertising or other marketing campaigns, albeit such exploration would significantly increase the analytical complexity of our current framework. Third, it may also be worthwhile to study a competitive market to explore additional insights regarding the effects of consumer fairness concerns on channels. Note again that our current model is already very difficult to analyze since it has four decision stages alternating between two firms in the channel; so, to analyze competition with more players in the game, one would have to simplify the model framework in some aspects to ensure analytical tractability. We leave it to future research to explore additional insights from competitive markets, which is worth a separate and systematic study. Finally, the empirical literature on dynamic pricing in a channel is still evolving, so we outline several testable hypotheses for future empirical research based on our analytical results.

- A retailer charging a smaller mark-up is more likely to receive a lower wholesale price and a higher retail mark-up in the future.
- In markets where the demand growth is small (large), the wholesale price is more likely to decrease (increase) over time, while the retail price is more likely to remain stable (increase) over time.
- In markets or product categories where consumers are more likely to exhibit loss-aversion or fairness concerns, channel coordination between the manufacturer and the retailer is more efficient, leading to less double marginalization and higher profits.

References

Amaldoss W, He C (2018) Reference-dependent utility, product variety, and price competition. *Management Sci.* 64(9):4302-4316.

Amaldoss W, Jain S (2008) Research note—Trading up: A strategic analysis of reference group effects. *Marketing Sci.* 27(5):932-942.

Anderson E, Jaimovich N, Simester D (2015) Price stickiness: Empirical evidence of the menu cost channel. *Review of Econ. and Statistics* 97(4):813-826.

Blinder A, Canetti E, Lebow D, Rudd J (1998) Asking About Prices: A New Approach to Understanding Price Stickiness. *Russell Sage Foundation*.

Bolton LE, Alba JW (2006) Price fairness: Good and service differences and the role of vendor costs. *J. of Consumer Research* 33(2):258-265.

Bolton GE, Ockenfels A (2000) ERC: A theory of equity, reciprocity, and competition. *The American Econ. Review* 90(1):166-193.

Bolton LE, Warlop L, Alba JW (2003) Consumer perceptions of price (un) fairness. *J. of Consumer Research* 29(4):474-491.

Bondi S, Goldrick M, Wilkie J, Sen B, Reasor E (2021) The dos and don'ts of dynamic pricing in retail. *McKinsey & Company,* April 28. Access link: https://www.mckinsey.com/business-functions/marketing-and-sales/our-insights/the-dos-and-donts-of-dynamic-pricing-in-retail

Campbell MC (1999) Perceptions of price unfairness: antecedents and consequences. *J. of Marketing Research* 36(2):187-199.

Charness G, Rabin M (2002) Understanding social preferences with simple tests. *The Quarterly J. of Econ.* 117(3):817-869.

Chen Y, Cui TH (2013) The benefit of uniform price for branded variants. *Marketing Sci.* 32(1):36-50.

Coombs CH, Avrunin GS (1977) Single-peaked functions and the theory of preference. *Psych. review* 84(2): 216.

Cui TH, Raju JS, Zhang ZJ (2007) Fairness and channel coordination. *Management Sci.* 53(8):1303-1314.

Dickson PR, Kalapurakal R (1994) The use and perceived fairness of price-setting rules in the bulk electricity market. *J. of Econ. Psychology* 15:427-448.

Fehr E, Schmidt KM (1999) A theory of fairness, competition, and cooperation. *The Quarterly J. of Econ.* 114(3):817-868.

Friedman M (2002) Consumer boycotts: Effecting change through the marketplace and media. *Routledge*.

Greenleaf EA (1995) The impact of reference price effects on the profitability of price promotions. *Marketing Sci.* 14(1):82-104.

Guo X, Jiang B (2016) Signaling through price and quality to consumers with fairness concerns. *J. of Marketing Research* 53(6):988-1000.

Guo L (2015) Inequity aversion and fair selling. *J. of Marketing Research* 52(1):77-89.

Gurumurthy K, Little JDC (1988) A Price Response Model Based on Perceptual Theories. Working paper, Massachusetts Institute of Technology.

Hardie BGS, Johnson EJ, Fader PS (1993) Modeling loss aversion and reference dependence effects on brand choice. *Marketing Sci.* 12(4):378-394.

Harutyunyan M, Jiang B (2019) The bright side of hiving an enemy. *J. of Marketing Research* 56(4):679-690.

Hayward S, Rodger J (2017) Tesco increases price of Marmite just months after row with Unilever. *Birmingham Mail*, 14 May. Access link: www.birminghammail.co.uk/news/midlands-news/tesco-increases-price-marmite-just-13032795

Ho TH, Su X (2009) Peer-induced fairness in games. *American Economic Review* 99(5):2022-49.

Ho TH, Zhang J (2008) Designing pricing contracts for boundedly rational customers: Does the framing of the fixed fee matter? *Management Sci.* 54(4):686-700.

Homburg C, Koschate N, Hoyer WD (2005) Do satisfied customers really pay more? A study of the relationship between customer satisfaction and willingness to pay. *J. of Marketing* 69(2):84-96.

Irving A (2020) SPAR blasted as 'shameful and greedy' after increasing £1 hand soap price… it says it was an "honest mistake." *Manchester Evening News,* March 11. Access link: www.manchestereveningnews.co.uk/whats-on/shopping/spar-markup-hand-soap-price-17898473

Jiang B, Liu C (2019) Managerial optimism in a competitive market. *Production and Operations Management* 28(4):833-846.

Jiang B, Narasimhan C, Turut O (2017) Anticipated regret and product innovation. *Management Sci.* 63(12):4308-4323.

Jiang B, Ni J, Srinivasan K (2014) Signaling through pricing by service providers with social preferences. *Marketing Sci.* 33(5):641-654.

Kahneman D, Knetsch JL, Thaler R (1986) Fairness as a constraint on profit seeking: entitlements in the market. *The American Economic Review* 76(4):728-741.

Kalwani MU, Yim CK, Rinne HJ, Sugita Y (1990) A price expectations model of customer brand choice. *J. of Marketing Research* 27(3):251-262.

Kopalle PK, Rao AG, Assuncao JL (1996) Asymmetric reference price effects and dynamic pricing policies. *Marketing Sci.* 15(1):60-85.

Kuksov D, Wang K (2014) The bright side of loss aversion in dynamic and competitive markets. *Marketing Sci.* 33(5):693-711.

Levy D, Bergen M, Dutta S, Venable R (1997) The magnitude of menu costs: direct evidence from large US supermarket chains. *The Quarterly J. of Econ.s* 112(3):791-824.

Li KJ, Jain S (2015) Behavior-based pricing: An analysis of the impact of peer-induced fairness. *Management Sci.* 62(9):2705-2721.

Linning S, Baker K, Burke D, Glanfield E, Poulter S (2016) Victory for Britain in great Marmite war! Tesco comes out on top as Unilever withdraws blanket price rises for 200 top brands after threats of a boycott by shoppers. *Daily Mail*, 12 October. Access link: www.dailymail.co.uk/news/article-3835034/Major-brands-axed-Tesco-shelves-row-supplier-Unilever-demanded-10-cent-price-rise-falling-pound.html

Maxwell S (2002) Rule-based price fairness and it effect on willingness to purchase. *J. of Econ. Psychology* 23(2):191-212.

Maxwell S, Mayer VF, Stamer HH, Diller H, Avila MG (2013) Reactions to a price increase: what makes it seem fair. *Global J. of Management And Business Research*.

Mazumdar T, Raj SP, Sinha I (2005) Reference price research: review and propositions. *J. of Marketing* 69(4):84-102.

McGuire TW, Staelin R (1983) An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.e* 2(2):161-191.

Moorthy KS (1988) Strategic decentralization in channels. *Marketing Sci.e* 7(4):335-355.

Narasimhan C, Turut O (2013) Differentiation or imitate? The role of context dependent preferences. *Marketing Sci.* 32(3):393-410.

Peltzman S (2000) Prices rise faster than they fall. *J. of political economy*. 108(3):466-502.

Popescu I, Wu Y (2007) Dynamic pricing strategies with reference effects. *Operations Research* 55(3):413-429.

Raju J, Zhang ZJ (2005) Channel coordination in the presence of a dominant retailer. *Marketing Sci.* 24(2):254-262.

Rinne HJ (1981) An empirical investigation of the effects of reference prices on sales. *Working paper*, Purdue University.

Selove M (2019) Dynamic pricing with fairness concerns and a capacity constraint. *Quant. Marketing and Econ.* 17(1):385-413.

Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. *The Rand J. of Econ.* 546-554.

Sinha I, Batra R (1999) The effect of consumer price consciousness on private label purchase. *International J. of Research in Marketing* 16(3):237-251.

Slade ME (1998) Optimal pricing with costly adjustment: evidence from retail-grocery prices. *The Review of Econ. Studies* 65(1):87-107.

Stanley JO (2016) A Call for Fairness in Drug Prices. *Missouri medicine* 113(1):26.

Tripathi S (2017) Et tu, brute? how unfair! *J. of Retailing and Consumer Services* 39:79-92.

Yi Z, Wang Y, Liu Y, Chen YJ (2018) The impact of consumer fairness seeking on distribution channel selection: Direct selling vs. agent selling. *Production and Operations Management* 27(6):1148-1167.

Zhang J, Chiang WK, Liang L (2014) Strategic pricing with reference effects in a competitive supply chain. *Omega* 44:126-135.

Zou T, Zhou B, Jiang B (2020) Product-line design in the presence of consumers' anticipated regret. *Management Sci.* 66(12):5665-5682.

ONLINE APPENDIX

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Part I

Proofs of Lemmas and Propositions

PROOF OF LEMMA 1. In the main paper, we found that the manufacturer's and retailer's equilibrium prices in the first and second periods are as follows: $p_1^{*NF} = \frac{3}{4\beta}$, $p_2^{*NF} = \frac{3(1+\delta)}{4\beta}$, $w_1^{*NF} = \frac{1}{2\beta}$ and $w_2^{*NF} = \frac{1+\delta}{2\beta}$. It is easy to see that $p_2^{*NF} > p_1^{*NF}$ and $w_2^{*NF} > w_1^{*NF}$. ■

ANALYSIS OF CENTRALIZED CHANNEL AND PROOF OF LEMMA 2. We solve the game by backward induction. In the second period, for a given first-period price p_1, the manufacturer chooses its second-period price p_2 to maximize its second-period profit, $\pi_2 = D_2\, p_2$. The second-period demand function is as follows.

$$D_2 = \begin{cases} 1+\delta-\beta p_2 & \text{if } \ 0 < p_2 \le p_1 \\ 1+\delta-\beta p_2 - \lambda\gamma(p_2 - p_1) & \text{if } \ p_1 < p_2 \le \frac{1+\delta+\gamma p_1}{\beta+\gamma} \\ (1-\lambda)(1+\delta-\beta p_2) & \text{if } \ \frac{1+\delta+\gamma p_1}{\beta+\gamma} < p_2 \le \frac{1+\delta}{\beta} \\ 0 & \text{if } \ p_2 > \frac{1+\delta}{\beta} \end{cases}$$

Note that the manufacturer's profit function is continuous and piecewise concave on each interval. We first obtain the optimal prices within each of the following three intervals $I_1 \equiv (0, p_1]$, $I_2 \equiv (p_1, \frac{1+\delta+\gamma p_1}{\beta+\gamma}]$, and $I_3 \equiv (\frac{1+\delta+\gamma p_1}{\beta+\gamma}, \frac{1+\delta}{\beta})$; among the three, the price that yields the highest profit is the manufacturer's optimal second-period retail price. Let $p_2^{I_i}$ denote the manufacturer's optimal price within the interval I_i. For non-trivial analysis, we assume $\beta > \gamma$ in this model.

- Suppose that $p_2 \in I_1$. If $0 < p_1 \le \frac{1+\delta}{2\beta}$, then one can show that the optimal price is $p_2^{I_1} = p_1$; if $\frac{1+\delta}{2\beta} < p_1 < \frac{1+\delta}{\beta}$, then one can show that the optimal price is $p_2^{I_1} = \frac{1+\delta}{2\beta}$.

- Suppose that $p_2 \in I_2$. If $0 < p_1 \le \frac{1+\delta}{2\beta+\gamma\lambda}$, the optimal price is $p_2^{I_2} = \frac{1+\delta+\lambda\gamma p_1}{2(\beta+\gamma)}$; if $p_1 > \frac{1+\delta}{2\beta+\gamma\lambda}$, the optimal price is $p_2^{I_2} = p_1$.

- Suppose that $p_2 \in I_3$. If $0 < p_1 < \frac{1+\delta}{\beta}$, the optimal price is $p_2^{I_2} = \frac{1+\delta+\gamma p_1}{\beta+\gamma}$.

Comparing the profits corresponding to prices $p_2^{I_1}$, $p_2^{I_2}$ and $p_2^{I_3}$, one can show that the manufacturer's optimal second-period price is as follows.

$$\tilde{p}_2(p_1) = \begin{cases} \frac{1+\delta+\lambda\gamma p_1}{2(\beta+\gamma)} & \text{if } 0 < p_1 \leq \frac{1+\delta}{2\beta+\gamma\lambda} \\ p_1 & \text{if } \frac{1+\delta}{2\beta+\gamma\lambda} \leq p_1 \leq \frac{1+\delta}{2\beta}. \\ \frac{1+\delta}{2\beta} & \text{if } p_1 > \frac{1+\delta}{2\beta} \end{cases}$$

Using $\tilde{p}_2$, we can obtain the manufacturer's second-period subgame equilibrium profits, where $\tilde{\pi}_2 = D_2\, \tilde{p}_2$.

$$\tilde{\pi}_2 = \begin{cases} \frac{(1+\delta+p_1\gamma\lambda)^2}{4(\beta+\gamma\lambda)} & \text{if } p_1 \in (0, \frac{1+\delta}{2\beta+\gamma\lambda}] \\ p_1(1+\delta-\beta p_1) & \text{if } p_1 \in (\frac{1+\delta}{2\beta+\gamma\lambda}, \frac{1+\delta}{2\beta}] \\ \frac{(1+\delta)^2}{4\beta} & \text{if } p_1 \in (\frac{1+\delta}{2\beta}, \infty) \end{cases}$$

In the first period, the manufacturer chooses p_1 to maximize its profit $\tilde{\pi} = D_1 p_1 + \tilde{\pi}_2$, where $D_1 = 1 - \beta p_1$ if $p_1 < \frac{1}{\beta}$ and $D_1 = 0$ if otherwise.

To find the optimal p_1, we first find the locally optimal p_1 within each of the intervals $K_1 \equiv (0, \frac{1+\delta}{2\beta+\gamma\lambda}]$, $K_2 \equiv (\frac{1+\delta}{2\beta+\gamma\lambda}, \frac{1+\delta}{2\beta}]$ and $K_3 \equiv (\frac{1+\delta}{2\beta}, \infty]$. Then, comparing the profits under each local optimum, we can find the globally optimal price, $\tilde{p}_1$. The analysis is straightforward. Specifically, one can show that

$$\tilde{p}_1 = \begin{cases} \frac{2+\delta}{4\beta} & \text{if } 0 \leq \delta < \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \\ \frac{2\beta+\gamma\lambda(3+\delta)}{4\beta^2+4\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \leq \delta < \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}. \\ \frac{1+\delta}{2\beta} & \text{if } \delta \geq \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \end{cases}$$

Plugging $\tilde{p}_1$ into the expression for $\tilde{p}_2$, we find that

$$\tilde{p}_2 = \begin{cases} \frac{2+\delta}{4\beta} & \text{if } 0 \le \delta < \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \\ \frac{2\beta(1+\delta)+\gamma\lambda}{4\beta^2+4\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } \frac{2\gamma\lambda}{2\beta-\gamma\lambda} \le \delta < \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \\ \frac{1+\delta}{2\beta} & \text{if } \delta \ge \frac{2\beta^2+\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \end{cases}.$$

This finishes the proof of Lemma 2. ■

PROOF OF LEMMA 3. The proof of Lemma 3 is similar to the proof of Lemma 2, but is more technical and much lengthier. Hence, we present it separately in Part II of the Online Appendix in order to keep Part I of the Online Appendix more compact and easier to follow.

PROOF OF PROPOSITION 1. In Part II of the Online Appendix, we show that the manufacturer's subgame wholesale price in the second period is given by:

$$w_2^*(p_1) = \begin{cases} \frac{1+\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ \frac{2p_1\beta-1-\delta+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \rho_a \le p_1 \le \rho_b \\ \frac{1+\delta}{2\beta} & \text{if } p_1 > \rho_b \end{cases}$$

where $\rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$ and $\rho_b = \frac{(1+\delta)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. Taking the derivative with respect to p_1, one can easily show that in each region of $p_1 \le \rho_b$, w_2^* is always increasing in p_1. ■

PROOF OF PROPOSITION 2. The manufacturer's equilibrium wholesale prices in each period (w_1^* and w_2^*) are given in Lemma 3; Recall that when consumers do not have fairness concerns, the manufacturer's equilibrium prices are given by: $w_1^{*NF} = \frac{1}{2\beta}$ and $w_2^{*NF} = \frac{1+\delta}{2\beta}$. Define $\Delta w_1 = w_1^* - w_1^{*NF}$ and $\Delta w_2 = w_2^* - w_2^{*NF}$ and let $\delta_m = \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$. One can show that when $0 \le \delta \le \delta_m$, we have $\Delta w_1 = w_1^* - w_1^{*NF} = \frac{16\beta^2(3\delta+1)+24\beta\gamma\lambda\delta+\gamma^2\lambda^2(\delta-3)}{4\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)} + \frac{(1+\delta)(4\beta+3\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{2(\beta+\gamma\lambda)}} - \frac{1}{2\beta} > 0$; when $\delta_m < \delta \le \delta^*$, $\Delta w_1 = w_1^* - w_1^{*NF} = \frac{(1+\delta)(48\beta^2(3\delta-1)-8\beta\gamma\lambda(7+\delta)+(1+\delta)\gamma^2\lambda^2)}{16\beta(\beta(3\delta-1)-\gamma\lambda)(4\beta+\gamma\lambda)} - \frac{1}{2\beta} > 0$; when $\delta^* < \delta < \delta^{**}$, $\Delta w_1 = w_1^* - w_1^{*NF} = \frac{128\beta^4+16\beta^3\gamma(17+\delta)\lambda+8\beta^2\gamma^2(17+2\delta)\lambda^2+\beta\gamma^3(\delta-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} - \frac{1}{2\beta} > 0$. In sum, $w_1^* > w_1^{*NF}$. Further, when $0 < \delta \le \delta^*$, $\Delta w_2 = w_2^* - w_2^{*NF} = \frac{2(1+\delta)}{4\beta+\gamma\lambda} - \frac{1+\delta}{2\beta} < 0$; when $\delta^* < \delta < \delta^{**}$, $\Delta w_2 = w_2^* - w_2^{*NF} = \frac{32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)-\beta\gamma^2\lambda^2(\delta-23)-\gamma^3\lambda^3}{8\beta(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} - \frac{1+\delta}{2\beta} < 0$. Hence, $w_2^* < w_2^{*NF}$. ■

PROOF OF PROPOSITION 3. The retailer's equilibrium prices in two periods are given in Lemma 3. Recall that when no consumers have fairness concerns, the retailer's equilibrium prices are given by: $p_1^{*NF} = \frac{3}{4\beta}$ and $p_2^{*NF} = \frac{3(1+\delta)}{4\beta}$. Define $\Delta p_1 = p_1^* - p_1^{*NF}$ and $\Delta p_2 = p_2^* - p_2^{*NF}$. Since $\frac{\gamma\lambda}{4\beta} < \delta^*$, one can show that when $\delta < \frac{\gamma\lambda}{4\beta}$, we have $\Delta p_1 = p_1^* - p_1^{*NF} = \frac{3(1+\delta)}{4\beta+\gamma\lambda} - \frac{3}{4\beta} < 0$, $\Delta p_2 = p_2^* - p_2^{*NF} = \frac{3(1+\delta)}{4\beta+\gamma\lambda} - \frac{3(1+\delta)}{4\beta} < 0$. That is, $p_1^* < p_1^{*NF}$ and $p_2^* < p_2^{*NF}$. ■

PROOF OF PROPOSITION 4. The manufacturer's and the retailer's equilibrium prices in each period are given in Lemma 3. To show a higher demand increase in the second period can lead to a decrease in the retail and wholesale prices in both periods, we prove that there is a discrete drop in the firms' equilibrium pricing strategies at $\delta = \delta^*$, as illustrated in Figures 1 and 2. For example, to show that an increase in δ can lead to a decrease in p_1^*, it suffices to show $\lim_{\delta\to\delta^{*-}} p_1^*(\delta) > \lim_{\delta\to\delta^{*+}} p_1^*(\delta)$. Since $\lim_{\delta\to\delta^{*-}} p_1^*(\delta) = \frac{3(1+\delta^*)}{4\beta+\gamma\lambda}$ and $\lim_{\delta\to\delta^{*+}} p_1^*(\delta) = \frac{24\beta^2+3\beta\gamma(9+\delta^*)\lambda-\gamma^2\lambda^2}{4\beta(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)}$, $\lim_{\delta\to\delta^{*-}} p_1^*(\delta) - \lim_{\delta\to\delta^{*+}} p_1^*(\delta) = \frac{3(1+\delta^*)}{4\beta+\gamma\lambda} - \frac{24\beta^2+3\beta\gamma(9+\delta^*)\lambda-\gamma^2\lambda^2}{4\beta(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} > 0$, one can show that $\lim_{\delta\to\delta^{*-}} p_1^*(\delta) > \lim_{\delta\to\delta^{*+}} p_1^*(\delta)$. Using a similar analysis, we can also prove that $\lim_{\delta\to\delta^{*-}} p_2^*(\delta) > \lim_{\delta\to\delta^{*+}} p_2^*(\delta)$, and $\lim_{\delta\to\delta^{*-}} w_1^*(\delta) > \lim_{\delta\to\delta^{*+}} w_1^*(\delta)$, $\lim_{\delta\to\delta^{*-}} w_2^*(\delta) > \lim_{\delta\to\delta^{*+}} w_2^*(\delta)$. ■

PROOF OF LEMMA 4. When consumers do not have fairness concerns, the firms' equilibrium demand in the second period is given by: $D_2^{*NF} = \frac{1+\delta}{4}$. With a segment of consumers having fairness concerns in the market, the firms' equilibrium demand in the second period is given by:

$$D_2^* = \begin{cases} \frac{(1+\delta)(\beta+\gamma\lambda)}{4\beta+\gamma\lambda} & \text{if } 0 \le \delta < \delta^* \\ \frac{32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta)-\gamma^3\lambda^3}{16\beta(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} & \text{if } \delta^* \le \delta < \delta^{**}, \end{cases}$$

Define $\Delta D_2 = D_2^* - D_2^{*NF}$. Specifically,

$$\Delta D_2=\begin{cases}\frac{3\gamma\lambda(1+\delta)}{4(4\beta+\gamma\lambda)} & \text{if } 0\le\delta<\delta^*\\ \frac{\gamma\lambda(24\beta^2+3\beta\gamma\lambda(9+\delta)-\gamma^2\lambda^2)}{16\beta(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} & \text{if } \delta^*\le\delta<\delta^{**}\end{cases}.$$

Since $\delta<\delta^{**}$, once can show that $\Delta D_2>0$. In words, consumers' fairness concerns will increase the second-period subgame equilibrium sales in the channel. ■

PROOF OF PROPOSITION 5. Recall that when there are *no* consumers with fairness concerns in the market, the manufacturer's profits in the first period and the second period are given by $\pi_{M1}^{*NF}=\frac{1}{8\beta}$ and $\pi_{M2}^{*NF}=\frac{(1+\delta)^2}{8\beta}$. Also, in Part II of the Online Appendix, we show that when there are some consumers with fairness concerns, the manufacturer's per-period profits π_{M1}^* and π_{M2}^* are given by

$$\pi_{M1}^*=\begin{cases}\hat{\pi}_{M1} & \text{if } 0\le\delta<\delta^*\\ \frac{(8\beta^2+\beta\gamma(5-3\delta)\lambda-3\gamma^2\lambda^2)(128\beta^4+16\beta^3\gamma\lambda(17+\delta)+8\beta^2\gamma^2\lambda^2(17+2\delta)-\beta\gamma^3\lambda^3(7-\delta)+\gamma^4\lambda^4)}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^*\le\delta<\delta^{**}\end{cases}$$

where $\hat{\pi}_{M1}$ is given below

$$\hat{\pi}_{M1}=\begin{cases}\frac{\beta-3\beta\delta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{(1+\delta)(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}+\frac{2(16\beta^2(1+3\delta)+24\beta\gamma\lambda\delta+\gamma^2\lambda^2(\delta-3))}{\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)}\right) & \text{if } 0<\delta\le\delta_m\\ \frac{(1+\delta)(48\beta^2(1-3\delta)+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta))}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_m\le\delta<\delta^*,\end{cases}$$

$$\pi_{M2}^*=\begin{cases}\frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } 0\le\delta<\delta^*\\ \frac{(\gamma^3\lambda^3-32\beta^3(1+\delta)-8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta))^2}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^*\le\delta<\delta^{**}.\end{cases}$$

We will first show that the manufacturer's profit in the first period can be higher than that when no consumers have fairness concerns. More specifically, we will show when δ is not too large, $\pi_{M1}^*>\pi_{M1}^{*NF}$. Define $\Delta\pi_{M1}=\pi_{M1}^*-\pi_{M1}^{*NF}$. If $\delta\in[0,\delta_m]$, where $\delta_m=\frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$, then $\Delta\pi_{M1}=\frac{\beta-3\beta\delta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{(1+\delta)(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}+\frac{2\left(16\beta^2(1+3\delta)+24\beta\gamma\lambda\delta+(\delta-3)\gamma^2\lambda^2\right)}{\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)}\right)-\frac{1}{8\beta}$. We can show that $\Delta\pi_{M1}|_{\delta=0}=\frac{(\beta+\gamma\lambda)\left(\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}(4\beta+3\gamma\lambda)+\frac{32\beta^2-6\gamma^2\lambda^2}{4\beta+\gamma\lambda}\right)}{8\beta\,(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)}-\frac{1}{8\beta}>0$. By continuity of $\Delta\pi_{M1}$ in δ, there exists some $\bar{\delta}\in(0,\delta_m]$ such that if $\delta\in[0,\bar{\delta})$, then $\Delta\pi_{M1}>0$, i.e., $\pi_{M1}^*>\pi_{M1}^{*NF}$.

Second, let us show that the manufacturer's profit in the second period can be higher than when no consumers have fairness concerns. Let us show that when δ is sufficiently small, $\pi_{M2}^{*} > \pi_{M2}^{*NF}$. Define $\Delta\pi_{M2} = \pi_{M2}^{*} - \pi_{M2}^{*NF}$. If $\delta \in [0, \delta^{*}]$, then $\Delta\pi_{M2} = \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} - \frac{(1+\delta)^2}{8\beta}$. Since $\gamma < \frac{\beta}{3}$, one can readily show that $\Delta\pi_{M2} > 0$ for any δ. Therefore, if $\delta \in [0, \delta^{*}]$, then $\Delta\pi_{M2} > 0$, i.e., $\pi_{M2}^{*} > \pi_{M2}^{*NF}$. Since $\bar{\delta} < \delta^{*}$, it follows that whenever $\delta \in [0, \bar{\delta})$, the existence of consumers with fairness concerns will lead to higher profit for the manufacturer in each time period.

Third, we prove that the manufacturer's *overall* profit can be higher with a segment of consumers having fairness concerns. In Part II of the Online Appendix, we show that when a fraction of consumers have fairness concerns, the manufacturer's and the retailer's total profits are given by $\pi_M^{*} = \pi_{M1}^{*} + \pi_{M2}^{*} =$

$$\begin{cases} \hat{\pi}_M & \text{if } 0 \le \delta < \delta^{*} \\ \frac{64\beta^4(2+2\delta+\delta^2)+16\beta^3\gamma\lambda(17+13\delta+4\delta^2)+\beta^2\gamma^2\lambda^2(161+82\delta+\delta^2)+2\beta\gamma^3\lambda^3(9+\delta)+\gamma^4\lambda^4}{64\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^{*} \le \delta < \delta^{**} \end{cases}$$

where $\hat{\pi}_M$ is given by $\hat{\pi}_M = \hat{\pi}_{M1} + \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$. In the benchmark case without consumer fairness concerns, the manufacturer's total profit is given by $\pi_M^{*NF} = \frac{1+(1+\delta)^2}{8\beta}$. Define $\Delta\pi_M = \pi_M^{*} - \pi_M^{*NF}$. When $0 \le \delta < \delta^{*}$, $\Delta\pi_M = \pi_M^{*} - \pi_M^{*NF} = \hat{\pi}_{M1} + \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} - \frac{1+(1+\delta)^2}{8\beta} > 0$; when $\delta^{*} \le \delta < \delta^{**}$, $\Delta\pi_M = \pi_M^{*} - \pi_M^{*NF} = \frac{64\beta^4(2+2\delta+\delta^2)+16\beta^3\gamma\lambda(17+13\delta+4\delta^2)+\beta^2\gamma^2\lambda^2(161+82\delta+\delta^2)+2\beta\gamma^3\lambda^3(9+\delta)+\gamma^4\lambda^4}{64\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} - \frac{1+(1+\delta)^2}{8\beta} =$
$\frac{\gamma\lambda(-16\beta^3(1+\delta)(4\delta-1)+\beta^2\gamma(49-5\delta(6+11\delta))\lambda+2\beta\gamma^2(17+\delta(9+4\delta))\lambda^2+\gamma^3\lambda^3)}{64\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2}$. One can show that, in this subcase, $\Delta\pi_M > 0$ (i.e., $\pi_M^{*} > \pi_M^{*NF}$) if and only if $\delta^{*} \le \delta < \tilde{\delta}$, where

$$\tilde{\delta} = 2\sqrt{\frac{2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)(25\beta^2+24\beta\gamma\lambda+\gamma^2\lambda^2)}{\beta(64\beta^2+55\beta\gamma\lambda-8\gamma^2\lambda^2)}} - \frac{3(8\beta^2+5\beta\gamma\lambda-3\gamma^2\lambda^2)}{64\beta^2+55\beta\gamma\lambda-8\gamma^2\lambda^2}. \blacksquare$$

PROOF OF PROPOSITION 6. In this Proposition, we show that when $\delta^{*} < \delta < \tilde{\delta}$, consumer fairness concerns can increase *both* the manufacturer's and the retailer's profits. Recall in Proposition 5, we have already proved that if $\delta^{*} < \delta < \tilde{\delta}$, then $\pi_M^{*} > \pi_M^{*NF}$. Next, we will show in region $\delta^{*} < \delta < \tilde{\delta}$, the retailer will also become better off. Recall that when consumers do not have fairness concerns, the retailer's total profits are given by $\pi_R^{*NF} = \frac{1+(1+\delta)^2}{16\beta}$. Moreover, in Part II of the Online Appendix, we show that when a fraction of consumers have fairness concerns, the retailer's total profit is given by

$$\pi_R^* = \pi_{R1}^* + \pi_{R2}^* =$$

$$\begin{cases} \hat{\pi}_R & \text{if } 0 \le \delta < \delta^* \\ \frac{(8\beta^2+\beta\gamma(5-3\delta)\lambda-3\gamma^2\lambda^2)(8\beta^2+\beta\gamma\lambda(9+\delta)+\gamma^2\lambda^2)}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} + \frac{(32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta)-\gamma^3\lambda^3)^2}{256\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

where $\hat{\pi}_R$ is given by $\hat{\pi}_R = \hat{\pi}_{R1} + \frac{(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$ and

$$\hat{\pi}_{R1} = \begin{cases} \frac{(\beta(3\delta-1)-\gamma\lambda)(8\sigma\beta^3-2\gamma^2\lambda^2(3-\delta)+\beta\gamma\lambda(24\delta+\sigma\gamma\lambda-24)+2\beta^2(24\delta+3\sigma\gamma\lambda-8))}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2} & \text{if } 0 < \delta \le \delta_m \\ \frac{\gamma(1+\delta)^2\lambda(\gamma\lambda-8\beta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_m \le \delta < \delta^* \end{cases}$$

where $\sigma \equiv \frac{(1+\delta)(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}$ and $\delta_m = \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$.

Define $\Delta\pi_R = \pi_R^* - \pi_R^{*NF}$. When $\delta^* \le \delta < \delta^{**}$, we have $\Delta\pi_R = \pi_R^* - \pi_R^{*NF} = \frac{(8\beta^2+\beta\gamma(5-3\delta)\lambda-3\gamma^2\lambda^2)(8\beta^2+\beta\gamma\lambda(9+\delta)+\gamma^2\lambda^2)}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} + \frac{(32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta)-\gamma^3\lambda^3)^2}{256\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} - \frac{1+(1+\delta)^2}{16\beta}$. One can show that $\Delta\pi_R > 0$ (i.e, $\pi_R^* > \pi_R^{*NF}$) if and only if $\delta^* \le \delta < \delta_T$, where $\delta_T =$

$$\frac{-3(8\beta-3\gamma\lambda)(\beta+\gamma\lambda)(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2)\sqrt{\beta}+4(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)\sqrt{(\beta+\gamma\lambda)(400\beta^4+848\beta^3\gamma\lambda+375\beta^2\gamma^2\lambda^2-80\beta\gamma^3\lambda^3+7\gamma^4\lambda^4)}}{(1024\beta^4+1904\beta^3\gamma\lambda+624\beta^2\gamma^2\lambda^2-247\beta\gamma^3\lambda^3+16\gamma^4\lambda^4)\sqrt{\beta}}$$

satisfies $\Delta\pi_R|_{\delta=\delta_T} = 0$ and $\delta_T < \delta^{**}$. Since $\delta_T > \tilde{\delta}$ (recall that $\tilde{\delta}$ is defined in the proof of Proposition 5), it follows that when $\delta^* < \delta < \tilde{\delta}$, we have $\pi_R^* > \pi_R^{*NF}$ and $\pi_M^* > \pi_M^{*NF}$. That is, consumer fairness concerns can increase both the manufacturer's and the retailer's profits if $\delta^* < \delta < \tilde{\delta}$. ■

Part II

Technical Analysis (Proof of Lemma 3)

This part analyzes the subgame perfect equilibrium of the game. More specifically, we present the detailed proof of Lemma 3 in the main paper. We solve the game by backward induction. That is, we will first find the second-period subgame equilibrium outcome. Then, we will solve for the first-period equilibrium.

Derivation of the second-period subgame perfect equilibrium.

Given the first-period retail price p_1, in the second period, the manufacturer chooses its second-period wholesale price w_2, followed by the retailer choosing its second-period retail price p_2. More specifically, the retailer chooses p_2 to maximize its second-period profit, $\pi_{R2} = D_2(p_2 - w_2)$, where D_2 is as follows:

$$D_2 = \begin{cases} 1+\delta-\beta p_2 & \text{if } 0 \le p_2 \le p_1 \\ \lambda(1+\delta-\beta p_2-\gamma(p_2-p_1))+(1-\lambda)(1+\delta-\beta p_2) & \text{if } p_1 \le p_2 \le \frac{1+\delta+\gamma p_1}{\beta+\gamma} \\ (1-\lambda)(1+\delta-\beta p_2) & \text{if } \frac{1+\delta+\gamma p_1}{\beta+\gamma} \le p_2 \le \frac{1+\delta}{\beta} \\ 0 & \text{if } p_2 > \frac{1+\delta}{\beta} \end{cases}$$

Let p_2^* denote the retailer's optimal price: $p_2^* \equiv \underset{p_2}{\operatorname{argmax}}\, \pi_{R2}$. Note that π_{R2} is continuous. We will first obtain the optimal prices within each of the following three intervals $I_1 \equiv [0, p_1]$, $I_2 \equiv [p_1, \frac{1+\delta+\gamma p_1}{\beta+\gamma}]$, and $I_3 \equiv [\frac{1+\delta+\gamma p_1}{\beta+\gamma}, \frac{1+\delta}{\beta}]$; among the three, the price that yields the highest profit is the retailer's optimal second-period retail price. Let $p_2^{I_i}$ denote the retailer's optimal price within the interval I_i. Note that π_{R2} is piecewise concave on each interval I_i, $i = 1,2,3$. Furthermore, the interval I_i is compact. Hence, the local maximizer in the interval I_i is either the interior solution (satisfying the first-order condition), or one of the bounds of the interval I_i (i.e., a corner solution). In either case, the analysis is rather straightforward, so we will not provide too much detail.

- Suppose that $p_2 \in I_1$. The retailer's profit function is given by $\pi_{R2} = (1+\delta-\beta p_2)(p_2 - w_2)$. In this case, the second-period retail price is smaller or equal than the first-period retail price, so consumers will not have any fairness concerns. There are two subcases to analyze: First, if $0 < p_1 \le \frac{1+\delta}{2\beta}$, then π_{R2} is increasing in p_2 on the interval I_1. Hence, the optimal price within the interval I_1 is $p_2^{I_1} = p_1$(i.e., the corner solution); Second, if $\frac{1+\delta}{2\beta} < p_1 < \frac{1+\delta}{\beta}$, then one can show that the optimal price is $p_2^{I_1} = \frac{1+w_2\beta+\delta}{2\beta}$ (i.e., the interior solution) when $0 < w_2 \le \frac{-1+2p_1\beta-\delta}{\beta}$, and $p_2^{I_1} = p_1$ when $\frac{-1+2p_1\beta-\delta}{\beta} < w_2 < \frac{1+\delta}{\beta}$.

- Suppose that $p_2 \in I_2$. The retailer's profit function is given by $\pi_{R2} = (\lambda\big(1+\delta-\beta p_2-\gamma(p_2-p_1)\big)+(1-\lambda)(1+\delta-\beta p_2))(p_2-w_2)$. In this case, the second-period retail price exceeds the first-period retail price, and hence, the retailer will lose some extra sales because of consumers' fairness concerns. There are two subcases to analyze: First, if $0 < p_1 \le \frac{1+\delta}{2\beta+\gamma\lambda}$, the optimal price is $p_2^{I_2} = \frac{1+w_2\beta+\delta+p_1\gamma\lambda+w_2\gamma\lambda}{2\beta+2\gamma\lambda}$ (i.e., the interior solution) when $0 < w_2 \le \frac{2p_1\beta\gamma+(\beta-\gamma)(\delta+1)+2\gamma\lambda-p_1\beta\gamma\lambda+p_1\gamma^2\lambda+2\gamma\lambda\delta}{\beta^2+\beta\gamma+\beta\gamma\lambda+\gamma^2\lambda}$; the optimal price is $p_2^{I_2} = \frac{1+\delta+\gamma p_1}{\beta+\gamma}$ (i.e., the corner solution) when $w_2 > \frac{2p_1\beta\gamma+(\beta-\gamma)(\delta+1)+2\gamma\lambda-p_1\beta\gamma\lambda+p_1\gamma^2\lambda+2\gamma\lambda\delta}{\beta^2+\beta\gamma+\beta\gamma\lambda+\gamma^2\lambda}$. Second, if $\frac{1+\delta}{2\beta+\gamma\lambda} < p_1 < \frac{1+\delta}{\beta}$, the optimal price is $p_2^{I_1} = p_1$ when $0 < w_2 \le \frac{2p_1\beta-\delta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}$; the optimal price is $p_2^{I_2} = \frac{1+w_2\beta+\delta+p_1\gamma\lambda+w_2\gamma\lambda}{2\beta+2\gamma\lambda}$, when $\frac{2p_1\beta-\delta-1+p_1\gamma\lambda}{\beta+\gamma\lambda} < w_2 \le \frac{2p_1\beta\gamma+(\beta-\gamma)(\delta+1)+2\gamma\lambda-p_1\beta\gamma\lambda+p_1\gamma^2\lambda+2\gamma\delta\lambda}{\beta^2+\beta\gamma+\beta\gamma\lambda+\gamma^2\lambda}$; the optimal price is $p_2^{I_2} = \frac{1+\delta+\gamma p_1}{\beta+\gamma}$ when $w_2 > \frac{\beta-\gamma+2p_1\beta\gamma+\beta\delta-\gamma\delta+2\gamma\lambda-p_1\beta\gamma\lambda+p_1\gamma^2\lambda+2\gamma\delta\lambda}{\beta^2+\beta\gamma+\beta\gamma\lambda+\gamma^2\lambda}$.

- Suppose that $p_2 \in I_3$. The retailer's profit function is given by $\pi_{R2} = (1-\lambda)(1+\delta-\beta p_2)(p_2-w_2)$. In this case, p_2 is so high that the segment of consumers with fairness concerns will not buy the product. One can show that if $0 < p_1 < \frac{1+\delta}{\beta}$, then the optimal price is $p_2^{I_3} = \frac{1+\delta+\gamma p_1}{\beta+\gamma}$ when $0 < w_2 < \frac{\beta-\gamma+2p_1\beta\gamma+\beta\delta-\gamma\delta}{\beta^2+\beta\gamma}$; the optimal price is $p_2^{I_3} = \frac{1+\mathrm{w}2\beta+\delta}{2\beta}$ when $\frac{\beta-\gamma+2p_1\beta\gamma+\beta\delta-\gamma\delta}{\beta^2+\beta\gamma} \le w_2 < \frac{1+\delta}{\beta}$. Comparing the profits corresponding to prices $p_2^{I_1}$, $p_2^{I_2}$ and $p_2^{I_3}$, we get the retailer's optimal second-period price:

$$p_2^*(w_2) = \begin{cases} \frac{1+w_2\beta+\delta}{2\beta} & \text{if } 0 \le w_2 < \omega_a \\ p_1 & \text{if } \omega_a \le w_2 < \omega_b \\ \frac{1+w_2(\beta+\gamma\lambda)+\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } \omega_b \le w_2 \le \omega_c \\ \frac{1+w_2\beta+\delta}{2\beta} & \text{if } \omega_c < w_2 \le \frac{1+\delta}{\beta} \end{cases}$$

where the expressions for $\omega_a = \max\{0, \frac{2p_1\beta-1-\delta}{\beta}\}$, $\omega_b = \max\{0, \frac{2p_1\beta+p_1\gamma\lambda-1-\delta}{\beta+\gamma\lambda}\}$ and $\omega_c = \frac{1+\delta+p_1\gamma}{\beta+\gamma} - \frac{\gamma(1+\delta-p_1\beta)}{\beta+\gamma}\sqrt{\frac{(1-\lambda)}{\beta(\beta+\gamma\lambda)}}$. ω_c satisfies $\pi_{R2}^*|_{p_2^*=\frac{1+\omega_c\beta+\delta+p_1\gamma\lambda+W_1\gamma\lambda}{2\beta+2\gamma\lambda}} = \pi_{R2}^*|_{p_2^*=\frac{1+\omega_c\beta+\delta}{2\beta}}$ i.e., $\frac{(1+\delta+p_1\gamma\lambda-\omega_c(\beta+\gamma\lambda))^2}{4(\beta+\gamma\lambda)} = \frac{(1-\lambda)(1-\omega_c\beta+\delta)^2}{4\beta}$. The retailer's second-period profits is given by

$$\pi_{R2}^* = \begin{cases} \frac{(1-w_2\beta+\delta)^2}{4\beta} & \text{if } 0 \le w_2 < \omega_a \\ (p_1 - w_2)(1+\delta-\beta p_1) & \text{if } \omega_a \le w_2 < \omega_b \\ \frac{(1+\delta+p_1\gamma\lambda-w_2(\beta+\gamma\lambda))^2}{4(\beta+\gamma\lambda)} & \text{if } \omega_b \le w_2 \le \omega_c \\ \frac{(1-\lambda)(1-w_2\beta+\delta)^2}{4\beta} & \text{if } \omega_c < w_2 \le \frac{1+\delta}{\beta} \end{cases}.$$

Next, we will find the manufacturer's optimal second-period wholesale price. The manufacturer correctly anticipates the retailer's best response p_2^* to w_2 and chooses its optimal wholesale price to maximize its profit, $\pi_{M2} = D_2\, w_2$. Let w_2^* denote the manufacturer's optimal wholesale price, i.e., $w_2^* \equiv \underset{w_2}{\operatorname{argmax}}\, \pi_{M2}$. We will first obtain the manufacturer's optimal wholesale prices within each of the following four intervals: $M_1 \equiv [0, \omega_a]$, $M_2 \equiv [\omega_a, \omega_b]$, $M_3 \equiv [\omega_b, \omega_c]$, and $M_4 \equiv (\omega_c, \frac{1+\delta}{\beta}]$. Among the four, the price that yields the highest profit is the manufacturer's optimal second-period wholesale price. Let $w_2^{M_i}$ denote the manufacturer's optimal second-period wholesale price within the interval M_i, $i = 1,2,3,4$. Note that the manufacturer's profit function π_{M2} is piecewise concave on each interval M_i, and is discontinuous at ω_c: it jumps down at ω_c since the retailer switches from serving both segments to serving only the segment without fairness concerns, so the manufacturer will not charge a w_2^* larger than ω_c, because a tiny drop from ω_c will give the manufacturer a profit increase.

- Suppose that $w_2 \in M_1$ (where $M_1 \equiv [0, \omega_a]$), the manufacturer's second-period profit function is given by $\pi_{M2} = \frac{w_2(1+\delta-w_2\beta)}{2}$. One can show that if $\frac{3+3\delta}{4\beta} < p_1 < \frac{1+\delta}{\beta}$, the solution is interior at $w_2^{M_1} = \frac{1+\delta}{2\beta}$, and if $\frac{1+\delta}{2\beta} < p_1 \le \frac{3+3\delta}{4\beta}$, then the solution is at the right corner $w_2^{M_1} = \frac{2p_1\beta-1-\delta}{\beta}$.

- Suppose that $w_2 \in M_2$ (where $M_2 \equiv [\omega_a, \omega_b]$), the manufacturer's second-period profit function is given by $\pi_{M2} = (1+\delta-\beta p_1)w_2$. Clearly, $\frac{d\pi_{M2}}{dw_2} > 0$, π_{M2} is increasing in w_2. Hence, if M_2 is non-empty (i.e., $\frac{1+\delta}{2\beta+\gamma\lambda} < p_1 < \frac{1+\delta}{\beta}$), then the optimal wholesale price in M_2 is at the right corner $w_2^{M_2} = \frac{2p_1\beta-1-\delta+p_1\gamma\lambda}{\beta+\gamma\lambda}$.

- Suppose that $w_2 \in M_3$ (where $M_3 \equiv [\omega_b, \omega_c]$), the manufacturer's second-period profit function is given by $\pi_{M2} = \frac{w_2(1+\delta+p_1\gamma\lambda-w_2(\beta+\gamma\lambda))}{2}$. One can show that if $0 < p_1 \leq \frac{3+3\delta}{4\beta+\gamma\lambda}$, then the solution is interior at $w_2^{M_3} = \frac{1+\delta+p_1\gamma\lambda}{2\beta+2\gamma\lambda}$, and if $\frac{3+3\delta}{4\beta+\gamma\lambda} < p_1 < \frac{1+\delta}{\beta}$, then the solution is at the left corner at $w_2^{M_3} = \frac{2p_1\beta-\delta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}$.

- Suppose that $w_2 \in M_4$ (where $M_4 \equiv (\omega_c, \frac{1+\delta}{\beta}]$), the manufacturer's second-period profit function is given by $\pi_{M2} = \frac{w_2(1-\lambda)(1+\delta-w_2\beta)}{2}$. Then π_{M2} is decreasing in w_2 on the interval M_4. One can show that the solution is at the left corner $w_2^{M4} = \omega_c$.

Comparing the manufacturer's profits corresponding to each wholesale price $w_2^{M_i}$, one can show that the manufacturer's optimal second-period price is given by

$$w_2^*(p_1) = \begin{cases} \frac{1+\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 \leq p_1 \leq \rho_a \\ \frac{2p_1\beta-1-\delta+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \rho_a \leq p_1 \leq \rho_b \\ \frac{1+\delta}{2\beta} & \text{if } p_1 > \rho_b \end{cases}$$

where $\rho_a = \frac{3(1+\delta)}{4\beta+\gamma\lambda}$ and $\rho_b = \frac{(1+\delta)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. Note that the manufacturer's wholesale price jumps down at ρ_b in order to induce the retailer to sufficiently reduce its second-period price to become perceived as fair. The retailer's optimal second-period price is given by

$$p_2^*(p_1) = \begin{cases} \frac{3(1+\delta+p_1\gamma\lambda)}{4(\beta+\gamma\lambda)} & \text{if } 0 \leq p_1 \leq \rho_a \\ p_1 & \text{if } \rho_a \leq p_1 \leq \rho_b. \\ \frac{3(1+\delta)}{4\beta} & \text{if } p_1 > \rho_b \end{cases}$$

Using w_2^* and p_2^*, we can obtain the manufacturer's and the retailer's second-period subgame equilibrium profits. Note that the manufacturer's profit is continuous at ρ_b. The retailer's profit jumps up at ρ_b, i.e., $\pi_{R2}^*|_{p_1=\rho_b,\, w_2=\frac{2\rho_b\beta-1-\delta+\rho_b\gamma\lambda}{\beta+\gamma\lambda}} < \pi_{R2}^*|_{p_1=\rho_b,\, w_2=\frac{1+\delta}{2\beta}}$.

$$\pi_{M2}^* = \begin{cases} \frac{(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ \frac{(1-p_1\beta+\delta)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } \rho_a \le p_1 \le \rho_b, \\ \frac{(1+\delta)^2}{8\beta} & \text{if } p_1 > \rho_b \end{cases}$$

$$\pi_{R2}^* = \begin{cases} \frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ \frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda} & \text{if } \rho_a \le p_1 \le \rho_b \\ \frac{(1+\delta)^2}{16\beta} & \text{if } p_1 > \rho_b \end{cases}$$

Table AII-1 summarizes the second-period equilibrium outcome.

Table AII-1: Second-period Equilibrium Results

p_1 Range	w_2^*	p_2^*	π_{M2}^*	π_{R2}^*
$0 \le p_1 \le \rho_a$	$\frac{1+\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)}$	$\frac{3(1+\delta+p_1\gamma\lambda)}{4(\beta+\gamma\lambda)}$	$\frac{(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)}$	$\frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$
$\rho_a \le p_1 \le \rho_b$	$\frac{(2\beta+\gamma\lambda)p_1-(1+\delta)}{\beta+\gamma\lambda}$	p_1	$\frac{(1-p_1\beta+\delta)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda}$	$\frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda}$
$p_1 > \rho_b$	$\frac{1+\delta}{2\beta}$	$\frac{3(1+\delta)}{4\beta}$	$\frac{(1+\delta)^2}{8\beta}$	$\frac{(1+\delta)^2}{16\beta}$

Derivation of the first-period equilibrium.

In this section, we will solve for the first-period equilibrium. The manufacturer's and the retailer's second-period equilibrium profits are given by π_{M2}^* and π_{R2}^* as in equations (AI-3) and (AI-4), respectively. In the first period, given w_1, the retailer chooses its first-period price p_1^* to maximize its total profit $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^*$, where $D_1 = \begin{cases} 1-\beta p_1 & \text{if } 0 < p_1 < \frac{1}{\beta} \\ 0 & \text{if } \frac{1}{\beta} < p_1 \le \frac{1+\delta}{\beta} \end{cases}$.

Let p_1^* denote the retailer's optimal price, i.e., $p_1^* \equiv \underset{p_1}{\operatorname{argmax}}\, \pi_R$. We will obtain the optimal first-period retail prices within each of the following three intervals $K_1 \equiv [0, \rho_a]$, $K_2 \equiv [\rho_a, \rho_b]$, and $K_3 \equiv (\rho_b, \frac{1+\delta}{\beta}]$; among the three, the price that yields the highest total profit is the retailer's optimal first-period retail price. Denote the retailer's optimal first-period price within the interval K_i by $p_1^{K_i}$, $i = 1,2,3$. Note that if $p_1 \geq \frac{1}{\beta}$, then there will be no sales in the first period (i.e., $D_1 = 0$), so the retailer's total profit π_R equals to its second-period profit π_{R2}^*. Note that π_R is piecewise concave on each interval K_i. Hence, the local maximizer in the interval K_i is either in the interior of K_i (and hence satisfied the first-order condition) or it is at one of the endpoints of K_i. Since the analysis is rather straightforward, below we will directly provide the solution to the profit maximization problem within each K_i. There are three cases to analyze: $\frac{1}{\beta} \in K_1$, $\frac{1}{\beta} \in K_2$ and $\frac{1}{\beta} \in K_3$.

Case 1: $\frac{1}{\beta} \in K_3$ (i.e., $\rho_b \leq \frac{1}{\beta} \leq \frac{1+\delta}{\beta}$). Note that $\frac{1}{\beta} \in K_3$ if and only if $0 \leq \delta \leq \delta_m = \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$. First, if $p_1 \in K_1$ (where $K_1 \equiv [0, \rho_a]$), the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$. One can show that if $0 < w_1 < \omega_A = \frac{4\beta+12\beta\delta-3\gamma\lambda-\gamma\delta\lambda}{8\beta^2+2\beta\gamma\lambda}$, the solution is interior at $p_1^{K_1} = \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}$; and if $w_1 \geq \omega_A$, the solution is at the right corner $p_1^{K_1} = \rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$. Second, if $p_1 \in K_2$ (where $K_2 \equiv [\rho_a, \rho_b]$), the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda}$. If $0 < w_1 < \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$, and if $\frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \leq w_1 \leq \frac{(1+\delta)\gamma\lambda\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}+4\beta+8\beta\delta+2\gamma\delta\lambda}{2\beta(2\beta+\gamma\lambda)}$, then the solution is interior at $p_1^{K_2} = \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda}$, and if $w_1 \geq \frac{(1+\delta)\gamma\lambda\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}+4\beta+8\beta\delta+2\gamma\delta\lambda}{2\beta(2\beta+\gamma\lambda)}$, then the solution is at the right corner $p_1^{K_2} = \rho_b$. Third, if $p_1 \in K_3$ (where $K_3 \equiv (\rho_b, \frac{1+\delta}{\beta}]$) and $p_1 \leq \frac{1}{\beta}$, then the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1+\delta)^2}{16\beta}$; if $p_1 \in K_3$

and $p_1 > \frac{1}{\beta}$, then the retailer's total profit is: $\pi_R = \pi_{R2}^* = \frac{(1+\delta)^2}{16\beta}$. One can show that if $0 < w_1 < \omega_\mathrm{C}$, where $\omega_\mathrm{C} = \frac{\beta+3\beta\delta+\gamma\delta\lambda}{2\beta^2+\beta\gamma\lambda} + \frac{1+\delta}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}}$, then the solution is at the left corner at $p_1^{K_3} = \rho_b$, and if $\omega_\mathrm{C} \leq w_1 < \frac{1}{\beta}$, then the solution is at $p_1^{K_3} = \frac{1+w_1\beta}{2\beta}$, and if $w_1 \geq \frac{1}{\beta}$, then the solution is $p_1^{K_3} = \frac{1}{\beta}$. Comparing the profits corresponding to each of the prices $p_1^{K_i}$ for $i = 1,2,3$, one can show that, when $0 \leq \delta \leq \delta_m$, the retailer's first-period optimal price $p_1^*(w_1)$ is as follows.

$$p_1^* = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 < w_1 < \omega_\mathrm{A} \\ \rho_a & \text{if } \omega_\mathrm{A} \leq w_1 \leq \omega_\mathrm{B} \\ \rho_b & \text{if } \omega_\mathrm{B} < w_1 \leq \omega_\mathrm{C} \\ \frac{1+w_1\beta}{2\beta} & \text{if } \omega_\mathrm{C} \leq w_1 < \frac{1}{\beta} \\ \frac{1}{\beta} & \text{if } w_1 \geq \frac{1}{\beta} \end{cases}$$

where $\omega_\mathrm{A} = \frac{4\beta+12\beta\delta-3\gamma\lambda-\gamma\lambda\delta}{8\beta^2+2\beta\gamma\lambda}$, $\omega_\mathrm{B} = \frac{16\beta^2(1+3\delta)+24\beta\gamma\delta\lambda-\gamma^2(3-\delta)\lambda^2}{4\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)} + \frac{(1+\delta)(4\beta+3\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{2(\beta+\gamma\lambda)}}$, which satisfies $\pi_R^*|_{p_1^*=\rho_a} = \pi_R^*|_{p_1^*=\rho_b}$, i.e., $\frac{4\omega_\mathrm{B}\beta^2(3\delta-1)+\gamma\lambda(\delta^2+5\delta+4-\omega_\mathrm{B}\gamma\lambda)+\beta(4-8\delta^2-5\gamma\lambda\omega_\mathrm{B}+\delta(3\gamma\lambda\omega_\mathrm{B}-4))}{(4\beta+\gamma\lambda)^2} = \frac{(1+\delta)^2}{16\beta} + (\frac{(1+\delta)\left(6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}\right)}{4\beta(2\beta+\gamma\lambda)} - \omega_\mathrm{B})(1 - \frac{(1+\delta)\left(6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}\right)}{4(2\beta+\gamma\lambda)})$, and $\omega_C = \frac{\beta+3\beta\delta+\gamma\delta\lambda}{2\beta^2+\beta\gamma\lambda} + \frac{1+\delta}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}}$ (i.e., the corner solution) above which $p_1^* = \frac{1+w_1\beta}{2\beta}$ will be the optimal solution. Note that p_1^* is not continuous at ω_B: it jumps up to ρ_b. The manufacturer's profit jumps down at ω_B. Plugging in p_1^* , we can get the retailer's profit $\pi_R^*(w_1)$:

$\pi_R^*(w_1) =$

$$\begin{cases} \frac{\beta(4w_1\beta(w_1\beta-2)+\delta(\delta+2)+5)+\gamma\lambda\left(5+\delta+w_1\beta(4w_1\beta+\delta-7)\right)+w_1\gamma^2\lambda^2}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 < w_1 < \omega_\mathrm{A} \\ \frac{4w_1\beta^2(3\delta-1)+\gamma\lambda(\delta^2+5\delta+4-w_1\gamma\lambda)+\beta(4-8\delta^2-5\gamma\lambda w_1+\delta(3\gamma\lambda w_1-4))}{(4\beta+\gamma\lambda)^2} & \text{if } \omega_\mathrm{A} \leq w_1 \leq \omega_\mathrm{B} \\ \frac{(1+\delta)^2}{16\beta} + (\frac{(1+\delta)\left(6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}\right)}{4\beta(2\beta+\gamma\lambda)} - w_1)(1 - \frac{(1+\delta)\left(6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}\right)}{4(2\beta+\gamma\lambda)}) & \text{if } \omega_\mathrm{B} \leq w_1 \leq \omega_\mathrm{C} \\ \frac{4w_1\beta(\beta w_1-2)+\delta(\delta+2)+5}{16\beta} & \text{if } \omega_\mathrm{C} \leq w_1 < \frac{1}{\beta} \\ \frac{(1+\delta)^2}{16\beta} & \text{if } w_1 \geq \frac{1}{\beta} \end{cases}$$

Plugging p_1^*, we can also obtain the manufacturer's total profit $\pi_M^*(w_1)$:

$$\pi_M^*(w_1) = \begin{cases} \frac{8(\beta+\gamma\lambda)(\gamma\lambda+\beta(2+2\delta+w_1\gamma\lambda))^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2)^2} + w_1(1 - \frac{\beta(8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}) & \text{if } 0 < w_1 < \omega_A \\ \frac{2(1+\delta)^2(\beta+\gamma\lambda)+w_1(4\beta+\gamma\lambda)(\beta-3\beta\delta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } \omega_A \le w_1 \le \omega_B \\ \frac{(1+\delta)^2}{8\beta} + w_1(1-\beta\frac{(1+\delta)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}) & \text{if } \omega_B < w_1 \le \omega_C \\ \frac{4w_1\beta-4w_1^2\beta^2+(1+\delta)^2}{8\beta} & \text{if } \omega_C \le w_1 < \frac{1}{\beta} \\ \frac{(1+\delta)^2}{8\beta} & \text{if } w_1 \ge \frac{1}{\beta} \end{cases}$$

Case 2: $\frac{1}{\beta} \in K_2$ (i.e., $\rho_a \le \frac{1}{\beta} \le \rho_b$). Note that $\frac{1}{\beta} \in K_2$ if and only if $\delta_m < \delta \le \frac{\beta+\gamma\lambda}{3\beta}$.

First, if $p_1 \in K_1$ (where $K_1 \equiv [0, \rho_a]$), then the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$. If $0 < w_1 < \omega_A$, one can show that the solution is interior at $p_1^{K_1} = \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}$, and if $w_1 \ge \omega_A$, the solution is at the right corner $p_1^{K_1} = \rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$. Second, if $p_1 \in K_2$ (where $K_2 \equiv [\rho_a, \rho_b]$), the retailer's total profit is as follows: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda}$ when $p_1 < \frac{1}{\beta}$, and $\pi_R = \pi_{R2}^* = \frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda}$ when $p_1 \ge \frac{1}{\beta}$. One can show that if $0 < w_1 < \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda}$, then the solution is at the left corner $p_1^{K_2} = \rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$, and if $\frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \le w_1 < \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda}$, then the solution is at $p_1^{K_2} = \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda}$, and if $w_1 \ge \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda}$, then the solution is at $p_1^{K_2} = \frac{1}{\beta}$. Third, if $p_1 \in K_3$ (where $K_3 \equiv [\rho_b, \frac{1+\delta}{\beta}]$), the retailer's total profit is equal to its second-period profit, i.e., $\pi_R = \pi_{R2}^* = \frac{(1+\delta)^2}{16\beta}$. One can show that $\frac{(1+\delta)^2}{16\beta}$ is less than the maximal profit the retailer can get by charging $p_1^{K_2}$. Thus, when $\delta_m < \delta < \frac{\beta+\gamma\lambda}{3\beta}$, in equilibrium, the retailer's price will never belong to the interval K_3. Comparing the retailer's profits corresponding to prices $p_1^{K_1}$ and $p_1^{K_2}$, we obtain the retailer's optimal p_1^*.

If $\delta_m < \delta < \frac{4\beta+3\gamma\lambda}{12\beta+\gamma\lambda}$, then

$$p_1^* = \begin{cases} \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \le w_1 \le \omega_A \\ \rho_a & \text{if } \omega_A \le w_1 \le \omega_D \\ \rho_b & \text{if } w_1 \ge \omega_D \end{cases},$$

where $\omega_A = \frac{4\beta+12\beta\delta-3\gamma\lambda-\gamma\lambda\delta}{8\beta^2+2\beta\gamma\lambda}$ and $\omega_D = \frac{(1+\delta)(48\beta^2(3\delta-1)-8\beta\gamma\lambda(7+\delta)+(1+\delta)\gamma^2\lambda^2)}{16\beta(\beta(3\delta-1)-\gamma\lambda)(4\beta+\gamma\lambda)}$.

If $\frac{4\beta+3\gamma\lambda}{12\beta+\gamma\lambda} \le \delta < \frac{\beta+\gamma\lambda}{15\beta-\gamma\lambda} + 4\sqrt{\frac{\beta^2+\beta\gamma\lambda}{(15\beta-\gamma\lambda)^2}}$, then

$$p_1^* = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \le w_1 \le \omega_A \\ \rho_a & \text{if } \omega_A \le w_1 \le \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \\ \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda} & \text{if } \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \le w_1 \le \omega_E \\ \rho_b & \text{if } w_1 \ge \omega_E \end{cases},$$

where $\omega_E \equiv \frac{\beta+2\beta\delta+\gamma\lambda}{\beta(\beta+\gamma\lambda)} - \frac{\sqrt{\gamma\lambda(\beta(1+2\delta-15\delta^2)+\gamma\lambda(1+\delta)^2)}}{2\beta(\beta+\gamma\lambda)}$.

If $\frac{\beta+\gamma\lambda}{15\beta-\gamma\lambda} + 4\sqrt{\frac{\beta^2+\beta\gamma\lambda}{(15\beta-\gamma\lambda)^2}} \le \delta \le \frac{\beta+\gamma\lambda}{3\beta}$, then

$$p_1^* = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \le w_1 \le \omega_A \\ \rho_a & \text{if } \omega_A \le w_1 \le \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \\ \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda} & \text{if } \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \le w_1 < \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda} \\ \frac{1}{\beta} & \text{if } w_1 \ge \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda} \end{cases}.$$

Note that p_1^* jumps up at ω_D and ω_E, and thus, the manufacturer's profit jumps down at these points.

Plugging in p_1^*, we can obtain the manufacturer's profit $\pi_M^*(w_1)$:

$$\pi_M^* =$$

$$\begin{cases} \frac{8(\beta+\gamma\lambda)(\gamma\lambda+\beta(2+2\delta+w_1\gamma\lambda))^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2)^2} + w_1\left(1 - \frac{\beta(8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}\right) & \text{if } p_1^* = \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} \\ \frac{2(1+\delta)^2(\beta+\gamma\lambda)+w_1(4\beta+\gamma\lambda)(\beta-3\beta\delta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } p_1^* = \rho_a \\ \frac{w_1(\gamma\lambda+\beta(1+2\delta-w_1\gamma\lambda)-w_1\beta^2)}{2\gamma\lambda} - \frac{(w_1\beta-2\delta-1)(\beta+\gamma\lambda)(2w_1\beta^2+\gamma\lambda-\beta(2+4\delta-w_1\gamma\lambda))}{4\beta\gamma^2\lambda^2} & \text{if } p_1^* = \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda} \\ \frac{\delta(1-\delta+\frac{\gamma\lambda}{\beta})}{\beta+\gamma\lambda} & \text{if } p_1^* = \frac{1}{\beta} \\ \frac{(1+\delta)^2}{8\beta} & \text{if } p_1^* = \rho_b \end{cases}$$

Case 3: $\frac{1}{\beta} \in K_1$ (i.e., $0 < \frac{1}{\beta} \leq \rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$). Note that $\frac{1}{\beta} \in K_1$ if and only if $\delta \geq \frac{\beta+\gamma\lambda}{3\beta}$.

First, if $p_1 \in K_1$ (where $K_1 \equiv [0, \rho_a]$), then the retailer's total profit is given by: $\pi_R = D_1 \cdot (p_1 - w_1) + \pi_{R2}^* = (1 - \beta p_1)(p_1 - w_1) + \frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$ when $p_1 < \frac{1}{\beta}$, and $\pi_R = \pi_{R2}^* = \frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$ when $p_1 \geq \frac{1}{\beta}$. One can show that if $\frac{\beta+\gamma\lambda}{3\beta} \leq \delta < \frac{8\beta^2+7\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}$ and $0 < w_1 \leq \frac{8\beta^2+7\beta\gamma\lambda-\beta\gamma\lambda\delta-\gamma^2\lambda^2}{8\beta^3+8\beta^2\gamma\lambda}$, then the retailer's profit is maximized at the interior solution $p_1^{K_1} = \frac{8w_1\beta^2+\gamma\lambda(9+\delta)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}$; if $\delta > \frac{8\beta^2+7\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}$ or $\frac{\beta+\gamma\lambda}{3\beta} \leq \delta \leq \frac{8\beta^2+7\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda}$ and $w_1 > \frac{8\beta^2+7\beta\gamma\lambda-\beta\gamma\lambda\delta-\gamma^2\lambda^2}{8\beta^3+8\beta^2\gamma\lambda}$, then the solution is $p_1^{K_1} = \frac{1}{\beta}$. Second, if $p_1 \in K_2$ (where $K_2 \equiv [\rho_b, \rho_b]$), in this case, $\pi_R^* = \pi_{R2}^* = \frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda}$. Note that there are no sales in the first period (i.e., $D_1 = 0$). When $p_1 \leq \frac{1+\delta}{\beta}$, one can show that $\frac{\partial \pi_R^*}{\partial p_1} < 0$, i.e., π_R^* is decreasing in p_1, and hence, the solution is at the left corner at $p_1^{K_2} = \rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$. Plugging in $p_1^{K_2}$, we can get the retailer's total profit: $\pi_R^* = \frac{(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$. Third, if $p_1 \in K_3$ (where $K_3 \equiv (\rho_b, \frac{1+\delta}{\beta}]$), in this case, $\pi_R^* = \pi_{R2}^* = \frac{(1+\delta)^2}{16\beta}$ and there are no sales in the first period, then any price $p_1 \in K_3$ yields the same profit to the retailer, i.e., any $p_1 \in K_3$ is a solution to the retailer's maximization problem on the constraint set K_3. One can show that $\frac{(1+\delta)^2}{16\beta}$ is less than the profit the retailer can get by charging $p_1^{K_2}$. Thus, when $\delta \geq \frac{\beta+\gamma\lambda}{3\beta}$, in equilibrium, the retailer's price will never belong to the interval K_3. Comparing the retailer's profits corresponding to the prices $p_1^{K_i}$ for $i = 1,2,3$, one can show that the retailer's optimal price is given by

$$p_1^* = \begin{cases} \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 < w_1 \leq \omega_F \text{ and } \frac{\beta+\gamma\lambda}{3\beta} < \delta < \delta^\circ \\ \frac{3+3\delta}{4\beta+\gamma\lambda} & \text{if } w_1 > \omega_F \text{ and } \frac{\beta+\gamma\lambda}{3\beta} < \delta \leq \delta^\circ, \text{ or } \delta > \delta^\circ \end{cases},$$

where $\delta^\circ \equiv \frac{-32\beta^2-20\beta\gamma\lambda+3\gamma^2\lambda^2}{2(24\beta^2+14\beta\gamma\lambda-\gamma^2\lambda^2)} + \frac{4\beta+\gamma\lambda}{2(24\beta^2+14\beta\gamma\lambda-\gamma^2\lambda^2)}\sqrt{\frac{3(128\beta^3+208\beta^2\gamma\lambda+72\beta\gamma^2\lambda^2-5\gamma^3\lambda^3)}{\gamma\lambda}}$, and $\omega_F \equiv \frac{8\beta^2+\beta\gamma(7-\delta)\lambda-\gamma^2\lambda^2}{8\beta^2(\beta+\gamma\lambda)} - \frac{\sqrt{\gamma\lambda(\gamma^2\lambda^2-16\beta^2-16\beta\gamma\lambda)(\beta^2\gamma(17-14\delta-15\delta^2)\lambda-8\beta^3(3\delta^2+2\delta-1)+2\beta\gamma^2(5+\delta)\lambda^2+\gamma^3\lambda^3)}}{8\beta^2(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, which satisfies $\pi_R^*|_{p_1^*=\frac{8\omega_F\beta^2+\gamma(9+\delta)\lambda+8\beta(1+\omega_F\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}} = \pi_R^*|_{p_1^*=\rho_a}$, i.e.,

$$\frac{\beta(4\omega_F\beta(\omega_F\beta-2)+\delta(\delta+2)+5)+\gamma\lambda\big(5+\delta+\omega_F\beta(4\omega_F\beta+\delta-7)\big)+\omega_F\gamma^2\lambda^2}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} = \frac{(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}.$$

Plugging in p_1^*, we can get the retailer's profit $\pi_R^*(w_1) = \pi_R^* =$

$$\begin{cases} \frac{\beta(4w_1\beta(w_1\beta-2)+\delta(\delta+2)+5)+\gamma\lambda\big(5+\delta+w_1\beta(4w_1\beta+\delta-7)\big)+w_1\gamma^2\lambda^2}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 < w_1 \le \omega_F \text{ and } \frac{\beta+\gamma\lambda}{3\beta} < \delta < \delta^\circ \\ \frac{(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } w_1 > \omega_F \text{ and } \frac{\beta+\gamma\lambda}{3\beta} < \delta \le \delta^\circ, \text{ or } \delta > \delta^\circ \end{cases}$$

Plugging in p_1^*, we can also obtain the manufacturer's profit $\pi_M^*(w_1) = \pi_M^* =$

$$\begin{cases} \frac{8(\beta+\gamma\lambda)(\gamma\lambda+\beta(2+2\delta+w_1\gamma\lambda))^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2)^2} + w_1\left(1-\frac{\beta(8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}\right) & \text{if } 0 < w_1 \le \omega_F \text{ and } \frac{\beta+\gamma\lambda}{3\beta} < \delta < \delta^\circ \\ \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } w_1 > \omega_F \text{ and } \frac{\beta+\gamma\lambda}{3\beta} < \delta \le \delta^\circ, \text{or } \delta > \delta^\circ \end{cases}$$

Finally, we proceed to finding the manufacturer's optimal price in the first period (i.e., w_1^*). For a given δ, define $\Omega_1 \equiv \{w_1 | \, p_1^* = \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}\}$, $\Omega_2 \equiv \{w_1 | \, p_1^* = \rho_a\}$, $\Omega_3 \equiv \{w_1 | \, p_1^* = \rho_b\}$, $\Omega_4 \equiv \{w_1 | \, p_1^* = \frac{1+w_1\beta}{2\beta}\}$, $\Omega_5 \equiv \{w_1 | \, p_1^* = \frac{1}{\beta}\}$, and $\Omega_6 \equiv \{w_1 | \, p_1^* = \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda}\}$. Note that any $w_1 \ge 0$ satisfies $w_1 \in \bigcup_{i=1}^{6}\Omega_i$. We will first find the manufacturer's optimal price $w_1^{\Omega_i}$ within each of the sets Ω_i. The price corresponding to the highest profit will be the manufacturer's equilibrium wholesale price in the first period.

First, suppose that $w_1 \in \Omega_1$ (where $\Omega_1 \equiv \{w_1 | \, p_1^* = \frac{8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}\}$). Then, the manufacturer's profit is given by $\pi_M(w_1) = \frac{8(\beta+\gamma\lambda)(\gamma\lambda+\beta(2+2\delta+w_1\gamma\lambda))^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2)^2} + w_1(1 - \frac{\beta(8w_1\beta^2+\gamma(9+\delta)\lambda+8\beta(1+w_1\gamma\lambda))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2})$. One can show that the maximizer of π_M in Ω_1 is given by

$$w_1^{\Omega_1} = \begin{cases} \frac{4\beta+12\beta\delta-3\gamma\lambda-\gamma\delta\lambda}{8\beta^2+2\beta\gamma\lambda} & \text{if } 0 < \delta \le \frac{36\beta^2\gamma\lambda+35\beta\gamma^2\lambda^2-\gamma^3\lambda^3}{96\beta^3+84\beta^2\gamma\lambda-15\beta\gamma^2\lambda^2} \\ \frac{128\beta^4+16\beta^3\gamma(17+\delta)\lambda+8\beta^2\gamma^2(17+2\delta)\lambda^2+\beta\gamma^3(\delta-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \frac{36\beta^2\gamma\lambda+35\beta\gamma^2\lambda^2-\gamma^3\lambda^3}{96\beta^3+84\beta^2\gamma\lambda-15\beta\gamma^2\lambda^2} < \delta \le \check{\delta} \\ \omega_F & \text{if } \check{\delta} < \delta \le \delta^\circ \\ \text{any price} & \text{if } \delta^\circ < \delta < \frac{8\beta^2+7\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} \end{cases},$$

where, as defined earlier, $\omega_F \equiv \frac{8\beta^2+\beta\gamma(7-\delta)\lambda-\gamma^2\lambda^2}{8\beta^2(\beta+\gamma\lambda)} -$

$\frac{\sqrt{\gamma\lambda(\gamma^2\lambda^2-16\beta^2-16\beta\gamma\lambda)(\beta^2\gamma(17-14\delta-15\delta^2)\lambda-8\beta^3(3\delta^2+2\delta-1)+2\beta\gamma^2(5+\delta)\lambda^2+\gamma^3\lambda^3)}}{8\beta^2(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, $\check{\delta}$ is implicitly defined by solving the equation $\omega_{\mathrm{F}} = \frac{128\beta^4+16\beta^3\gamma(17+\delta)\lambda+8\beta^2\gamma^2(17+2\delta)\lambda^2+\beta\gamma^3(\delta-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)}$, and $\delta^\circ \equiv \frac{-32\beta^2-20\beta\gamma\lambda+3\gamma^2\lambda^2}{2(24\beta^2+14\beta\gamma\lambda-\gamma^2\lambda^2)} + \frac{4\beta+\gamma\lambda}{2(24\beta^2+14\beta\gamma\lambda-\gamma^2\lambda^2)}\sqrt{\frac{3(128\beta^3+208\beta^2\gamma\lambda+72\beta\gamma^2\lambda^2-5\gamma^3\lambda^3)}{\gamma\lambda}}$.

Second, suppose that $w_1 \in \Omega_2$ (where $\Omega_2 \equiv \{w_1 | \, p_1^* = \rho_a\}$). Then, if $\delta \leq \frac{\beta+\gamma\lambda}{3\beta}$, the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{2(1+\delta)^2(\beta+\gamma\lambda)+\mathrm{w}_1(4\beta+\gamma\lambda)(\beta-3\beta\delta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$, and if $\delta \geq \frac{\beta+\gamma\lambda}{3\beta}$, we have $\pi_M^*(w_1) = \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$. For $\delta \leq \frac{\beta+\gamma\lambda}{3\beta}$, π_M^* is increasing in w_1, i.e., $\frac{\partial \pi_M^*}{\partial w_1} > 0$. Thus, the maximizer of Ω_2 is the largest w_1 in Ω_2. By contrast, for $\delta \geq \frac{\beta+\gamma\lambda}{3\beta}$, $\pi_M^*(w_1)$ does not depend on w_1. So, one can show that the maximizer of π_M in Ω_2 is given by

$$w_1^{\Omega_2} = \begin{cases} \omega_{\mathrm{B}} & \text{if } 0 < \delta \leq \delta_m \\ \omega_{\mathrm{D}} & \text{if } \delta_m \leq \delta < \frac{4\beta+3\gamma\lambda}{12\beta+\gamma\lambda} \\ \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} & \text{if } \frac{4\beta+3\gamma\lambda}{12\beta+\gamma\lambda} \leq \delta \leq \frac{\beta+\gamma\lambda}{3\beta} \\ \text{any price larger than } \omega_{\mathrm{F}} & \text{if } \frac{\beta+\gamma\lambda}{3\beta} < \delta \leq \delta^\circ \\ \text{any price} & \text{if } \delta > \delta^\circ \end{cases},$$

where, as defined earlier, $\omega_{\mathrm{B}} \equiv \frac{16\beta^2(1+3\delta)+24\beta\gamma\delta\lambda-\gamma^2(3-\delta)\lambda^2}{4\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)} + \frac{(1+\delta)(4\beta+3\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{2(\beta+\gamma\lambda)}}$, and $\omega_{\mathrm{D}} \equiv \frac{(1+\delta)(48\beta^2(3\delta-1)-8\beta\gamma\lambda(7+\delta)+(1+\delta)\gamma^2\lambda^2)}{16\beta(\beta(3\delta-1)-\gamma\lambda)(4\beta+\gamma\lambda)}$.

Third, suppose that $w_1 \in \Omega_3$ (where $\Omega_3 \equiv \{w_1 | \, p_1^* = \rho_b\}$). Then, the manufacturer's profit is given by

$$\pi_M^*(w_1) = \begin{cases} \frac{(1+\delta)^2}{8\beta} + w_1\left(1 - \frac{\beta(1+\delta)\left(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda\right)}{4\beta(2\beta+\gamma\lambda)}\right) & \text{if } 0 < \delta \leq \delta_m \\ \frac{(1+\delta)^2}{8\beta} & \text{if } \delta_m < \delta < \frac{4\beta+3\gamma\lambda}{12\beta+\gamma\lambda} \end{cases}.$$

One can show $w_1^{\Omega_3} = \begin{cases} \frac{\beta+3\beta\delta+\gamma\delta\lambda}{2\beta^2+\beta\gamma\lambda} + \frac{1+\delta}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}} & \text{if } 0 < \delta \leq \delta_m \\ \text{any price, e.g., } \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda} & \text{if } \delta_m \leq \delta < \frac{4\beta+3\gamma\lambda}{12\beta+\gamma\lambda} \end{cases}.$

Fourth, if $w_1 \in \Omega_4$ (where $\Omega_4 \equiv \{w_1 | \, p_1^* = \frac{1+w_1\beta}{2\beta}\}$), then the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{4w_1\beta - 4w_1^2\beta^2 + (1+\delta)^2}{8\beta}$. The manufacturer's optimal price in Ω_4 is given by $w_1^{\Omega_4} = \frac{\beta+3\beta\delta+\gamma\delta\lambda}{2\beta^2+\beta\gamma\lambda} + \frac{1+\delta}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}}$.

Fifth, if $w_1 \in \Omega_5$ (where $\Omega_5 \equiv \{w_1 | \, p_1^* = \frac{1}{\beta}\}$), the manufacturer's profit is given by

$$\pi_M^*(w_1) = \begin{cases} \frac{(1+\delta)^2}{8\beta} & \text{if } \; 0 < \delta \le \frac{3(\beta+\gamma\lambda) - 2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda} & \text{and } \; w_1 \ge \frac{1}{\beta} \\ \frac{\delta(1-\delta+\frac{\gamma\lambda}{\beta})}{\beta+\gamma\lambda} & \text{if } \; \frac{\beta+\gamma\lambda}{15\beta-\gamma\lambda} + 4\sqrt{\frac{\beta^2+\beta\gamma\lambda}{(15\beta-\gamma\lambda)^2}} \le \delta \le \frac{\beta+\gamma\lambda}{3\beta} & \text{and } \; w_1 \ge \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda} \\ \frac{\left(1-\delta+\frac{\gamma\lambda}{\beta}\right)^2}{8(\beta+\gamma\lambda)} & \text{if } \; \frac{\beta+\gamma\lambda}{3\beta} \le \delta \le \frac{8\beta^2+7\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} & \text{and } \; w_1 > \frac{8\beta^2+7\beta\gamma\lambda-\beta\gamma\lambda\delta-\gamma^2\lambda^2}{8\beta^3+8\beta^2\gamma\lambda} \\ \frac{\left(1-\delta+\frac{\gamma\lambda}{\beta}\right)^2}{8(\beta+\gamma\lambda)} & \text{if } \; \delta > \frac{8\beta^2+7\beta\gamma\lambda-\gamma^2\lambda^2}{\beta\gamma\lambda} & \end{cases}$$

Since $\delta_m < \frac{\beta+\gamma\lambda}{15\beta-\gamma\lambda} + 4\sqrt{\frac{\beta^2+\beta\gamma\lambda}{(15\beta-\gamma\lambda)^2}}$, it follows that $\pi_M^*(w_1)$ is constant in w_1 as long as $w_1 \in \Omega_5$. Hence, we can take, for example, $w_1^{\Omega_5} = \frac{1}{\beta}$ if $0 < \delta \le \delta_m$ and $w_1^{\Omega_5} = \frac{\beta+2\beta\delta+\gamma\lambda}{\beta^2+\beta\gamma\lambda}$ if $\delta \ge \frac{\beta+\gamma\lambda}{15\beta-\gamma\lambda} + 4\sqrt{\frac{\beta^2+\beta\gamma\lambda}{(15\beta-\gamma\lambda)^2}}$. Finally, if $w_1 \in \Omega_6$ (where $\Omega_6 \equiv \{w_1 | \, p_1^* = \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta)}{2\beta\gamma\lambda}\}$), the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{w_1(\gamma\lambda+\beta(1+2\delta-w_1\gamma\lambda)-w_1\beta^2)}{2\gamma\lambda} - \frac{(w_1\beta-2\delta-1)(\beta+\gamma\lambda)(2w_1\beta^2+\gamma\lambda-\beta(2+4\delta-w_1\gamma\lambda))}{4\beta\gamma^2\lambda^2}$. We obtain that, in Ω_6, π_M is maximized at $w_1^{\Omega_6} = \frac{4\beta+8\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda}$.

To find the manufacturer's equilibrium first-period price, it remains to compare the manufacturer's profits under the wholesale prices $w_1^{\Omega_i}$ for $i \in \{1, \dots, 6\}$. The price that yields the highest profit will correspond to the manufacturer's equilibrium price. We will do the comparison in 2 steps.

Step 1. One can show that the solution in Ω_1 dominates the solutions in Ω_3, Ω_4, Ω_5 and Ω_6, i.e., the manufacturer will earn higher profit by setting $w_1 = w_1^{\Omega_1}$ than by setting $w_1 = w_1^{\Omega_i}$ for any $i \in \{3,4,5,6\}$.

Step 2. It remains to compare the manufacturer's profits under $w_1^{\Omega_1}$ and $w_1^{\Omega_2}$. One can show that the equilibrium first-period wholesale price will be $w_1^{\Omega_1}$ if δ is above a threshold (denoted by δ^*), and $w_1^{\Omega_2}$ if

δ below that threshold. For notational convenience, we first implicitly define two thresholds δ_A and δ_B, where δ_A satisfies

$$\pi_M^*|_{w_1^{\Omega_2*}=\omega_B=\frac{16\beta^2(3\delta_A+1)+24\beta\gamma\lambda\delta_A+\gamma^2\lambda^2(\delta_A-3)}{4\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)}+\frac{(1+\delta_A)(4\beta+3\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{2(\beta+\gamma\lambda)}}} =$$

$$\pi_M^*|_{w_1^{\Omega_1*}=\frac{128\beta^4+16\beta^3\gamma\lambda(17+\delta_A)+8\beta^2\gamma^2(17+2\delta_A)\lambda^2+\beta\gamma^3(\delta_A-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)}}$$

, i.e., $\frac{\beta-3\beta\delta_A+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{(1+\delta_A)(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}+\frac{2\left(16\beta^2(1+3\delta_A)+24\beta\gamma\lambda\delta_A+\gamma^2\lambda^2(\delta_A-3)\right)}{\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)}\right)+\frac{2(1+\delta_A)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}=$

$\frac{64\beta^4\left(2+2\delta_A+\delta_A^2\right)+16\beta^3\gamma\lambda\left(17+13\delta_A+4\delta_A^2\right)+\beta^2\gamma^2\lambda^2\left(161+82\delta_A+\delta_A^2\right)+2\beta\gamma^3\lambda^3(9+\delta_A)+\gamma^4\lambda^4}{64\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)}$, and δ_B satisfies

$$\pi_M^*|_{w_1^{\Omega_2*}=\omega_D=\frac{(1+\delta_B)(48\beta^2(3\delta_B-1)-8\beta\gamma\lambda(7+\delta_B)+\gamma^2\lambda^2(1+\delta_B))}{16\beta(\beta(3\delta_B-1)-\gamma\lambda)(4\beta+\gamma\lambda)}} =$$

$$\pi_M^*|_{w_1^{\Omega_1*}=\frac{128\beta^4+16\beta^3\gamma\lambda(17+\delta_B)+8\beta^2\gamma^2(17+2\delta_B)\lambda^2+\beta\gamma^3(\delta_B-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)}},$$

i.e. $\frac{(1+\delta_B)(16\beta^2(5-7\delta_B)+8\beta\gamma\lambda(7+\delta_B)-\gamma^2\lambda^2(1+\delta_B))}{16\beta(4\beta+\gamma\lambda)^2}=$

$\frac{64\beta^4\left(2+2\delta_B+\delta_B^2\right)+16\beta^3\gamma\lambda\left(17+13\delta_B+4\delta_B^2\right)+\beta^2\gamma^2\lambda^2\left(161+82\delta_B+\delta_B^2\right)+2\beta\gamma^3\lambda^3(9+\delta_B)+\gamma^4\lambda^4}{64\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)}$.

We define δ^* as follows.

$$\delta^* = \begin{cases} \delta_A & \text{if } \delta_A \le \delta_m = \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda} \\ \delta_B & \text{otherwise} \end{cases}.$$

Also, let δ^{**} be the solution for δ to the following equation that falls in the interval $(\frac{\beta+\gamma\lambda}{3\beta},\delta^\circ)$:

$\pi_M^*|_{w_1^*=\frac{128\beta^4+16\beta^3\gamma\lambda(17+\delta)+8\beta^2\gamma^2(17+2\delta)\lambda^2+\beta\gamma^3(\delta-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)}} = \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$. More specifically, $\delta^{**} = \frac{8}{3}\sqrt{\frac{2(\beta+\gamma\lambda)^2(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2(6\beta+\gamma\lambda)(8\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)}{\beta^2\gamma\lambda\,(512\beta^3+784\beta^2\gamma\lambda+232\beta\gamma^2\lambda^2-43\gamma^3\lambda^3)^2}}-\frac{(4\beta-\gamma\lambda)(\beta+\gamma\lambda)(224\beta^2+180\beta\gamma\lambda+\gamma^2\lambda^2)}{3\beta(512\beta^3+784\beta^2\gamma\lambda+232\beta\gamma^2\lambda^2-43\gamma^3\lambda^3)}$. Note that if $\delta > \delta^{**}$, then $p_1^* = \frac{3+3\delta}{4\beta+\gamma\lambda} > \frac{1}{\beta}$, which implies no first-period sales. Finally, using $w_1^{\Omega_1}$ and $w_1^{\Omega_2}$ expressions, one can show that

$$w_1^* = \begin{cases} \hat{w} & \text{if } 0 \le \delta < \delta^* \\ \frac{128\beta^4+16\beta^3\gamma\lambda(17+\delta)+8\beta^2\gamma^2(17+2\delta)\lambda^2+\beta\gamma^3(\delta-7)\lambda^3+\gamma^4\lambda^4}{32\beta^2(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

where $\widehat{w} = \begin{cases} \omega_B & \text{if } 0 < \delta \le \delta_m \\ \omega_D & \text{if } \delta_m \le \delta < \delta^* \end{cases}$,

where, as defined earlier, $\delta_m = \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$, $\omega_B \equiv \frac{16\beta^2(1+3\delta)+24\beta\gamma\delta\lambda-\gamma^2\lambda^2(3-\delta)}{4\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)} + \frac{(1+\delta)(4\beta+3\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{2(\beta+\gamma\lambda)}}$, and $\omega_D \equiv \frac{(1+\delta)(48\beta^2(3\delta-1)-8\beta\gamma\lambda(7+\delta)+(1+\delta)\gamma^2\lambda^2)}{16\beta(\beta(3\delta-1)-\gamma\lambda)(4\beta+\gamma\lambda)}$.

Using the manufacturer's first-period equilibrium price, we can readily derive the equilibrium outcome of the entire game. In particular, the retailer's first-period equilibrium price is:

$$p_1^* = \begin{cases} \frac{3(1+\delta)}{4\beta+\gamma\lambda} & \text{if } 0 \le \delta < \delta^* \\ \frac{24\beta^2+3\beta\gamma(9+\delta)\lambda-\gamma^2\lambda^2}{4\beta(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

From expressions of w_1^* and p_1^*, we can obtain the second-period equilibrium prices:

$$w_2^* = \begin{cases} \frac{2(1+\delta)}{4\beta+\gamma\lambda} & \text{if } 0 \le \delta < \delta^* \\ \frac{32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)-\beta\gamma^2\lambda^2(\delta-23)-\gamma^3\lambda^3}{8\beta(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \delta^* \le \delta < \delta^{**}, \end{cases}$$

$$p_2^* = \begin{cases} \frac{3(1+\delta)}{4\beta+\gamma\lambda} & \text{if } 0 \le \delta < \delta^* \\ \frac{96\beta^3(1+\delta)+24\beta^2\gamma\lambda(7+4\delta)-3\beta\gamma^2\lambda^2(\delta-23)-3\gamma^3\lambda^3}{16\beta(8\beta^3+16\beta^2\gamma\lambda+7\beta\gamma^2\lambda^2-\gamma^3\lambda^3)} & \text{if } \delta^* \le \delta < \delta^{**}. \end{cases}$$

Using these prices, one can derive the manufacturer's and the retailer's profits in each period.

$$\pi_{M1}^* = \begin{cases} \hat{\pi}_{M1} & \text{if } 0 \le \delta < \delta^* \\ \frac{(8\beta^2+\beta\gamma(5-3\delta)\lambda-3\gamma^2\lambda^2)(128\beta^4+16\beta^3\gamma\lambda(17+\delta)+8\beta^2\gamma^2\lambda^2(17+2\delta)-\beta\gamma^3\lambda^3(7-\delta)+\gamma^4\lambda^4)}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

where $\hat{\pi}_{M1}$ is as follows:

$$\hat{\pi}_{M1} = \begin{cases} \frac{\beta-3\beta\delta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{(1+\delta)(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{2(16\beta^2(1+3\delta)+24\beta\gamma\lambda\delta+\gamma^2\lambda^2(\delta-3))}{\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)}\right) & \text{if } 0 < \delta \le \delta_m \\ \frac{(1+\delta)(48\beta^2(1-3\delta)+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta))}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_m \le \delta < \delta^* \end{cases}$$

$$\pi_{M2}^* = \begin{cases} \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } 0 \le \delta < \delta^* \\ \frac{(\gamma^3\lambda^3-32\beta^3(1+\delta)-8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta))^2}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

$$\pi_{R1}^* = \begin{cases} \hat{\pi}_{R1} & \text{if } 0 \le \delta < \delta^* \\ \frac{(8\beta^2+\beta\gamma(5-3\delta)\lambda-3\gamma^2\lambda^2)(8\beta^2+\beta\gamma\lambda(9+\delta)+\gamma^2\lambda^2)}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

where

$$\hat{\pi}_{R1} = \begin{cases} \frac{(\beta(3\delta-1)-\gamma\lambda)(8\sigma\beta^3-2\gamma^2\lambda^2(3-\delta)+\beta\gamma\lambda(24\delta+\sigma\gamma\lambda-24)+2\beta^2(24\delta+3\sigma\gamma\lambda-8))}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2} & \text{if } 0 < \delta \le \delta_m \\ \frac{\gamma(1+\delta)^2\lambda(\gamma\lambda-8\beta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_m \le \delta < \delta^* \end{cases},$$

with $\sigma \equiv \frac{(1+\delta)(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}$ and $\delta_m = \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$.

$$\pi_{R2}^* = \begin{cases} \frac{(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } 0 \le \delta < \delta^* \\ \frac{(32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta)-\gamma^3\lambda^3)^2}{256\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

The manufacturer's and the retailer's total profits are given by

$$\pi_M^* = \pi_{M1}^* + \pi_{M2}^* =$$

$$\begin{cases} \hat{\pi}_M & \text{if } 0 \le \delta < \delta^* \\ \frac{64\beta^4(2+2\delta+\delta^2)+16\beta^3\gamma\lambda(17+13\delta+4\delta^2)+\beta^2\gamma^2\lambda^2(161+82\delta+\delta^2)+2\beta\gamma^3\lambda^3(9+\delta)+\gamma^4\lambda^4}{64\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

where $\hat{\pi}_M$ is defined as $\hat{\pi}_M = \hat{\pi}_{M1} + \frac{2(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$. Finally,

$$\pi_R^* = \pi_{R1}^* + \pi_{R2}^* =$$

$$\begin{cases} \hat{\pi}_R & \text{if } 0 \le \delta < \delta^* \\ \frac{(8\beta^2+\beta\gamma(5-3\delta)\lambda-3\gamma^2\lambda^2)(8\beta^2+\beta\gamma\lambda(9+\delta)+\gamma^2\lambda^2)}{128\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)} + \frac{(32\beta^3(1+\delta)+8\beta^2\gamma\lambda(7+4\delta)+\beta\gamma^2\lambda^2(23-\delta)-\gamma^3\lambda^3)^2}{256\beta^2(\beta+\gamma\lambda)(8\beta^2+8\beta\gamma\lambda-\gamma^2\lambda^2)^2} & \text{if } \delta^* \le \delta < \delta^{**} \end{cases}$$

where $\hat{\pi}_R$ is given by $\hat{\pi}_R = \hat{\pi}_{R1} + \frac{(1+\delta)^2(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$. ■

Part III

Models with a Micro-Foundation

In the main paper, we analyze a core model with a reduced-form demand function. As we demonstrate below, our reduced-form demand function can be obtained from a setting in which consumers' decisions are explicitly modeled with utility functions. We provide two such examples.

III-(1) Let us consider a model in which a unit measure of consumers shop for a product. The consumer's expected utility from buying the product in the first period is $v + x - p_1$, where p_1 is the product's first-period price and $v + x$ is the product's consumption value, which consists of two parts: observable (v) and experiential (x). Each consumer learns her v before making a purchase; v is assumed to be uniformly distributed on the interval $[0,1]$ in the consumer population. The firm knows that its product's experiential value x is $\delta > 0$. This is reasonable because, in practice, firms usually spend significant time and money to conduct market research to learn consumers' preferences about their products. By contrast, consumers ex-ante do not know that the true value of x, but they learn it after consuming the product. More specifically, in the first period, consumers believe that $x = \delta$ with probability q, and $x = -\delta$ with probability $(1 - q)$. Intuitively, consumers often face uncertainty about products that they have not used before, especially new products in the market. Hence, one can think of x as the experiential component of the product value, which is revealed to consumers only after they consume the product. For simplicity and to avoid any ex ante bias against or for product purchase, we let $q = \frac{1}{2}$ such that the consumer's expected value from the experiential attribute (x) is zero. Each consumer has an outside option that gives zero utility.

In the first period, a consumer will buy the product only if the consumer's expected utility from buying the product exceeds zero, i.e., $v + \frac{-\delta+\delta}{2} - p_1 \geq 0$. Since v is uniformly distributed on $[0,1]$, one can readily show that the product's first-period demand is given by $D_1 = 1 - p_1$. In the second period, all consumers learn about the product's true experiential value $x = \delta$. Intuitively, consumers tend to share a lot of information on social media, review web-sites, etc. So, it is reasonable that over time a consumer will learn x even if she did not purchase the product before. A fraction $\lambda \in (0,1)$ of consumers have fairness concerns when the firm's price increases. Such consumers' utility from purchasing the product in the second period is $v + \delta - p_2 - \gamma \max\{p_2 - p_1; 0\}$, where $-\gamma \max\{p_2 - p_1; 0\}$ represents the consumer's disutility due to fairness concerns when the firm increases its price. The remaining fraction $(1 - \lambda)$ of consumers do not have any fairness concerns, and such a consumer's utility from purchasing the product is $v + \delta - p_2$. Since v is uniformly distributed on $[0,1]$, the second-period demand for the product is as follows: $D_2 = \lambda(1 + \delta - p_2 - \gamma \max\{p_2 - p_1; 0\}) + (1 - \lambda)(1 + \delta - p_2)$. Comparing D_1 and D_2 in the above equations, one can see that they coincide with those in our main model, provided that $\beta = 1$. Thus, our original reduced-form demand functions in the main model can be rationalized in the above utility-based model with micro-foundations for consumer choice.

There are also other ways to rationalize the linear demand functions in our core model. For example, we can have a model in which the consumers' valuations for quality are uniformly distributed on the interval $[0, 1]$ in the first period whereas, in the second period, the upper bound of that distribution increases by some amount (e.g., due to positive reviews, which may increase the product's perceived value and also encourage more consumers to consider the product).

III-(2) Our reduced-form general model can also be obtained from a setting where each consumer can buy multiple units instead of just one unit. To show this, we adapt the modeling framework from Singh and Vives (1984). Specifically, there is a unit measure of consumers. In the first period, a consumer will obtain a utility $U_1(q) = \alpha q - \frac{1}{2}q^2 - p_1 q$ from purchasing q units of the product, where the parameter α captures the marginal benefit that the consumer receives from each unit of consumption, the quadratic disutility term captures the consumer's satiation as the consumption increases, and p_1 is the first-period retail price. Without loss of generality, we normalize $\alpha = 1$. Note that U_1 is concave in q to capture the reality that consumers derive declining marginal utility with every additional unit of the product (Coombs and Avrunin

1977, Horowitz et al. 2007). Solving the first-order condition, one can readily show that the first-period demand for the firm's product will be $D_1 = 1 - p_1$.

In the second period, the consumer's marginal benefit from the product increases from α to $\alpha + \delta$. For example, past research has shown that consumers' satisfaction with a product or service can increase their future willingness to pay for it (Homburg et al. 2005). As we discussed in the paper, increased willingness to pay can also be driven by positive word-of-mouth or reviews about the product, improved brand reputation and image, celebrity or influencer endorsements, etc. Consistent with our core model, there are two types of consumers. Fraction $\lambda \in (0,1)$ of consumers have fairness concerns when the firm's price increases in the second period. Their utility from buying q units of the firm's product is $U_2^F(q) = (1+\delta)q - \frac{1}{2}q^2 - p_2 q - \gamma \max\{p_2 - p_1; 0\}\, q$. Note that consumers' total disutility due to fairness concerns increases with q. This is reasonable because a consumer who buys very few units is likely to get less disutility due to fairness concerns than a consumer who buys many units at the increased price. The remaining fraction $1 - \lambda$ of consumers do not have fairness concerns, and their utility from buying q units of the product is $U_2^{NF}(q) = (1+\delta)q - \frac{1}{2}q^2 - p_2 q$. Solving the first-order conditions, we find that consumers with and without fairness concerns will buy $q_2^{*F} = 1 + \delta - p_2 - \gamma \max\{p_2 - p_1; 0\}$ and $q_2^{*NF} = 1 + \delta - p_2$ units of the product, respectively. Hence, the aggregate second-period demand for the firm's product will be $D_2 = \lambda(1 + \delta - p_2 - \gamma \max\{p_2 - p_1; 0\}) + (1 - \lambda)(1 + \delta - p_2)$. Finally, it is easy to see that D_1 and D_2 in the main paper reduce to demand functions in this specific model with a micro-foundation if we normalize $\beta = 1$ in our main model.

Part IV

Analysis of Multiplicative Demand Model

Note that in our core model, the second-period demand function has an upward shift in intercept. In this part of the Online Appendix, we analyze an alternative *multiplicative* demand growth model. More specifically, the second-period demand is $D_2^{NF} = (1+\delta)(1-\beta p_2)$ in a market without consumer fairness concerns. Accordingly, in the presence of consumer fairness concerns, the second-period demand function is given by $D_2 = \lambda(1+\delta)(1-\beta p_2 - \gamma \max\{p_2 - p_1, 0\}) + (1-\lambda)(1+\delta)(1-\beta p_2)$. Other aspects of the model are the same as those in our core model. We adopt the same notations here.

IV-I. Benchmark Case Analysis (No fairness concerns case)

In this section, let us start with a benchmark model without fairness concerns. We use a superscript "NF" on the demand and equilibrium variables to indicate this "no fairness concern" benchmark. In the first period, the aggregate market demand is given by $D_1^{NF} = 1 - \beta p_1$, and the market is expected to grow in the second period. To capture the market growth, we assume that the second-period market demand will become $D_2^{NF} = (1+\delta)(1-\beta p_2)$, where δ represents the demand increase in the second period. One can solve the equilibrium outcome using backward induction. We first show the following lemma.

LEMMA IV-1. *In equilibrium, the manufacturer's first-period and second-period wholesale prices are given by* $w_1^{*NF} = w_2^{*NF} = \frac{1}{2\beta}$, *and the retailer's prices are* $p_1^{*NF} = p_2^{*NF} = \frac{3}{4\beta}$. *The equilibrium unit sales in the two periods are* $D_1^{*NF} = \frac{1}{4}$ *and* $D_2^{*NF} = \frac{1+\delta}{4}$, *respectively. The manufacturer's and the retailer's equilibrium profits are* $\pi_M^{*NF} = \frac{2+\delta}{8\beta}$ *and* $\pi_R^{*NF} = \frac{2+\delta}{16\beta}$, *respectively.*

PROOF OF LEMMA IV-1. Lemma IV-1 shows the manufacturer's and the retailer's first-period and second-period wholesale and retail prices in a decentralized channel with no consumers having fairness concerns. The results of Lemma IV-1 can be easily solved using backward induction. We will first find the second-period subgame equilibrium outcome and then solve for the first-period equilibrium. In the second period, for a given second-period wholesale price w_2, the retailer chooses p_2 to maximize its second-period profit $\pi_{R2}^{NF} = (p_2 - w_2)D_2^{NF}$, where $D_2^{NF} = (1+\delta)(1-\beta p_2)$. One can show that the retailer's optimal second-period price is interior at $p_2^{*NF}(w_2) = \frac{1+w_2\beta}{2\beta}$. Then the second-period market demand has become $D_2^{*NF} = \frac{(1+\delta)(1-\beta w_2)}{2}$. Next, the manufacturer chooses w_2 to maximize its second-period profit $\pi_{M2}^{NF} = w_2\, D_2^{*NF} = w_2 \frac{(1+\delta)(1-\beta w_2)}{2}$. One can show that the manufacturer's optimal second-period wholesale price is $w_2^{*NF} = \frac{1}{2\beta}$. Plugging in w_2^{*NF}, we can get the retailer's second-period retail price: $p_2^{*NF} = \frac{3}{4\beta}$. The retailer's and the manufacturer's second-period profits are given by: $\pi_{R2}^{*NF} = \frac{1+\delta}{16\beta}$, $\pi_{M2}^{*NF} = \frac{1+\delta}{8\beta}$.

In the first period, for a given first-period wholesale price w_1, the retailer chooses p_1 to maximize its total profit $\pi_R^{NF} = (p_1 - w_1)\, D_1^{NF} + \pi_{R2}^{*NF}$, where $D_1^{NF} = 1 - \beta p_1$. One can show that the retailer's optimal first-period price is $p_1^{*NF} = \frac{1+w_1\beta}{2\beta}$. Thus, the first-period market demand has become $D_1^{*NF} = \frac{1-\beta w_2}{2}$. Then the manufacturer chooses w_1 to maximize its total profit $\pi_M^{NF} = w_1 \cdot D_1^{NF} + \pi_{M2}^{*NF}$. The manufacturer's optimal first-period wholesale price is $w_1^{*NF} = \frac{1}{2\beta}$. Plugging in w_1^{*NF}, we can get the retailer's retail price $p_1^{*NF} = \frac{3}{4\beta}$. In equilibrium, the manufacturer's and the retailer's equilibrium profits are $\pi_M^{*NF} = \frac{2+\delta}{8\beta}$ and $\pi_R^{*NF} = \frac{2+\delta}{16\beta}$, respectively. ■

IV-II. Benchmark Case Analysis (A Centralized Channel)

In this section, we analyze another benchmark where the manufacturer sells its product directly to the consumers (i.e., a centralized channel) in a market with a positive fraction of consumers having fairness concerns. We use a tilde on a variable to indicate the current case of a centralized channel; other aspects of the model are the same as before. The first-period and second-period demand functions are same as what we have specified in the alternative model: $\widetilde{D}_1 = 1 - \beta\tilde{p}_1$ and $\widetilde{D}_2 = \lambda(1+\delta)(1-\beta\tilde{p}_2 - \gamma\max\{\tilde{p}_2 - \tilde{p}_1, 0\}) + (1-\lambda)(1+\delta)(1-\beta\tilde{p}_2)$. The centralized manufacturer's total profit can be expressed as $\tilde{\pi} = \widetilde{D}_1\,\tilde{p}_1 + \widetilde{D}_2\,\tilde{p}_2$. This game can be solved by backward induction.

LEMMA IV-2. *In a market with consumer fairness concerns, the centralized manufacturer's equilibrium first-period and second-period prices are given by* $\tilde{p}_1^* = \tilde{p}_2^* = \frac{1}{2\beta}$, and the manufacturer's equilibrium profits is $\tilde{\pi}_M^{*NF} = \frac{2+\delta}{4\beta}$.

PROOF OF LEMMA IV-2. Lemma IV-2 shows the centralized manufacturer's equilibrium first-period and second-period retail prices in a market with consumer fairness concerns. We solve the game by backward induction. We will first find the second-period subgame equilibrium outcome and then solve for the first-period equilibrium. In the second period, for a given first-period price $\tilde{p}_1$, the manufacturer chooses its second-period price $\tilde{p}_2^*$ to maximize its second-period profit, $\tilde{\pi}_2 = \widetilde{D}_2\, \tilde{p}_2$. The second-period demand function is as follows:

$$\widetilde{D}_2 = \begin{cases} (1+\delta)(1-\beta p_2) & \text{if } 0 \le p_2 \le p_1 \\ \lambda(1+\delta)(1-\beta p_2 - \gamma(p_2 - p_1)) + (1-\lambda)(1+\delta)(1-\beta p_2) & \text{if } p_1 \le p_2 \le \frac{1+\gamma p_1}{\beta+\gamma} \\ (1-\lambda)(1+\delta)(1-\beta p_2) & \text{if } \frac{1+\gamma p_1}{\beta+\gamma} \le p_2 < \frac{1}{\beta} \\ 0 & \text{if } p_2 \ge \frac{1}{\beta} \end{cases}$$

Note that the manufacturer's profit function is continuous and piecewise concave on each interval. We first obtain the optimal prices within each of the following three intervals $I_1 \equiv [0, \tilde{p}_1]$, $I_2 \equiv [\tilde{p}_1, \frac{1+\gamma p_1}{\beta+\gamma}]$, and $I_3 \equiv [\frac{1+\gamma p_1}{\beta+\gamma}, \frac{1}{\beta})$; among the three, the price that yields the highest profit is the manufacturer's optimal second-period retail price. Let $\tilde{p}_2^{I_i}$ denote the manufacturer's optimal price within the interval I_i.

- Suppose that $\tilde{p}_2 \in I_1$, if $0 < \tilde{p}_1 \le \frac{1}{2\beta}$, then one can show that the optimal price is $\tilde{p}_2^{I_1} = \tilde{p}_1$; if $\frac{1}{2\beta} \le \tilde{p}_1$, then one can show that the optimal price is $\tilde{p}_2^{I_1} = \frac{1}{2\beta}$.
- Suppose that $\tilde{p}_2 \in I_2$, if $0 < \tilde{p}_1 \le \frac{1}{2\beta+\gamma\lambda}$, the optimal price is $\tilde{p}_2^{I_2} = \frac{1+\gamma\lambda\ \tilde{p}_1}{2(\beta+\gamma\lambda)}$; if $\frac{1}{2\beta+\gamma\lambda} < \tilde{p}_1 < \frac{1}{\beta}$, the optimal price is $\tilde{p}_2^{I_1} = \tilde{p}_1$.

- Suppose that $\tilde{p}_2 \in I_3$, If $0 < \tilde{p}_1 < \frac{1}{\beta}$, the optimal price is $\tilde{p}_2^{I_3} = \frac{1+\gamma\tilde{p}_1}{\beta+\gamma}$. Comparing the profits corresponding to prices $\tilde{p}_2^{I_1}$, $\tilde{p}_2^{I_2}$ and $\tilde{p}_2^{I_3}$, one can show that the manufacturer's optimal second-period price is as follows.

$$\tilde{p}_2^* = \begin{cases} \frac{1+\lambda\gamma\,\tilde{p}_1}{2(\beta+\gamma\lambda)} & \text{if } 0 < \tilde{p}_1 \leq \frac{1}{2\beta+\gamma\lambda} \\ \tilde{p}_1 & \text{if } \frac{1}{2\beta+\gamma\lambda} \leq \tilde{p}_1 \leq \frac{1}{2\beta} \\ \frac{1}{2\beta} & \text{if } \frac{1}{2\beta} \leq \tilde{p}_1 < \infty \end{cases}.$$

Using $\tilde{p}_2^*$, we can obtain the manufacturer's second-period subgame equilibrium profits, where $\tilde{\pi}_2^* = \widetilde{D}_2\,\tilde{p}_2$.

$$\tilde{\pi}_2^* = \begin{cases} \frac{(1+\delta)(1+\tilde{p}_1\,\gamma\lambda)^2}{4(\beta+\gamma\lambda)} & \text{if } \tilde{p}_1 \in [0, \frac{1}{2\beta+\gamma\lambda}] \\ \tilde{p}_1(1+\delta)(1-\beta\tilde{p}_1) & \text{if } \tilde{p}_1 \in [\frac{1}{2\beta+\gamma\lambda}, \frac{1}{2\beta}] \\ \frac{1+\delta}{4\beta} & \text{if } \tilde{p}_1 \in [\frac{1}{2\beta}, \infty) \end{cases}$$

In the first period, the manufacturer chooses its first-period price $\tilde{p}_1$ to maximize its profit $\tilde{\pi} = \widetilde{D}_1\tilde{p}_1 + \tilde{\pi}_2^*$, where

$$\widetilde{D}_1 = \begin{cases} 1-\beta\tilde{p}_1 & \text{if } 0 < \tilde{p}_1 < \frac{1}{\beta} \\ 0 & \text{if } \tilde{p}_1 \geq \frac{1}{\beta} \end{cases}.$$

Denote the retailer's optimal first-period price within the interval K_i by $\tilde{p}_1^{K_i}$, where $K_1 \equiv [0, \frac{1}{2\beta+\gamma\lambda}]$, $K_2 \equiv [\frac{1}{2\beta+\gamma\lambda}, \frac{1}{2\beta}]$, and $K_3 \equiv [\frac{1}{2\beta}, \infty)$. To find the optimal first-period price $\tilde{p}_1^*$, we first find locally optimal prices in each of the intervals K_i. We denote these local optima by $\tilde{p}_1^{K_i}$ for $i = 1,2,3$. Then we compare the manufacturer's profit corresponding to each $\tilde{p}_1^{K_i}$. The price corresponding to the highest profit will be the manufacturer's optimal (subgame equilibrium) price. The analysis is similar to the one in our main model. We find that the centralized manufacturer's equilibrium first-period price is given by $\tilde{p}_1^* = \frac{1}{2\beta}$. Plugging this into the expression for $\tilde{p}_2^*$, we find that on the equilibrium path $\tilde{p}_2^* = \frac{1}{2\beta}$. Under these prices, the manufacturer's profit is $\tilde{\pi}_M^{*NF} = \frac{2+\delta}{4\beta}$.■

IV-III. Analysis and Results

We now return to analyze our alternative multiplicative demand model where the manufacturer sells its product through an independent retailer in a market that has a segment ($\lambda > 0$) of consumers having fairness concerns. More specifically, in a market with consumer fairness concerns, the second-period demand function is given by: $D_2 = \lambda(1+\delta)(1-\beta p_2 - \gamma \max\{p_2 - p_1, 0\}) + (1-\lambda)(1+\delta)(1-\beta p_2)$. We will examine the impact of consumer fairness concerns on the manufacturer's and the retailer's pricing decisions and the firms' profits. Proposition IV-1 identifies an important strategic link between the two periods that is not present in the benchmark model where consumers do not have fairness concerns, which is qualitatively the same as Proposition 1 in the main paper.

PROPOSITION IV-1. *If* $p_1 < \hat{P}$, *then* $\frac{\partial w_2^*(p_1)}{\partial p_1} > 0$, *where* $\hat{P} \equiv \frac{6\beta + 2\gamma\lambda + \sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{4\beta(2\beta+\gamma\lambda)}$. *In words, a higher* (*lower*) *first-period retail price will lead to a higher* (*lower*) *second-period subgame equilibrium wholesale price, provided that the first-period retail price is not too high.*

PROOF OF PROPOSITION IV-1. The manufacturer's subgame wholesale price in the second period are given by:

$$w_2^* = \begin{cases} \frac{1+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 < p_1 \leq \frac{3}{4\beta+\gamma\lambda} \\ \frac{2p_1\beta - 1 + p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \frac{3}{4\beta+\gamma\lambda} \leq p_1 \leq \hat{P} \\ \frac{1}{2\beta} & \text{if } p_1 \geq \hat{P} \end{cases}$$

where $\hat{P} = \frac{6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{4\beta(2\beta+\gamma\lambda)}$ satisfies $\pi_{M2}^*\big|_{w_2 = \frac{2\hat{P}\beta - 1 + \hat{P}\gamma\lambda}{\beta+\gamma\lambda}} = \pi_{M2}^*\big|_{w_2 = \frac{1}{2\beta}}$, i.e., $\frac{(1+\delta)(1-\hat{P}\beta)(2\hat{P}\beta - 1 + \hat{P}\gamma\lambda)}{\beta+\gamma\lambda} = \frac{1+\delta}{8\beta}$. Taking the derivative with respect to p_1, one can easily show that in each region of $p_1 \leq \hat{P}$, w_2^* is always increasing in p_1. ■

Lemma IV-3 shows the manufacturer's and the retailer's full-equilibrium pricing strategies.

LEMMA IV-3. *In a market with consumer fairness concerns, the equilibrium wholesale and retail prices are given by*:

$$w_1^* = \begin{cases} \frac{1}{8}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2)}\right), & \text{if } 0 \le \delta \le \delta_1 \\ \frac{2\beta-\gamma\lambda}{\beta(4\beta+\gamma\lambda)} + \frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)} & \text{if } \delta_1 < \delta \le \delta_2, \\ \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)} & \text{if } \delta_2 < \delta \end{cases}$$

$w_2^* = \frac{2}{4\beta+\gamma\lambda}$, $p_1^* = \frac{3}{4\beta+\gamma\lambda}$, $p_2^* = \frac{3}{4\beta+\gamma\lambda}$, *where,* $\delta_1 \equiv \frac{\gamma\lambda\,(24\,\beta^2+22\beta\gamma\lambda+7\gamma^2\lambda^2)}{(8\beta-\gamma\lambda)(2\beta+\gamma\lambda)^2} + \frac{4(\beta+\gamma\lambda)(4\beta+\gamma\lambda)\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{(8\beta-\gamma\lambda)(2\beta+\gamma\lambda)^2}$ *and* $\delta_2 \equiv \frac{16\beta^2+24\beta\gamma\lambda+17\gamma^2\lambda^2}{8\beta\gamma\lambda-\gamma^2\lambda^2}$.

PROOF OF LEMMA IV-3. We solve the game by backward induction. That is, we will first find the second-period subgame equilibrium outcome. Then, we will solve for the first-period equilibrium.

Derivation of the second-period subgame perfect equilibrium.

Given the first-period retail price p_1, in the second period, the manufacturer chooses its second-period wholesale price w_2, followed by the retailer choosing its second-period retail price p_2. More specifically, the retailer chooses p_2 to maximize its second-period profit, $\pi_{R2} = D_2(p_2 - w_2)$, where D_2 is as follows:

$$D_2 = \begin{cases} (1+\delta)(1-\beta p_2) & \text{if } 0 \le p_2 \le p_1 \\ \lambda(1+\delta)(1-\beta p_2 - \gamma(p_2-p_1)) + (1-\lambda)(1+\delta)(1-\beta p_2) & \text{if } p_1 \le p_2 \le \frac{1+\gamma p_1}{\beta+\gamma} \\ (1-\lambda)(1+\delta)(1-\beta p_2) & \text{if } \frac{1+\gamma p_1}{\beta+\gamma} \le p_2 < \frac{1}{\beta} \\ 0 & \text{if } p_2 \ge \frac{1}{\beta} \end{cases}$$

Let p_2^* denote the retailer's optimal price, i.e., $p_2^* \equiv \underset{p_2}{\operatorname{argmax}}\, \pi_{R2}$. Note that the retailer's profit function is continuous. We will first obtain the optimal prices within each of the following three intervals $I_1 \equiv [0, p_1]$, $I_2 \equiv [p_1, \frac{1+\gamma p_1}{\beta+\gamma}]$, and $I_3 \equiv [\frac{1+\gamma p_1}{\beta+\gamma}, \frac{1}{\beta})$; among the three, the price that yields the highest profit is the retailer's optimal second-period retail price. Let $p_2^{I_i}$ denote the retailer's optimal price within the interval I_i. Note that π_{R2} is piecewise concave on each interval I_i, $i = 1,2,3$. Furthermore, the interval I_i is compact.

Hence, the local maximizer in the interval I_i is either the interior solution (satisfying the first-order condition), or one of the bounds of the interval I_i (i.e., a corner solution). In either case, the analysis is rather straightforward, so we will not provide too much detail.

- Suppose that $p_2 \in I_1$. The retailer's profit function is given by $\pi_{R2} = (1+\delta)(1-\beta p_2)(p_2 - w_2)$. In this case, the second-period retail price is smaller or equal than the first-period retail price, so consumers will not have any fairness concerns. There are three subcases to analyze:

First, if $0 < p_1 \leq \frac{1}{2\beta}$, then π_{R2} is increasing in p_2 on the interval I_1. Hence, the optimal price within the interval I_1 is $p_2^{I_1} = p_1$(i.e., the corner solution);

Second, if $\frac{1}{2\beta} < p_1 \leq \frac{1}{\beta}$, then one can show that the optimal price is $p_2^{I_1} = \frac{1+w_2\beta}{2\beta}$ (i.e., the interior solution) when $0 < w_2 \leq \frac{2p_1\beta-1}{\beta}$, and $p_2^{I_1} = p_1$ when $\frac{2p_1\beta-1}{\beta} \leq w_2 \leq \frac{1}{\beta}$.

Third, if $p_1 > \frac{1}{\beta}$, then one can show that the optimal price is $p_2^{I_1} = \frac{1+w_2\beta}{2\beta}$ when $0 < w_2 \leq \frac{1}{\beta}$.

- Suppose that $p_2 \in I_2$. The retailer's profit function is given by $\pi_{R2} = \lambda(1+\delta)(1-\beta p_2 - \gamma(p_2 - p_1))\,(p_2 - w_2) + (1-\lambda)(1+\delta)(1-\beta p_2)(p_2 - w_2)$. In this case, the second-period retail price exceeds the first-period retail price (i.e., $p_2 > p_1$), and hence, the retailer will lose some extra sales because of consumers' fairness concerns.

If $0 < p_1 \leq \frac{1}{\beta}$, the optimal price is $p_2^{I_2} = p_1$ (i.e., the corner solution) when $0 < w_2 \leq max\{0, \frac{2p_1\beta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}\}$; the optimal price is $p_2^{I_2} = \frac{1+w_2\beta+(p_1+w_2)\gamma\,\lambda}{2(\beta+\gamma\lambda)}$ (i.e., the interior solution) when $\max\{0, \frac{2p_1\beta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}\} \leq w_2 \leq \frac{\beta-\gamma+2p_1\beta\gamma+2\gamma\lambda-p_1\beta\gamma\lambda+p_1\gamma^2\lambda}{\beta^2+\beta\gamma+\beta\gamma\lambda+\gamma^2\lambda}$; the optimal price is $p_2^{I_2} = \frac{1+\gamma p_1}{\beta+\gamma}$ (i.e., the corner solution) when $w_2 \geq \frac{\beta-\gamma+2p_1\beta\gamma+2\gamma\lambda-p_1\beta\gamma\lambda+p_1\gamma^2\lambda}{\beta^2+\beta\gamma+\beta\gamma\lambda+\gamma^2\lambda}$.

- Suppose that $p_2 \in I_3$. The retailer's profit function is given by $\pi_{R2} = (1-\lambda)(1+\delta)(1-\beta p_2)(p_2 - w_2)$. In this case, p_2 is so high that the segment of consumers with fairness concerns will not buy the product. One can show that if $0 < p_1 < \frac{1}{\beta}$, then the optimal price is $p_2^{I_3} = \frac{1+\gamma p_1}{\beta+\gamma}$ when $0 < w_2 < \frac{\beta-\gamma+2p_1\beta\gamma}{\beta^2+\beta\gamma}$; the optimal price is $p_2^{I_3} = \frac{1+w2\beta}{2\beta}$ when $\frac{\beta-\gamma+2p_1\beta\gamma}{\beta^2+\beta\gamma} \leq w_2 < \frac{1}{\beta}$.

Comparing the profits corresponding to prices $p_2^{I_1}$, $p_2^{I_2}$ and $p_2^{I_3}$, one can show that the retailer's optimal second-period price is as follows.

$$p_2^* = \begin{cases} \frac{1+w_2\beta}{2\beta} & \text{if } 0 \leq w_2 \leq \omega_1 \\ p_1 & \text{if } \omega_1 \leq w_2 \leq \omega_2 \\ \frac{1+w_2\beta+(p_1+w_2)\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } \omega_2 \leq w_2 \leq \omega_3 \\ \frac{1+w_2\beta}{2\beta} & \text{if } \omega_3 < w_2 \leq \frac{1}{\beta} \end{cases},$$

where $\omega_1 = \max\{0, \frac{2p_1\beta-1}{\beta}\}$, $\omega_2 = \max\{0, \frac{2p_1\beta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}\}$ and $\omega_3 = \frac{1+p_1\gamma+\delta}{\beta+\gamma+\beta\delta} - \frac{\gamma(1-p_1\beta)}{\beta+\gamma}\sqrt{\frac{1-\lambda}{\beta(\beta+\gamma\lambda)}}$ satisfies $\pi_{R2}^*|_{p_2^*=\frac{1+\omega_3\beta+(p_1+\omega_3)\gamma\lambda}{2(\beta+\gamma\lambda)}} = \pi_{R2}^*|_{p_2^*=\frac{1+\omega_3\beta}{2\beta}}$, i.e., $\frac{(1+\delta)(1+p_1\gamma\lambda-\omega_3(\beta+\gamma\lambda))^2}{4(\beta+\gamma\lambda)} = \frac{(1-\lambda)(1+\delta)(1-\omega_3\beta)^2}{4\beta}$. The retailer's second-period profits is given by

$$\pi_{R2}^* = \begin{cases} \frac{(1+\delta)(1-w_2\beta)^2}{4\beta} & \text{if } 0 \leq w_2 \leq \omega_1 \\ (1+\delta)(p_1-w_2)(1-\beta p_1) & \text{if } \omega_1 \leq w_2 \leq \omega_2 \\ \frac{(1+\delta)(1+p_1\gamma\lambda-w_2(\beta+\gamma\lambda))^2}{4(\beta+\gamma\lambda)} & \text{if } \omega_2 \leq w_2 \leq \omega_3 \\ \frac{(1-\lambda)(1+\delta)(1-w_2\beta)^2}{4\beta} & \text{if } \omega_3 < w_2 \leq \frac{1}{\beta} \end{cases}.$$

Next, we will find the manufacturer's optimal second-period wholesale price. The manufacturer correctly anticipates the retailer's best response p_2^* to w_2 and chooses its optimal wholesale price to maximize its profit, $\pi_{M2} = D_2\, w_2$. Let w_2^* denote the manufacturer's optimal wholesale price, i.e., $w_2^* \equiv \underset{w_2}{\operatorname{argmax}}\, \pi_{M2}$. We will first obtain the manufacturer's optimal wholesale prices within each of the following four intervals: $M_1 \equiv [0, \omega_1]$, $M_2 \equiv [\omega_1, \omega_2]$, $M_3 \equiv [\omega_2, \omega_3]$, and $M_4 \equiv (\omega_3, \frac{1}{\beta}]$. Among the four, the price that yields the highest profit is the manufacturer's optimal second-period wholesale price. Let $w_2^{M_i}$ denote the manufacturer's optimal second-period wholesale price within the interval M_i, $i = 1,2,3,4$. Note that the manufacturer's profit function π_{M2} is piecewise concave on each interval M_i, and is discontinuous at ω_3: it jumps down at ω_3 since the retailer switches from serving both segments to serving only the segment without fairness concerns, so the manufacturer will not charge a w_2^* larger than ω_3, because a tiny drop from ω_3 will give the manufacturer a profit increase.

- Suppose that $w_2 \in M_1$ (where $M_1 \equiv [0, \omega_1]$), the manufacturer's second-period profit function is given by $\pi_{M2} = \frac{w_2(1+\delta)(1-w_2\beta)}{2}$. One can show that if $\frac{3}{4\beta} < p_1 < \frac{1}{\beta}$, the solution is interior at $w_2^{M_1} = \frac{1}{2\beta}$, and if $0 < p_1 \leq \frac{3}{4\beta}$, then the solution is at the right corner $w_2^{M_1} = \frac{2p_1\beta-1}{\beta}$.
- Suppose that $w_2 \in M_2$ (where $M_2 \equiv [\omega_1, \omega_2]$, the manufacturer's second-period profit function is given by $\pi_{M2} = (1+\delta)(1-\beta p_1)w_2$. Clearly, $\frac{d\pi_{M2}}{dw_2} > 0$, π_{M2} is increasing in w_2. Hence, if M_2 is non-empty (i.e., $0 < p_1 < \frac{1}{\beta}$), then the optimal wholesale price in M_2 is at the right corner $w_2^{M_2} = \frac{2p_1\beta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}$.
- Suppose that $w_2 \in M_3$ (where $M_3 \equiv [\omega_2, \omega_3]$, the manufacturer's second-period profit function is given by $\pi_{M2} = \frac{w_2(1+\delta)(1-w_2\beta+(p_1-w_2)\gamma\lambda)}{2}$. One can show that if $0 < p_1 \leq \frac{3}{4\beta+\gamma\lambda}$, then the solution is interior at $w_2^{M_3} = \frac{1+p_1\gamma\lambda}{2(\beta+\gamma\lambda)}$, and if $\frac{3}{4\beta+\gamma\lambda} \leq p_1 < \frac{1}{\beta}$, then the solution is at the left corner at $w_2^{M_3} = \frac{2p_1\beta-1+p_1\gamma\lambda}{\beta+\gamma\lambda}$.
- Suppose that $w_2 \in M_4$ (where $M_4 \equiv (\omega_3, \frac{1}{\beta}]$), the manufacturer's second-period profit function is given by $\pi_{M2} = \frac{w_2(1-\lambda)(1+\delta)(1-w_2\beta)}{2}$. Then π_{M2} is decreasing in w_2 on the interval M_4. One can show that the solution is at the left corner $w_2^{M4} = \omega_3$.

Comparing the manufacturer's profits corresponding to each wholesale price $w_2^{M_i}$, one can show that the manufacturer's optimal second-period price is given by

$$w_2^* = \begin{cases} \frac{1+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 < p_1 \leq \frac{3}{4\beta+\gamma\lambda} \\ \frac{2p_1\beta-1+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \frac{3}{4\beta+\gamma\lambda} \leq p_1 \leq \hat{P} \\ \frac{1}{2\beta} & \text{if } p_1 \geq \hat{P} \end{cases}$$

where $\hat{P}=\frac{6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{4\beta(2\beta+\gamma\lambda)}$ satisfies $\pi_{M2}^{*}|_{w_2=\frac{2\hat{P}\beta-1+\hat{P}\gamma\lambda}{\beta+\gamma\lambda}}=\pi_{M2}^{*}|_{w_2=\frac{1}{2\beta}}$, i.e., $\frac{(1+\delta)(1-\hat{P}\beta)(2\hat{P}\beta-1+\hat{P}\gamma\lambda)}{\beta+\gamma\lambda}=\frac{1+\delta}{8\beta}$. Note that the manufacturer's wholesale price jumps down at $\hat{P}$ in order to induce the retailer to sufficiently reduce its second-period price to become perceived as fair. Using w_2^* and p_2^*, we can obtain the manufacturer's and the retailer's second-period subgame equilibrium profits. Note that the manufacturer's profit is continuous at $\hat{P}$. The retailer's profit jumps up at $\hat{P}$, i.e., $\pi_{R2}^{*}|_{p_1=\hat{P},w_2=\frac{2\hat{P}\beta-1+\hat{P}\gamma\lambda}{\beta+\gamma\lambda}}<\pi_{R2}^{*}|_{p_1=\hat{P},w_2=\frac{1}{2\beta}}$.

$$\pi_{M2}^{*}=\begin{cases}\frac{(1+\delta)(1+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } p_1\in(0,\frac{3}{4\beta+\gamma\lambda}]\\ \frac{(1+\delta)(1-p_1\beta)(2p_1\beta-1+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1\in[\frac{3}{4\beta+\gamma\lambda},\hat{P}]\\ \frac{1+\delta}{8\beta} & \text{if } p_1\in[\hat{P},\infty)\end{cases},$$

$$\pi_{R2}^{*}=\begin{cases}\frac{(1+\delta)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } p_1\in(0,\frac{3}{4\beta+\gamma\lambda}]\\ \frac{(1+\delta)(1-\beta p_1)^2}{\beta+\gamma\lambda} & \text{if } p_1\in[\frac{3}{4\beta+\gamma\lambda},\hat{P})\\ \frac{1+\delta}{16\beta} & \text{if } p_1\in[\hat{P},\infty)\end{cases}.$$

Derivation of the first-period equilibrium.

In this section, we will solve for the first-period equilibrium. The manufacturer's and the retailer's second-period equilibrium profits are given by π_{M2}^{*} and π_{R2}^{*}, respectively. In the first period, given w_1, the retailer chooses its first-period price p_1^* to maximize its total profit $\pi_R=D_1\cdot(p_1-w_1)+\pi_{R2}^{*}$, where

$$D_1=\begin{cases}1-\beta p_1 & \text{if } 0<p_1<\frac{1}{\beta}\\ 0 & \text{if } \frac{1}{\beta}\le p_1\end{cases}$$

Let p_1^* denote the retailer's optimal price, i.e., $p_1^*\equiv\underset{p_1}{\operatorname{argmax}}\,\pi_R$. We will obtain the optimal first-period retail prices within each of the following three intervals $K_1\equiv(0,\frac{3}{4\beta+\gamma\lambda}]$, $K_2\equiv[\frac{3}{4\beta+\gamma\lambda},\hat{P}]$, and $K_3\equiv[\hat{P},\infty)$; among the three, the price that yields the highest total profit is the retailer's optimal first-period retail price. Denote the retailer's optimal first-period price within the interval K_i by $p_1^{K_i}$, $i=1,2,3$. Note that if $p_1\ge\frac{1}{\beta}$, then there will be no sales in the first period (i.e., $D_1=0$), so the retailer's total profit π_R

equals to its second-period profit π_{R2}^*. One can show that $\hat{P} < \frac{1}{\beta}$, thus $\frac{1}{\beta} \in K_3$. Also note that π_R is piecewise concave on each interval K_i. Hence, the local maximizer in the interval K_i is either in the interior of K_i (and hence satisfied the first-order condition) or it is at one of the endpoints of K_i. Since the analysis is rather straightforward, below we will directly provide the solution to the profit maximization problem within each K_i.

First, if $p_1 \in K_1$ (where $K_1 \equiv (0, \frac{3}{4\beta+\gamma\lambda}]$), the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1+\delta)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$. One can show that when $0 \le \delta \le \frac{4\beta-3\gamma\lambda}{\gamma\lambda}$, if $0 < w_1 \le \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}$, the solution is interior at $p_1^{K_1} = \frac{8\beta+8w_1\beta^2+9\gamma\lambda+8w_1\beta\gamma\lambda+\gamma\lambda\,\delta}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2-\gamma^2\lambda^2\delta}$, and if $\frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda} \le w_1 \le \frac{4+\delta}{4\beta+\gamma\lambda}$, the solution is at the right corner $p_1^{K_1} = \frac{3}{4\beta+\gamma\lambda}$. When $\delta > \frac{4\beta-3\gamma\lambda}{\gamma\lambda}$ and $0 < w_1 \le \frac{4+\delta}{4\beta+\gamma\lambda}$, the optimal solution is at the right corner $p_1^{K_1} = \frac{3}{4\beta+\gamma\lambda}$.

Second, if $p_1 \in K_2$ (where $K_2 \equiv [\frac{3}{4\beta+\gamma\lambda}, \hat{P}]$), the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{(1+\delta)(1-\beta p_1)^2}{\beta+\gamma\lambda}$. There are two subcases to analyze:

(1) When $0 \le \delta \le \frac{\gamma\lambda}{\beta}$: if $0 < w_1 \le \frac{4\beta+2\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \frac{3}{4\beta+\gamma\lambda}$, and if $\frac{4\beta+2\beta\delta-\gamma\lambda}{4\beta^2+\beta\gamma\lambda} \le w_1 \le \frac{2+\delta}{2\beta+\gamma\lambda} + \frac{\gamma\lambda-\beta\delta}{2\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}$, then the solution is interior at $p_1^{K_2} = \frac{\beta-w_1\beta^2+2\beta\delta-\gamma\lambda-w_1\beta\gamma\lambda}{2\beta(\beta\delta-\gamma\lambda)}$, and if $\frac{2+\delta}{2\beta+\gamma\lambda} + \frac{\gamma\lambda-\beta\delta}{2\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} < w_1 < \frac{4\beta+\beta\delta+\gamma\lambda}{2\beta(2\beta+\gamma\lambda)} + \frac{\gamma\lambda-\beta\delta}{4\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}$, then the solution is at the right corner $p_1^{K_2} = \hat{P}$.

(2) When $\delta > \frac{\gamma\lambda}{\beta}$: if $0 < w_1 \le \frac{4+\delta}{4\beta+\gamma\lambda}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \frac{3}{4\beta+\gamma\lambda}$.

Third, if $p_1 \in K_3$ (where $K_3 \equiv [\hat{P}, \infty)$), when $p_1 \le \frac{1}{\beta}$, the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{1+\delta}{16\beta}$. One can show that if $0 < w_1 \le \frac{1}{2\beta+\gamma\lambda} + \frac{\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{2\beta(2\beta+\gamma\lambda)}$, then the solution is at the left corner at $p_1^{K_3}=\hat{P}$, and if $\frac{1}{2\beta+\gamma\lambda}+\frac{\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{2\beta(2\beta+\gamma\lambda)}\leq w_1<\frac{1}{\beta}$, then the solution is at $p_1^{K_3}=\frac{1+w_1\beta}{2\beta}$. If $w_1\geq\frac{1}{\beta}$, $\pi_R=\pi_{R2}^*=\frac{1+\delta}{16\beta}$, then the solution is $p_1^{K_3}=\frac{1}{\beta}$.

Comparing the profits corresponding to each of the prices $p_1^{K_i}$ for $i=1,2,3$, one can show that, in different δ regions, the retailer's first-period optimal price $p_1^*(w_1)$ is as follows:

(1) When $0\leq\delta\leq\delta_1$:

$$p_1^*(w_1)=\begin{cases}\frac{8\beta+8w_1\beta^2+9\gamma\lambda+8w_1\beta\gamma\lambda+\gamma\lambda\,\delta}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2-\gamma^2\lambda^2\delta} & \text{if } 0<w_1\leq\frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\\ \frac{3}{4\beta+\gamma\lambda} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\leq w_1\leq\omega_A\\ \hat{P} & \text{if } \omega_A<w_1\leq\omega_B\\ \frac{1+w_1\beta}{2\beta} & \text{if } \omega_B\leq w_1<\frac{1}{\beta}\\ \frac{1}{\beta} & \text{if } w_1\geq\frac{1}{\beta},\end{cases}$$

where $\delta_1\equiv\frac{\gamma\lambda\,(24\,\beta^2+22\beta\gamma\lambda+7\gamma^2\lambda^2)}{(8\beta-\gamma\lambda)(2\beta+\gamma\lambda)^2}+4\frac{(\beta+\gamma\lambda)(4\beta+\gamma\lambda)\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{(8\beta-\gamma\lambda)(2\beta+\gamma\lambda)^2}$, $\omega_A=\frac{1}{8}(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}+\frac{32\beta^2-4\beta\gamma\lambda\,\delta-2\gamma^2\lambda^2(3+\delta)}{\beta\left(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2\right)})$, which satisfies $\pi_R^*|_{p_1^*=\frac{3}{4\beta+\gamma\lambda}}=\pi_R^*|_{p_1^*=\hat{P}}$, i.e., $\frac{(\beta+\gamma\lambda)(4-4\omega_A\beta+\delta-\omega_A\gamma\lambda)}{(4\beta+\gamma\lambda)^2}=\frac{1+\delta}{16\beta}+(\hat{P}-\omega_A)(1-\beta\hat{P})$, and $\omega_B\equiv\frac{1}{2\beta+\gamma\lambda}+\frac{1}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}}$ (i.e., the corner solution) above which $p_1^*=\frac{1+w_1\beta}{2\beta}$ will be the optimal solution. Note that p_1^* is not continuous at ω_A: it jumps up to $\hat{P}$. The manufacturer's profit jumps down at ω_A. Plugging in p_1^*, we can get the retailer's profit $\pi_R^*(w_1)$:

$$\pi_R^*(w_1)=\begin{cases}\frac{(\beta+\gamma\lambda)(5-4w_1\beta(2-w_1\beta)+\delta+w_1\gamma\lambda(1+\delta))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)} & \text{if } 0<w_1\leq\frac{4\beta-3\gamma\lambda-\gamma\lambda\delta}{8\beta^2+2\beta\gamma\lambda}\\ \frac{(\beta+\gamma\lambda)(4+\delta-4w_1\beta-w_1\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\delta}{8\beta^2+2\beta\gamma\lambda}\leq w_1\leq\omega_A\\ \frac{1+\delta}{16\beta}+(\frac{6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{4\beta(2\beta+\gamma\lambda)}-w_1)(1-\frac{6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{4(2\beta+\gamma\lambda)}) & \text{if } \omega_A<w_1\leq\omega_B\\ \frac{5+4w_1\beta(\beta w_1-2)+\delta}{16\beta} & \text{if } \omega_B\leq w_1<\frac{1}{\beta}\\ \frac{1+\delta}{16\beta} & \text{if } w_1\geq\frac{1}{\beta}\end{cases}$$

Plugging p_1^*, we can also obtain the manufacturer's total profit $\pi_M^*(w_1)$:

$$\pi_M^*(w_1)=\begin{cases}\frac{8(1+\delta)(\beta+\gamma\lambda)\left(\gamma\lambda+\beta(2+w_1\gamma\lambda)\right)^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta))^2}+\frac{w_1(\beta+\gamma\lambda)(8\beta(1-w_1\beta)-\gamma\lambda(1+\delta))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)} & \text{if } 0<w_1\le\frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\\ \frac{(2(1+\delta)+4w_1\beta+w_1\gamma\lambda)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\le w_1\le\omega_\mathrm{A}\\ \frac{1+\delta}{8\beta}+w_1\left(1-\frac{6\beta+2\gamma\lambda+\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{4(2\beta+\gamma\lambda)}\right) & \text{if } \omega_\mathrm{A}<w_1\le\omega_\mathrm{B}\\ \frac{1+\delta+4w_1\beta(1-w_1\beta)}{8\beta} & \text{if } \omega_\mathrm{B}\le w_1<\frac{1}{\beta}\\ \frac{1+\delta}{8\beta} & \text{if } w_1\ge\frac{1}{\beta}\end{cases}$$

(2) When $\delta_1<\delta\le\delta_2$:

$$p_1^*(w_1)=\begin{cases}\frac{8\beta+8w_1\beta^2+9\gamma\lambda+8w_1\beta\gamma\lambda+\gamma\lambda\,\delta}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2-\gamma^2\lambda^2\delta} & \text{if } 0<w_1\le\frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\\ \frac{3}{4\beta+\gamma\lambda} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\le w_1\le\omega_\mathrm{C}\\ \frac{1+w_1\beta}{2\beta} & \text{if } \omega_\mathrm{C}\le w_1<\frac{1}{\beta}\\ \frac{1}{\beta} & \text{if } w_1\ge\frac{1}{\beta},\end{cases}$$

where $\delta_2\equiv\frac{16\beta^2+24\beta\gamma\lambda+17\gamma^2\lambda^2}{8\beta\gamma\lambda-\gamma^2\lambda^2}$, $\omega_\mathrm{C}=\frac{2\beta-\gamma\lambda}{\beta(4\beta+\gamma\lambda)}+\frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)}$, which satisfies $\pi_R^*|_{p_1^*=\frac{3}{4\beta+\gamma\lambda}}=\pi_R^*|_{p_1^*=\frac{1+w_1\beta}{2\beta}}$, i.e., $\frac{(\beta+\gamma\lambda)(4-4\omega_\mathrm{C}\beta+\delta-\omega_\mathrm{C}\gamma\lambda)}{(4\beta+\gamma\lambda)^2}=\frac{5-4\omega_\mathrm{C}\beta(2-\omega_\mathrm{C}\beta)+\delta}{16\beta}$. Note that p_1^* is not continuous at ω_C: it jumps up to $\frac{1+w_1\beta}{2\beta}$. The manufacturer's profit jumps down at ω_C (i.e., $\pi_M^*|_{w_1^*=\omega_\mathrm{C},\,p_1^*=\frac{3}{4\beta+\gamma\lambda}}>\pi_M^*|_{w_1^*=\omega_\mathrm{C},p_1^*=\frac{1+w_1\beta}{2\beta}}$).

Plugging p_1^*, we can get the manufacturer's total profit $\pi_M^*(w_1)$:

$$\pi_M^*(w_1)=\begin{cases}\frac{8(1+\delta)(\beta+\gamma\lambda)\left(\gamma\lambda+\beta(2+w_1\gamma\lambda)\right)^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta))^2}+\frac{w_1(\beta+\gamma\lambda)(8\beta(1-w_1\beta)-\gamma\lambda(1+\delta))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)} & \text{if } 0<w_1\le\frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\\ \frac{(2(1+\delta)+4w_1\beta+w_1\gamma\lambda)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\le w_1\le\omega_\mathrm{C}\\ \frac{1+\delta+4w_1\beta(1-w_1\beta)}{8\beta} & \text{if } \omega_\mathrm{C}\le w_1<\frac{1}{\beta}\\ \frac{1+\delta}{8\beta} & \text{if } w_1\ge\frac{1}{\beta}\end{cases}$$

(3) When $\delta_2<\delta\le\frac{4\beta-3\gamma\lambda}{\gamma\lambda}$:

$$p_1^*(w_1)=\begin{cases}\frac{8\beta+8w_1\beta^2+9\gamma\lambda+8w_1\beta\gamma\lambda+\gamma\lambda\,\delta}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2-\gamma^2\lambda^2\delta} & \text{if } 0<w_1\le\frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\\ \frac{3}{4\beta+\gamma\lambda} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}\le w_1\le\omega_\mathrm{D}\\ \frac{1}{\beta} & \text{if } w_1>\omega_\mathrm{D},\end{cases}$$

where $\omega_D = \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, which satisfies $\pi_R^*|_{p_1^*=\frac{3}{4\beta+\gamma\lambda}} = \pi_R^*|_{p_1^*=\frac{1}{\beta}}$, i.e., $\frac{(\beta+\gamma\lambda)(4-4\omega_D\beta+\delta-\omega_D\gamma\lambda)}{(4\beta+\gamma\lambda)^2} = \frac{1+\delta}{16\beta}$. Note that p_1^* is not continuous at ω_D: it jumps up to $\frac{1+w_1\beta}{2\beta}$. The manufacturer's profit jumps down at ω_D (i.e., $\pi_M^*|_{w_1^*=\omega_D,\, p_1^*=\frac{3}{4\beta+\gamma\lambda}} > \pi_M^*|_{w_1^*=\omega_D,\ p_1^*=\frac{1}{\beta}}$). Plugging p_1^*, we can get the manufacturer's total profit $\pi_M^*(w_1)$:

$$\pi_M^*(w_1) = \begin{cases} \frac{8(1+\delta)(\beta+\gamma\lambda)\left(\gamma\lambda+\beta(2+w_1\gamma\lambda)\right)^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta))^2} + \frac{w_1(\beta+\gamma\lambda)(8\beta(1-w_1\beta)-\gamma\lambda(1+\delta))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)} & \text{if } 0 < w_1 \le \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda} \\ \frac{(2(1+\delta)+4w_1\beta+w_1\gamma\lambda)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda} \le w_1 \le \omega_D \\ \frac{1+\delta}{8\beta} & \text{if } w_1 > \omega_D. \end{cases}$$

(4) When $\delta > \frac{4\beta-3\gamma\lambda}{\gamma\lambda}$:

$p_1^*(w_1) = \begin{cases} \frac{3}{4\beta+\gamma\lambda} & \text{if } 0 < w_1 \le \omega_D \\ \frac{1}{\beta} & \text{if } w_1 > \omega_D \end{cases}$, plugging p_1^*, we can get the manufacturer's total profit $\pi_M^*(w_1)$:

$$\pi_M^*(w_1) = \begin{cases} \frac{(2(1+\delta)+4w_1\beta+w_1\gamma\lambda)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} & \text{if } 0 < w_1 \le \omega_D \\ \frac{1+\delta}{8\beta} & \text{if } w_1 > \omega_D. \end{cases}$$

Finally, we proceed to finding the manufacturer's optimal price in the first period (i.e., w_1^*). For a given δ, define $\Omega_1 \equiv \{w_1 |\, p_1^* = \frac{8w_1\beta^2+\gamma\lambda(9+\delta)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)}\}$, $\Omega_2 \equiv \{w_1 |\, p_1^* = \frac{3}{4\beta+\gamma\lambda}\}$, $\Omega_3 \equiv \{w_1 |\, p_1^* = \hat{P}\}$, $\Omega_4 \equiv \{w_1 |\, p_1^* = \frac{1+w_1\beta}{2\beta}\}$, $\Omega_5 \equiv \{w_1 |\, p_1^* = \frac{1}{\beta}\}$. Note that any $w_1 \ge 0$ satisfies $w_1 \in \bigcup_{i=1}^{6} \Omega_i$. We will first find the manufacturer's optimal price $w_1^{\Omega_i}$ within each of the sets Ω_i. The price corresponding to the highest profit will be the manufacturer's equilibrium wholesale price in the first period.

First, suppose that $w_1 \in \Omega_1$ (where $\Omega_1 \equiv \{w_1 |\, p_1^* = \frac{8w_1\beta^2+\gamma\lambda(9+\delta)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)}\}$). Then, the manufacturer's profit is given by $\pi_M(w_1) = \frac{8(1+\delta)(\beta+\gamma\lambda)\left(\gamma\lambda+\beta(2+w_1\gamma\lambda)\right)^2}{(16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta))^2} + \frac{w_1(\beta+\gamma\lambda)(8\beta(1-w_1\beta)-\gamma\lambda(1+\delta))}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2(1+\delta)}$. For $\delta \le \frac{4\beta-3\gamma\lambda}{\gamma\lambda}$, π_M^* is increasing in w_1, i.e., $\frac{\partial\pi_M^*}{\partial w_1} > 0$. Thus, the maximizer of Ω_1 is the largest w_1 in Ω_1. One can show that if $\delta \le \frac{4\beta-3\gamma\lambda}{\gamma\lambda}$, the maximizer of π_M in Ω_1 is given by $w_1^{\Omega_1} = \frac{4\beta-3\gamma\lambda-\gamma\lambda\,\delta}{8\beta^2+2\beta\gamma\lambda}$.

Second, suppose that $w_1 \in \Omega_2$ (where $\Omega_2 \equiv \{w_1 | \, p_1^* = \frac{3}{4\beta+\gamma\lambda}\}$). Then, the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{(2(1+\delta)+4w_1\beta+w_1\gamma\lambda)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$, π_M^* is increasing in w_1, i.e., $\frac{\partial \pi_M^*}{\partial w_1} > 0$. Thus, the maximizer of Ω_2 is the largest w_1 in Ω_2. So, one can show that the maximizer of π_M in Ω_2 is given by

$$w_1^{\Omega_2} = \begin{cases} \omega_\mathrm{A} & \text{if } 0 \le \delta \le \delta_1 \\ \omega_\mathrm{C} & \text{if } \delta_1 < \delta \le \delta_2, \\ \omega_\mathrm{D} & \text{if } \delta_2 < \delta \end{cases}$$

where, as defined earlier, $\omega_\mathrm{A} = \frac{1}{8}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta\left(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2\right)}\right)$, $\omega_\mathrm{C} = \frac{2\beta-\gamma\lambda}{\beta(4\beta+\gamma\lambda)} + \frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)}$ and $\omega_\mathrm{D} = \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$.

Third, suppose that $w_1 \in \Omega_3$ (where $\Omega_3 \equiv \{w_1 | \, p_1^* = \hat{P}\}$). Then, the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{1+\delta}{8\beta} + \frac{w_1\left(2\beta+2\gamma\lambda-\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}\right)}{8\beta+4\gamma\lambda}$. When $0 \le \delta \le \delta_1$, $\pi_M^*(w_1)$ is increasing in w_1, i.e., $\frac{\partial \pi_M^*}{\partial w_1} > 0$. Thus, the maximizer of Ω_3 is the largest w_1 in Ω_2. So, one can show that the maximizer of π_M in Ω_3 is given by $w_1^{\Omega_3} = \omega_\mathrm{B} = \frac{1}{2\beta+\gamma\lambda} + \frac{1}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}}$.

Fourth, if $w_1 \in \Omega_4$ (where $\Omega_4 \equiv \{w_1 | \, p_1^* = \frac{1+w_1\beta}{2\beta}\}$), then the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{1+\delta+4w_1\beta(1-w_1\beta)}{8\beta}$. For $0 \le \delta \le \delta_2$, $\pi_M^*(w_1)$ is decreasing in w_1, i.e., $\frac{\partial \pi_M^*}{\partial w_1} < 0$. Thus, the maximizer of Ω_4 is the smallest w_1 in Ω_4. The manufacturer's optimal wholesale price in Ω_4 is given by

$$w_1^{\Omega_4} = \begin{cases} \omega_\mathrm{B} & \text{if } 0 \le \delta \le \delta_1 \\ \omega_\mathrm{C} & \text{if } \delta_1 < \delta \le \delta_2 \end{cases},$$

where, as defined earlier, $\omega_\mathrm{B} = \frac{1}{2\beta+\gamma\lambda} + \frac{1}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda(\beta+\gamma\lambda)}{2}}$ and $\omega_\mathrm{C} = \frac{2\beta-\gamma\lambda}{\beta(4\beta+\gamma\lambda)} + \frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)}$.

Fifth, if $w_1 \in \Omega_5$ (where $\Omega_5 \equiv \{w_1 | \, p_1^* = \frac{1}{\beta}\}$), then the manufacturer's profit is given by $\pi_M^*(w_1) = \frac{1+\delta}{8\beta}$ if $0 < \delta \le \delta_1$ and $w_1 \ge \frac{1}{\beta}$ or $\delta_1 < \delta \le \delta_2$ and $w_1 \ge \omega_\mathrm{C}$ or $\delta \le \delta_2$ and $w_1 \ge \omega_\mathrm{D}$. One can show that $\pi_M^*(w_1)$ is constant in w_1 as long as $w_1 \in \Omega_5$. Hence, we can take, for example, $w_1^{\Omega_5} = \frac{1}{\beta}$ if $0 < \delta \le \delta_1$ and $w_1^{\Omega_5} = \omega_\mathrm{C}$ if $\delta_1 < \delta \le \delta_2$.

To find the manufacturer's equilibrium first-period price, it remains to compare the manufacturer's profits under the wholesale prices $w_1^{\Omega_i}$ for $i \in \{1, \dots, 6\}$. The price that yields the highest profit will correspond to the manufacturer's equilibrium price. After the comparison, one can show that the manufacturer's optimal first-period wholesale price is as follows.

$$w_1^{\Omega_2} = \begin{cases} \omega_{\mathrm{A}} & \text{if } 0 \leq \delta \leq \delta_1 \\ \omega_{\mathrm{C}} & \text{if } \delta_1 < \delta \leq \delta_2 \\ \omega_{\mathrm{D}} & \text{if } \delta_2 < \delta \end{cases},$$

where, as defined earlier, $\omega_{\mathrm{A}} = \frac{1}{8}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta\left(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2\right)}\right)$, $\omega_{\mathrm{C}} = \frac{2\beta-\gamma\lambda}{\beta(4\beta+\gamma\lambda)} + \frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)}$ and $\omega_{\mathrm{D}} = \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, $\delta_1 \equiv \frac{\gamma\lambda(24\beta^2+22\beta\gamma\lambda+7\gamma^2\lambda^2)}{(8\beta-\gamma\lambda)(2\beta+\gamma\lambda)^2} + \frac{4(\beta+\gamma\lambda)(4\beta+\gamma\lambda)\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{(8\beta-\gamma\lambda)(2\beta+\gamma\lambda)^2}$ and $\delta_2 \equiv \frac{16\beta^2+24\beta\gamma\lambda+17\gamma^2\lambda^2}{8\beta\gamma\lambda-\gamma^2\lambda^2}$.

Using the manufacturer's first-period equilibrium price, we can readily derive the equilibrium outcome of the entire game. In particular, the retailer's first-period equilibrium price is: $p_1^* = \frac{3}{4\beta+\gamma\lambda}$, the manufacturer's and the retailer's second-period equilibrium prices are given by: $w_2^* = \frac{2}{4\beta+\gamma\lambda}$, $p_2^* = \frac{3}{4\beta+\gamma\lambda}$, respectively. The manufacturer's and the retailer's profits in each period are given by

$$\pi_{M1}^* = \begin{cases} \frac{\beta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2)}\right) & \text{if } 0 \leq \delta \leq \delta_1 \\ \frac{\beta+\gamma\lambda}{4\beta+\gamma\lambda}\left(\frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)} + \frac{2}{4\beta+\gamma\lambda} - \frac{\gamma\lambda}{4\beta^2+\beta\gamma\lambda}\right) & \text{if } \delta_1 < \delta \leq \delta_2 \\ \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_2 < \delta \end{cases}$$

$$\pi_{M2}^* = \frac{2(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}.$$

$$\pi_{R1}^* = \begin{cases} \frac{2\beta^2(3\sqrt{2}\gamma\lambda\sigma-8)-(\beta+\gamma\lambda)(8\sqrt{2}\beta^3\sigma-2\gamma^2\lambda^2(3+\delta)+\beta\gamma\lambda\left(\sqrt{2}\gamma\lambda\sigma-4\delta-24\right)}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2} & \text{if } 0 \leq \delta \leq \delta_1 \\ \frac{(\beta+\gamma\lambda)(2\gamma\lambda+2\beta-\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)})}{2\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_1 < \delta \leq \delta_2, \\ \frac{\gamma\lambda(1+\delta)(\gamma\lambda-8\beta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_2 < \delta \end{cases}$$

$$\pi_{R2}^* = \frac{(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2},$$

where $\sigma = \frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{\beta+\gamma\lambda}}$.

The manufacturer's and the retailer's total profits are given by

$\pi_M^* = \pi_{M1}^* + \pi_{M2}^* =$

$$\begin{cases} \frac{2(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} + \frac{\beta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2)}\right) & \text{if } 0 \le \delta \le \delta_1 \\ \frac{2(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} + \frac{\beta+\gamma\lambda}{4\beta+\gamma\lambda}\left(\frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)} + \frac{2}{4\beta+\gamma\lambda} - \frac{\gamma\lambda}{4\beta^2+\beta\gamma\lambda}\right) & \text{if } \delta_1 < \delta \le \delta_2 \\ \frac{2(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} + \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_2 < \delta, \end{cases}$$

$\pi_R^* = \pi_{R1}^* + \pi_{R2}^* =$

$$\begin{cases} \frac{(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} - \frac{(\beta+\gamma\lambda)(8\sqrt{2}\beta^3\sigma-2\gamma^2\lambda^2(3+\delta)+\beta\gamma\lambda(\sqrt{2}\gamma\lambda\sigma-4\delta-24)+2\beta^2(3\sqrt{2}\gamma\lambda\sigma-8))}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2} & \text{if } 0 \le \delta \le \delta_1 \\ \frac{(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} + \frac{(\beta+\gamma\lambda)(2\gamma\lambda+2\beta-\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)})}{2\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_1 < \delta \le \delta_2. \\ \frac{(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} + \frac{\gamma\lambda(1+\delta)(\gamma\lambda-8\beta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_2 < \delta \end{cases}$$

■

By comparing the equilibrium wholesale prices in Lemma IV-3 with those in the benchmark without fairness concerns (as in Lemma IV-1), Proposition IV-2 shows that the existence of consumers with fairness concerns in the market will, in equilibrium, increase the manufacturer's first-period wholesale price and decrease its second-period wholesale price, which are the same qualitative results as Proposition 2 in the main paper.

PROPOSITION IV-2. *When a fraction of consumers have fairness concerns, the manufacturer will, in equilibrium, charge a higher first-period wholesale price and a lower second-period wholesale price than when consumers do not have fairness concerns, i.e.* $w_1^* > w_1^{*NF}$ *and* $w_2^* < w_2^{*NF}$.

PROOF OF PROPOSITION IV-2. The manufacturer's equilibrium wholesale prices in two periods (w_1^* and w_2^*) are given in Lemma IV-3; From Lemma IV-1 we know that when consumers do not have fairness concerns, the manufacturer's equilibrium prices are given by: $w_1^{*NF} = w_2^{*NF} = \frac{1}{2\beta}$. Define $\Delta w_1 = w_1^* - w_1^{*NF}$ and $\Delta w_2 = w_2^* - w_2^{*NF}$. One can show that $\Delta w_1 = w_1^* - w_1^{*NF} > 0$, thus, $w_1^* > w_1^{*NF}$. Moreover, $\Delta w_2 = w_2^* - w_2^{*NF} = \frac{2}{4\beta+\gamma\lambda} - \frac{1}{2\beta} < 0$, thus, $w_2^* < w_2^{*NF}$. ■

Next let us turn to examine how the retailer's prices are affected by the existence of consumers with fairness concerns in this alternative model. Proposition IV-3 shows that the presence of a segment of consumers with fairness concerns can reduce the equilibrium retail prices in both periods when the market growth rate is relatively small, which is qualitatively the same as Proposition 3 in the main paper.

PROPOSITION IV-3. *The presence of a segment of consumers with fairness concerns will in equilibrium reduce both the first-period and second-period retail prices, i.e.,* $p_1^* < p_1^{*NF}$ *and* $p_2^* < p_2^{*NF}$.

PROOF OF PROPOSITION IV-3. The retailer's equilibrium prices in two periods are given in Lemma III-3, i.e., $p_1^* = p_2^* = \frac{3}{4\beta+\gamma\lambda}$. From Lemma IV-1 we know that when no consumers have fairness concerns, the retailer's equilibrium prices are given by: $p_1^{*NF} = p_2^{*NF} = \frac{3}{4\beta}$. Define $\Delta p_1 = p_1^* - p_1^{*NF}$ and $\Delta p_2 = p_2^* - p_2^{*NF}$. One can easily show that $\Delta p_1 = \Delta p_2 = \frac{3}{4\beta+\gamma\lambda} - \frac{3}{4\beta} < 0$, i.e., $p_1^* < p_1^{*NF}$ and $p_2^* < p_2^{*NF}$. ■

Having analyzed the manufacturer's and the retailer's equilibrium pricing strategies when facing a positive fraction of consumers with fairness concerns, we now examine how the existence of consumers with fairness concerns affects the firms' profits. Let us start with the manufacturer's profit. Proposition IV-4 shows that consumer fairness concerns can benefit the manufacturer in both time periods, which is qualitatively the same as Proposition 5 in the main paper.

PROPOSITION IV-4. *The existence of consumers with fairness concerns will, in equilibrium, increase both the manufacturer's first-period profit and its second-period profit ,i.e.,* $\pi_{M1}^* > \pi_{M1}^{*NF}$ *and* $\pi_{M2}^* > \pi_{M2}^{*NF}$.

PROOF OF PROPOSITION IV-4. Recall that when there are *no* consumers with fairness concerns in a market, the manufacturer's profits in the first period and the second period are given by $\pi_{M1}^{*NF} = \frac{1}{8\beta}$ and

$\pi_{M2}^{*NF} = \frac{1+\delta}{8\beta}$. Also, when there are some consumers with fairness concerns, the manufacturer's per-period profits π_{M1}^{*} and π_{M2}^{*} are given by

$$\pi_{M1}^{*} = \begin{cases} \frac{\beta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2)}\right) & \text{if } 0 \le \delta \le \delta_1 \\ \frac{\beta+\gamma\lambda}{4\beta+\gamma\lambda}\left(\frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)} + \frac{2}{4\beta+\gamma\lambda} - \frac{\gamma\lambda}{4\beta^2+\beta\gamma\lambda}\right) & \text{if } \delta_1 < \delta \le \delta_2 \\ \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_2 < \delta \end{cases},$$

$$\pi_{M2}^{*} = \frac{2(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}.$$

We will first show that the manufacturer's profit in the first period is higher than that when no consumers have fairness concerns. More specifically, we will show $\pi_{M1}^{*} > \pi_{M1}^{*NF}$. Define $\Delta\pi_{M1} = \pi_{M1}^{*} - \pi_{M1}^{*NF}$.

$$\text{Then } \Delta\pi_{M1} = \begin{cases} \frac{\beta+\gamma\lambda}{8(4\beta+\gamma\lambda)}\left(\frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}} + \frac{32\beta^2-4\beta\gamma\lambda\delta-2\gamma^2\lambda^2(3+\delta)}{\beta(8\beta^2+6\beta\gamma\lambda+\gamma^2\lambda^2)}\right) - \frac{1}{8\beta} & \text{if } 0 \le \delta \le \delta_1 \\ \frac{\beta+\gamma\lambda}{4\beta+\gamma\lambda}\left(\frac{\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)}}{2\beta(4\beta+\gamma\lambda)} + \frac{2}{4\beta+\gamma\lambda} - \frac{\gamma\lambda}{4\beta^2+\beta\gamma\lambda}\right) - \frac{1}{8\beta} & \text{if } \delta_1 < \delta \le \delta_2 \\ \frac{48\beta^2+8\beta\gamma\lambda(7+\delta)-\gamma^2\lambda^2(1+\delta)}{16\beta(4\beta+\gamma\lambda)^2} - \frac{1}{8\beta} & \text{if } \delta_2 > \delta \end{cases}$$

One can show that in each δ interval, $\Delta\pi_{M1} > 0$. Take $0 \le \delta \le \delta_1$ for example, in this interval, $\Delta\pi_{M1}|_{\delta=0} = \frac{(\beta+\gamma\lambda)\left(\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}(4\beta+3\gamma\lambda)+\frac{32\beta^2-6\gamma^2\lambda^2}{4\beta+\gamma\lambda}\right)}{8\beta\,(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)} - \frac{1}{8\beta} > 0$. One can also show that $\Delta\pi_{M1}$ increases in δ. Thus, as δ increase, $\Delta\pi_{M1}$increase. That is, if $\delta \in [0, \delta_1]$, then $\Delta\pi_{M1} > 0$. Similarly, we can also prove that within intervals $\delta_1 < \delta \le \delta_2$ and $\delta > \delta_2$, $\Delta\pi_{M1} > 0$, i.e., $\pi_{M1}^{*} > \pi_{M1}^{*NF}$.

Second, let us show that the manufacturer's profit in the second period can be higher than when no consumers have fairness concerns. Let us show that $\pi_{M2}^{*} > \pi_{M2}^{*NF}$. Define $\Delta\pi_{M2} = \pi_{M2}^{*} - \pi_{M2}^{*NF}$. Thus, $\Delta\pi_{M2} = \frac{2(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} - \frac{1+\delta}{8\beta}$. One can readily show that $\Delta\pi_{M2} > 0$ for any δ. Therefore, $\pi_{M2}^{*} > \pi_{M2}^{*NF}$. That is, the existence of consumers with fairness concerns will lead to higher profit for the manufacturer in each time period. ■

Next, we examine the effect of consumer fairness concerns on the retailer's profit. Proposition IV-5 shows that the retailer will become worse off with a segment of consumers having fairness concerns in the market.

PROPOSITION IV-5. $\pi_{R1}^{*} < \pi_{R1}^{*NF}$*,* $\pi_{R2}^{*} > \pi_{R2}^{*NF}$ *and* $\pi_{R}^{*} < \pi_{R}^{*NF}$*. That is, the existence of consumers with fairness concerns will in equilibrium decrease the retailer's first-period profit and increase its second-period profit, and the retailer's total profit will decrease.*

PROOF OF PROPOSITION IV-5. Recall that when there are *no* consumers with fairness concerns in a market, the retailer's profits in the first period and the second period are given by $\pi_{R1}^{*NF} = \frac{1}{16\beta}$ and $\pi_{R2}^{*NF} = \frac{1+\delta}{16\beta}$. Also, when there are some consumers with fairness concerns, the retailer's per-period profits π_{R1}^{*} and π_{R2}^{*} are given by

$$\pi_{R1}^{*} = \begin{cases} \frac{-(\beta+\gamma\lambda)(8\sqrt{2}\beta^3\sigma-2\gamma^2\lambda^2(3+\delta)+\beta\gamma\lambda(\sqrt{2}\gamma\lambda\sigma-4\delta-24)+2\beta^2(3\sqrt{2}\gamma\lambda\sigma-8))}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2} & \text{if } 0 \le \delta \le \delta_1 \\ \frac{(\beta+\gamma\lambda)(2\gamma\lambda+2\beta-\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)})}{2\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_1 < \delta \le \delta_2 \,, \\ \frac{\gamma\lambda(1+\delta)(\gamma\lambda-8\beta)}{16\beta(4\beta+\gamma\lambda)^2} & \text{if } \delta_2 < \delta \end{cases}$$

where $\sigma = \frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{\beta+\gamma\lambda}}$, and $\pi_{R2}^{*} = \frac{(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2}$.

We will first show that the retailer's profit in the first period is higher than that when no consumers have fairness concerns. More specifically, we will show $\pi_{R1}^{*} > \pi_{R1}^{*NF}$. Let us define $\Delta\pi_{R1} = \pi_{R1}^{*} - \pi_{R1}^{*NF}$.

$$\Delta\pi_{R1} = \begin{cases} \frac{-(\beta+\gamma\lambda)(8\sqrt{2}\beta^3\sigma-2\gamma^2\lambda^2(3+\delta)+\beta\gamma\lambda(\sqrt{2}\gamma\lambda\sigma-4\delta-24)+2\beta^2(3\sqrt{2}\gamma\lambda\sigma-8))}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2} - \frac{1}{16\beta} & \text{if } 0 \le \delta \le \delta_1 \\ \frac{(\beta+\gamma\lambda)(2\gamma\lambda+2\beta-\sqrt{\gamma\lambda(1+\delta)(8\beta-\gamma\lambda)})}{2\beta(4\beta+\gamma\lambda)^2} - \frac{1}{16\beta} & \text{if } \delta_1 < \delta \le \delta_2, \\ \frac{\gamma\lambda(1+\delta)(\gamma\lambda-8\beta)}{16\beta(4\beta+\gamma\lambda)^2} - \frac{1}{16\beta} & \text{if } \delta_2 < \delta \end{cases}$$

where $\sigma = \frac{2\beta(2+\delta)+\gamma\lambda(3+\delta)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{\beta+\gamma\lambda}}$, one can show that in each δ interval, $\Delta\pi_{R1} < 0$. Take $0 \leq \delta \leq \delta_1$ for example, in this interval, $\Delta\pi_{R1}|_{\delta=0} =$

$$\frac{-(\beta+\gamma\lambda)(\frac{8\sqrt{2}\beta^2(4\beta+3\gamma\lambda)}{2\beta+\gamma\lambda}\sqrt{\frac{\gamma\lambda}{\beta+\gamma\lambda}}-6\gamma^2\lambda^2+\beta\gamma\lambda(\frac{\sqrt{2}\gamma\lambda(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{\beta+\gamma\lambda}}-24)+2\beta^2(\frac{3\sqrt{2}\gamma\lambda(4\beta+3\gamma\lambda)}{\beta(2\beta+\gamma\lambda)}\sqrt{\frac{\gamma\lambda}{\beta+\gamma\lambda}}-8))}{8\beta(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)^2}$$

$\frac{-(\beta+\gamma\lambda)(\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}(4\beta+3\gamma\lambda)+\frac{32\beta^2-6\gamma^2\lambda^2}{4\beta+\gamma\lambda})}{8\beta\,(2\beta+\gamma\lambda)(4\beta+\gamma\lambda)} - \frac{1}{16\beta} < 0$. One can also show that $\Delta\pi_{R1}$ decreases in δ. Thus, if $\delta \in [0, \delta_1]$, then $\Delta\pi_{R1} < 0$. Similarly, we can also prove that within intervals $\delta_1 < \delta \leq \delta_2$ and $\delta > \delta_2$, $\Delta\pi_{R1} < 0$, i.e., $\pi^*_{R1} < \pi^{*NF}_{R1}$.

Second, let us show that the retailer's profit in the second period is higher than when no consumers have fairness concerns. Let us show that $\pi^*_{R2} > \pi^{*NF}_{R2}$. Define $\Delta\pi_{R2} = \pi^*_{R2} - \pi^{*NF}_{R2}$. Thus, $\Delta\pi_{R2} = \frac{(1+\delta)(\beta+\gamma\lambda)}{(4\beta+\gamma\lambda)^2} - \frac{1+\delta}{16\beta}$. One can readily show that $\Delta\pi_{R2} > 0$ for any δ. Therefore, $\pi^*_{R2} > \pi^{*NF}_{R2}$. That is, the existence of consumers with fairness concerns will lead to lower profit for the retailer in the first time period and higher profit in the second time period.

Third, let us compare the retailer's total profit with the benchmark case, we can show that the retailer will become worse off with a segment of fair-minded consumers in the market, i.e., $\pi^*_R < \pi^{*NF}_R$. Recall in the case without consumer fairness concerns, the retailer's total profit is given by: $\pi^{*NF}_R = \frac{2+\delta}{16\beta}$. Define $\Delta\pi_R = \pi^*_R - \pi^{*NF}_R$. One can show $\Delta\pi_R = \pi^*_{R1} + \pi^*_{R2} - \frac{2+\delta}{16\beta} < 0$, i.e., $\pi^*_R < \pi^{*NF}_R$. ■

In summary, similar to our core model, the existence of consumer fairness concerns can still give the retailer both the fairness-mitigation and cost-reduction incentives, albeit the fairness-mitigation incentive is weakened. As one can see, most of our main results hold across these two types of demand models. More specifically, we have shown that in the current model, the existence of a segment of consumers with fairness concerns can allow the retailer to lower its first-period price, which in essence helps the retailer to commit to not significantly raising its second-period price, to induce the manufacturer to lower its second-period wholesale price, making pricing in the channel more efficient. That is, the retailer's *cost-reduction incentive* illustrated in our core model still exists in the current model. Second, our analysis has shown that the presence of consumers with fairness concerns tends to make the manufacturer increase its first-period wholesale price and reduce its second-period wholesale price. Third, we have shown that the presence of consumers with fairness concerns can in equilibrium reduce both the first-period and second-period retail prices.

Moreover, we have also shown that, in the alternative multiplicative demand model, the presence of consumer fairness concerns can benefit the manufacturer. However, under the new model, the retailer can no longer benefit from the existence of fairness concerns because the retailer's fairness-mitigation incentive is significantly reduced relative to the core model. Put differently, the relevant range for the retailer's second-period price (that can result in positive sales) is much narrower in the alternative model (with a fixed upper bound of 1) than in the core model, which significantly reduces the manufacturer's incentive to decrease its first-period wholesale price to raise the retailer's cost-reduction incentive of choosing its first-period retail price. This also allows the manufacturer to more easily extract profits in the channel in the second period relative to the core model. Thus, the retailer is worse off overall even though it is better off in the second period due to the cost-reduction effect. So, in the multiplicative demand growth model, Proposition 6 in our core model—which shows an all-win outcome for the manufacturer, the retailer, and the consumers—will no longer hold.

Part V

Extensions with Demand Uncertainty

This part of the Online Appendix presents the results from an extension of our core model where, in the first period, the manufacturer and retailer are uncertain about second-period demand. Specifically, we investigate two scenarios.

In the first scenario (Part V-I), we consider the situation where second-period demand is expected to rise (e.g., due to increased brand awareness and image, celebrity endorsements or positive word-of-mouth), but there is uncertainty about whether the increase will be low or high. To model this situation, we assume that $\delta \in \{\delta_L, \delta_H\}$, where $0 \leq \delta_L < \delta_H$. The probability that $\delta = \delta_H$ is $\phi \in [0,1]$, and $\delta = \delta_L$ with the remaining probability $1 - \phi$. To simplify the analysis, without loss of generality, we normalize $\delta_L = 0$. In the first period, the manufacturer and retailer know the probability distribution of δ, but they observe the actual value of δ only at the beginning of the second period. We provide the analysis of this model in Part V-I of the Online Appendix and we demonstrate that our main qualitative results continue to hold even in the presence of uncertainty about δ.

In the second scenario (Part V-II), we explore the situation where second-period demand intercept will increase by $\delta > 0$ with probability ψ and may decrease by δ with probability $1 - \psi$.[10] Other aspects of the model are the same as in the first scenario that we described above.

Part V-I. Analysis of an extension with demand uncertainty. $\boldsymbol{\delta} \in \{\mathbf{0}, \boldsymbol{\delta_H}\}$

V-I (1). Benchmark with No Fairness Concerns

We will use a superscript "NF" on the variables to indicate this benchmark case without fairness concerns. In the first period, the aggregate market demand is given by $D_1^{NF} = 1 - \beta p_1$; in the second period, the potential market demand increase can be low or high. With probability $\phi \in [0,1]$, $\delta = \delta_H$ and

[10] In this model, we assume $\delta < 1$, so that if the marked demand in the second period is in low state, the demand intercept is still positive in the second period.

the second-period demand is $D_2^{NF}|_{\delta=\delta_H} = 1 + \delta_H - \beta p_2$, where $\delta_H > 0$ represents a positive demand change in the second period; with probability $1 - \phi$, we have $\delta = 0$ and the second-period demand is the same as in the first period: $D_2^{NF}|_{\delta=0} = 1 - \beta p_2$. The manufacturer's and the retailer's total expected profits can be written as $E(\pi_M^{NF}) = E(\pi_{M1}^{NF} + \pi_{M2}^{NF})$ and $E(\pi_R^{NF}) = E(\pi_{R1}^{NF} + \pi_{R2}^{NF})$. Note that π_{Mi}^{NF} and π_{Ri}^{NF} ($i \in \{1,2\}$) are the same as previously defined in Section 3 of the main paper, and the manufacturer's and the retailer's second-period profits, π_{M2}^{NF} and π_{R2}^{NF}, depend on the state of the second-period demand increase. More specifically, the expressions for $E(\pi_M^{NF})$ and $E(\pi_R^{NF})$ are given by: $E(\pi_M^{NF}) = \pi_{M1}^{NF} + \phi \cdot \pi_{M2}^{NF}|_{\delta=\delta_H} + (1 - \phi) \cdot \pi_{M2}^{NF}|_{\delta=0}$ and $E(\pi_R^{NF}) = \pi_{R1}^{NF} + \phi \cdot \pi_{R2}^{NF}|_{\delta=\delta_H} + (1 - \phi) \cdot \pi_{R2}^{NF}|_{\delta=0}$, respectively. One can readily solve the equilibrium outcome using backward induction. Since the analysis is very similar to that in our core model, we omit the details.

We find that, in equilibrium, the manufacturer's and the retailer's first-period wholesale and retail prices are given by $w_1^{*NF} = \frac{1}{2\beta}$ and $p_1^{*NF} = \frac{3}{4\beta}$, respectively. The firms' second-period prices are $w_2^{*NF}|_{\delta=\delta_H} = \frac{1+\delta_H}{2\beta}$, $p_2^{*NF}|_{\delta=\delta_H} = \frac{3(1+\delta_H)}{4\beta}$ when $\delta = \delta_H$, and $w_2^{*NF}|_{\delta=0} = \frac{1}{2\beta}$, $p_2^{*NF}|_{\delta=0} = \frac{3}{4\beta}$ when $\delta = 0$. The manufacturer's and the retailer's total expected equilibrium profits are given by $E(\pi_M^{*NF}) = \frac{\delta_H\, \phi\, (\delta_H+2)+2}{8\beta}$ and $E(\pi_R^{*NF}) = \frac{\delta_H\, \phi\, (\delta_H+2)+2}{16\beta}$, respectively.

V-I (2). Analysis with Fairness Concerns

We solve the game by backward induction. That is, we will first find the second-period subgame equilibrium outcome. Then, we will solve for the first-period equilibrium.

Derivation of the second-period subgame perfect equilibrium.

In the second period, given the first-period retail price p_1 and the realization of $\delta \in \{0, \delta_H\}$, the manufacturer chooses its second-period wholesale price w_2, followed by the retailer choosing its second-period retail price p_2. Since the firms observe δ when making their second-period decisions, the second-period analysis is identical to the one in our core model. Specifically, one can show that when $\delta=\delta_H$, the manufacturer's and the retailer's second-period subgame equilibrium prices are given by:

$$w_2^*|_{\delta=\delta_H}=\begin{cases}\frac{1+\delta_H+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0\le p_1\le\rho_a\\ \frac{2p_1\beta-1-\delta_H+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \rho_a\le p_1\le\rho_b\\ \frac{1+\delta_H}{2\beta} & \text{if } p_1>\rho_b\end{cases},$$

$$p_2^*|_{\delta=\delta_H}=\begin{cases}\frac{3(1+\delta_H+p_1\gamma\lambda)}{4(\beta+\gamma\lambda)} & \text{if } 0\le p_1\le\rho_a\\ p_1 & \text{if } \rho_a\le p_1\le\rho_b\\ \frac{3(1+\delta_H)}{4\beta} & \text{if } p_1>\rho_b\end{cases},$$

where $\rho_a=\frac{3+3\delta_H}{4\beta+\gamma\lambda}$ and $\rho_b=\frac{(1+\delta_H)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. Using $w_2^*|_{\delta=\delta_H}$ and $p_2^*|_{\delta=\delta_H}$, we readily find the manufacturer's and retailer's second-period subgame equilibrium profits when $\delta=\delta_H$:

$$\pi_{M2}^*|_{\delta=\delta_H}=\begin{cases}\frac{(1+\delta_H+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } 0\le p_1\le\rho_a\\ \frac{(1+\delta_H-p_1\beta)(2p_1\beta-1-\delta_H+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } \rho_a\le p_1\le\rho_b\\ \frac{(1+\delta_H)^2}{8\beta} & \text{if } p_1>\rho_b\end{cases},$$

$$\pi_{R2}^*|_{\delta=\delta_H}=\begin{cases}\frac{(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } 0\le p_1\le\rho_a\\ \frac{(1+\delta_H-p_1\beta)^2}{\beta+\gamma\lambda} & \text{if } \rho_a\le p_1\le\rho_b\\ \frac{(1+\delta_H)^2}{16\beta} & \text{if } p_1>\rho_b.\end{cases}$$

Similarly, if $\delta=0$, then one can show that the manufacturer's and retailer's second-period subgame equilibrium prices and profits are as follows.

$$p_2^*|_{\delta=0}=\begin{cases}\frac{3(1+p_1\gamma\lambda)}{4(\beta+\gamma\lambda)} & \text{if } 0\le p_1\le\rho_a'\\ p_1 & \text{if } \rho_a'\le p_1\le\rho_b'\\ \frac{3}{4\beta} & \text{if } p_1>\rho_b'\end{cases},$$

$$w_2^*|_{\delta=0}=\begin{cases}\frac{1+\gamma\lambda\, p_1}{2(\beta+\gamma\lambda)} & \text{if } 0\le p_1\le\rho_a'\\ \frac{(2\beta+\gamma\lambda)p_1-1}{\beta+\gamma\lambda} & \text{if } \rho_a'\le p_1\le\rho_b'\\ \frac{1}{2\beta} & \text{if } p_1>\rho_b'\end{cases},$$

$$\pi^*_{M2}|_{\delta=0} = \begin{cases} \frac{(1+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho'_a] \\ \frac{(1-p_1\beta)(2p_1\beta-1+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho'_a, \rho'_b] \\ \frac{1}{8\beta} & \text{if } p_1 \in (\rho'_b, +\infty) \end{cases},$$

$$\pi^*_{R2}|_{\delta=0} = \begin{cases} \frac{(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho'_a] \\ \frac{(1-p_1\beta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho'_a, \rho'_b] \\ \frac{1}{16\beta} & \text{if } p_1 \in (\rho'_b, +\infty) \end{cases},$$

where $\rho'_a = \frac{3}{4\beta+\gamma\lambda}$ and $\rho'_b = \frac{(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. For future reference, note that the manufacturer's wholesale price w^*_2 jumps down at ρ'_b . The manufacturer's second-period profit is continuous at ρ'_b and the retailer's profit jumps up at ρ'_b.

Derivation of the first-period equilibrium.

We proceed to solving for the first-period equilibrium. In the first period, the manufacturer and the retailer do not know the realization of δ but they know its probability distribution. Hence, the manufacturer and the retailer make their first-period pricing decisions to maximize the sum of their first-period and expected second-period profits. The manufacturer's and retailer's second-period expected profits are given by $E(\pi_{M2}) = \phi\, \pi^*_{M2}|_{\delta=\delta_H} + (1-\phi)\, \pi^*_{M2}|_{\delta=0}$ and $E(\pi_{R2}) = \phi\, \pi^*_{R2}|_{\delta=\delta_H} + (1-\phi)\, \pi^*_{R2}|_{\delta=0}$, respectively. More specifically, $E(\pi_{M2})$ and $E(\pi_{R2})$ are as follows.

(1) if $\delta_H \leq \delta_{02}$, where $\delta_{02} = \frac{2\beta\gamma\lambda+2\gamma^2\lambda^2+(4\beta+\gamma\lambda)\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{24\beta^2+12\beta\gamma\lambda}$, then

$$E(\pi_{M2}) = \begin{cases} \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\phi)(1+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho'_a] \\ \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)(2p_1\beta-1+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho'_a, \rho_a] \\ \frac{\phi(1-p_1\beta+\delta_H)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} + \frac{(1-\phi)(1-p_1\beta)(2p_1\beta-1+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a, \rho'_b] \\ \frac{\phi(1-p_1\beta+\delta_H)(2p_1\beta-1-\delta_H+p_1\gamma\lambda)}{\beta+\gamma\lambda} + \frac{1-\phi}{8\beta} & \text{if } p_1 \in [\rho'_b, \rho_b] \\ \frac{\phi(1+\delta_H)^2}{8\beta} + \frac{1-\phi}{8\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}] \end{cases},$$

$$E(\pi_{R2}) = \begin{cases} \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho_a'] \\ \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a', \rho_a] \\ \frac{\phi(1-p_1\beta+\delta_H)^2}{\beta+\gamma\lambda} + \frac{(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\, \rho_a, \rho_b'] \\ \frac{\phi(1-p_1\beta+\delta_H)^2}{\beta+\gamma\lambda} + \frac{1-\phi}{16\beta} & \text{if } p_1 \in [\rho_b', \rho_b] \\ \frac{\phi(1+\delta_H)^2}{16\beta} + \frac{1-\phi}{16\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}]. \end{cases} .$$

(2) if $\delta_H > \delta_{02}$, then

$$E(\pi_{M2}) = \begin{cases} \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\phi)(1+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho_a'] \\ \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)(2p_1\beta-1+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a', \rho_b'] \\ \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{1-\phi}{8\beta} & \text{if } p_1 \in [\rho_b', \rho_a] \\ \frac{\phi(1-p_1\beta+\delta_H)(2p_1\beta-1-\delta_H+p_1\gamma\lambda)}{\beta+\gamma\lambda} + \frac{1-\phi}{8\beta} & \text{if } p_1 \in [\rho_a, \rho_b] \\ \frac{\phi(1+\delta_H)^2}{8\beta} + \frac{1-\phi}{8\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}] \end{cases} ,$$

$$E(\pi_{R2}) = \begin{cases} \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho_a'] \\ \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a', \rho_b'] \\ \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta} & \text{if } p_1 \in [\rho_b', \rho_a] \\ \frac{\phi(1-p_1\beta+\delta_H)^2}{\beta+\gamma\lambda} + \frac{1-\phi}{16\beta} & \text{if } p_1 \in [\rho_a, \rho_b] \\ \frac{\phi(1+\delta_H)^2}{16\beta} + \frac{1-\phi}{16\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}] \end{cases} .$$

Given w_1, the retailer chooses its first-period price p_1 to maximize its total expected profit $\pi_R = (p_1 - w_1)\, D_1 + E(\pi_{R2})$, where $D_1 = \begin{cases} 1-\beta p_1 & \text{if } 0 < p_1 < \frac{1}{\beta} \\ 0 & \text{if } p_1 \geq \frac{1}{\beta}. \end{cases}$

Let p_1^* denote the retailer's best-response price, i.e., $p_1^* \equiv \underset{p_1}{\text{argmax}}\, \pi_R$. We will first obtain the optimal first-period retail prices within each of the following intervals $K_1 \equiv [0, \rho_a']$, $K_2 \equiv [\rho_a', \min\{\rho_a, \rho_b'\}]$, $K_{3a} \equiv [\rho_a, \rho_b']$, $K_{3b} \equiv [\rho_b', \rho_a]$, $K_4 \equiv [\max\{\rho_a, \rho_b'\}, \rho_b]$ and $K_5 \equiv [\rho_b, \frac{1+\delta_H}{\beta}]$. Note that, if $\delta_H \leq \delta_{02}$, then we have $\rho_a \leq \rho_b'$, and $\rho_a > \rho_b'$ if otherwise. After finding the locally optimal prices in each of these intervals, the price that yields the highest total profit is the retailer's globally optimal first-period retail price in response to w_1. Denote the retailer's optimal first-period price within the interval K_i by $p_1^{K_i}$, $i \in \{1, \ldots, 5\}$. Note that if $p_1 \geq \frac{1}{\beta}$, then there will be no sales in the first period (i.e., $D_1 = 0$), so the retailer's total profit π_R equals to its second-period expected profit $E(\pi_{R2}^*)$. Also note that $E(\pi_{R2}^*)$ is piecewise concave on each interval K_i. Hence, the local maximizer in the interval K_i is either in the interior of K_i (and hence satisfies the first-order condition) or it is at one of the endpoints of K_i. Since the analysis is rather straightforward, below we will directly provide the solution to the profit maximization problem within each K_i. One can verify that $\frac{1}{\beta} > \rho_b'$, and hence, there are three possible cases to analyze: $\frac{1}{\beta} \in K_{3b}$, $\frac{1}{\beta} \in K_4$ and $\frac{1}{\beta} \in K_5$.

Case 1: $\frac{1}{\beta} \in K_5$ (i.e., $\frac{1}{\beta} \geq \rho_b$). Note that $\frac{1}{\beta} \in K_5$ if and only if $0 \leq \delta_H \leq \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$.

First, if $p_1 \in K_1$ (where $K_1 \equiv [0, \rho_a']$), the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2+(1-\phi)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$. One can show that if $0 < w_1 < \omega_{b1}$, where $\omega_{b1} = \frac{16\beta^2+4\beta\gamma\lambda-12\gamma^2\lambda^2-4\beta\gamma\lambda\delta_H\phi-\gamma^2\lambda^2\delta_H\phi}{32\beta^3+40\beta^2\gamma\lambda+8\beta\gamma^2\lambda^2}$, the solution is interior at $p_1^{K_1} = \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}$; and if $w_1 \geq \omega_{b1}$, the solution is at the right corner $p_1^{K_1} = \rho_a'$.

Second, if $p_1 \in K_2$ (where $K_2 \equiv [\rho_a', \min\{\rho_a, \rho_b'\}]$), the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda}$. If $0 < w_1 < \omega_{b2} = \frac{16\beta^2(2-\phi)-\gamma^2\lambda^2(8+\phi(4+\delta_H))+4\beta\gamma\lambda(6-\phi(5+\delta_H))}{8\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \rho_a'$; if $\omega_{b2} \leq w_1 \leq \omega_{b3}$, where

$$\omega_{b3} = \begin{cases} \frac{\beta\gamma\lambda(6+\delta_H(12-\phi)-5\phi)-\gamma^2\lambda^2(2+\phi+\delta\phi)+4\beta^2(2+\phi(3\delta_H-1))}{2\beta(\beta+\gamma\lambda)(4\beta+\gamma\lambda)} & \text{if } \delta \leq \delta_{02} \\ \frac{(16\beta\gamma\lambda+16\phi\beta^2-\phi\gamma^2\lambda^2)\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}}{32\beta^2(2\beta+\gamma\lambda)} - \frac{2(16\beta^3(\phi-2)+\beta\gamma^2\lambda^2\phi(5+2\delta_H)+\gamma^3\lambda^3\phi+4\beta^2\gamma\lambda(\phi(5+\delta_H)-8))}{32\beta^2(2\beta+\gamma\lambda)(\beta+\gamma\lambda)} & \text{if } \delta > \delta_{02}, \end{cases}$$

then the solution is interior at $p_1^{K_2} = \frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\phi\beta^2+16\beta\gamma\lambda-\phi\,\gamma^2\lambda^2}$; if $w_1 \geq \omega_{b3}$, then the solution is at the right corner $p_1^{K_2} = \min\{\rho_a, \rho_b'\}$.

Third, if $p_1 \in K_{3a}$ (where $K_{3a} \equiv [\rho_a, \rho_b']$) and $\delta \leq \delta_{02}$, the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1-p_1\beta+\delta_H)^2+(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda}$. If $0 < w_1 < \omega_{b4} = \frac{4\beta^2+3\beta\gamma\lambda+6\beta\gamma\lambda\delta_H-\gamma^2\lambda^2+8\beta^2\delta_H\phi+2\beta\delta_H\phi\gamma\lambda}{4\beta^3+5\beta^2\gamma\lambda+\beta\gamma^2\lambda^2}$, then the solution can be shown to be at the left corner $p_1^{K_3} = \rho_a = \frac{3+3\delta_H}{4\beta+\gamma\lambda}$, and if $\omega_{b4} \leq w_1 \leq \omega_{b5} = \frac{2(\beta+\gamma\lambda+2\beta\delta_H\phi+\delta\phi\gamma\lambda)}{2\beta^2+3\beta\gamma\lambda+\gamma^2\lambda^2} + \sqrt{\frac{\gamma^3\lambda^3}{2\beta^2(\beta+\gamma\lambda)(2\beta+\gamma\lambda)^2}}$, then the solution is interior at $p_1^{K_3} = \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta_H\phi)}{2\beta\gamma\lambda}$, and if $w_1 \geq \omega_{b5}$, then the solution is at the right corner $p_1^{K_3} = \rho_b'$. If $p_1 \in K_{3b}$ (where $K_{3b} = [\rho_b', \rho_a]$) and $\delta > \delta_{02}$, the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta}$. If $0 < w_1 < \omega_{b4}'$, where $\omega_{b4}' = \frac{(16\beta\gamma\lambda+16\beta^2-\phi\gamma^2\lambda^2)\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}}{32\beta^2(2\beta+\gamma\lambda)} - \frac{32\beta^3-2\beta\phi\gamma^2\lambda^2(5+2\delta_H)-2\phi\gamma^3\lambda^3-8\beta^2\gamma\lambda(\phi-4+\delta_H\phi)}{32\beta^2(2\beta+\gamma\lambda)(\beta+\gamma\lambda)}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \rho_b'$, and if $\omega_{b4}' \leq w_1 \leq \omega_{b5}' = \frac{4\beta+12\beta\delta_H-(2+\phi+\delta_H\phi)\gamma\lambda}{8\beta^2+2\beta\gamma\lambda}$, then the solution is interior at $p_1^{K_{3b}} = \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta_H\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2}$, and if $w_1 \geq \omega_{b5}'$, then the solution is at the right corner $p_1^{K_{3b}} = \rho_a$.

Fourth, if $p_1 \in K_4$ (where $K_4 \equiv [\max\{\rho_a, \rho_b'\}, \rho_b]$), the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda} + \frac{1-\phi}{16\beta}$. If $0 < w_1 \leq \omega_{b6}$, where

$$\omega_{b6} = \begin{cases} \frac{\gamma\lambda(1+\phi+2\delta_H\phi)+\beta(1+\phi+4\delta_H\phi)}{2\beta^2+3\beta\gamma\lambda+\gamma^2\lambda^2} + \frac{\beta+\gamma\lambda-\beta\phi}{\beta(2\beta+\gamma\lambda)\sqrt{\frac{2\gamma\lambda+2\beta}{\gamma\lambda}}} & \text{if } \delta \leq \delta_{02} \\ \frac{2\beta(1+\phi+\delta_H(3+\phi))-\gamma\lambda}{\beta(4\beta+\gamma\lambda)} & \text{if } \delta > \delta_{02}, \end{cases}$$

then the solution can be shown to be at the left corner $p_1^{K_4} = \max\{\rho_a, \rho_b'\}$, and if $\omega_{b6} \leq w_1 \leq \omega_{b7} = \frac{(1+\delta_H)\gamma\lambda\sqrt{\frac{2\gamma\lambda(\beta+\gamma\lambda-\beta\phi)^2}{\beta+\gamma\lambda}}+2\beta(1+\phi+\delta_H(3+\phi))+2\gamma\delta_H\lambda}{2\beta(2\beta+\gamma\lambda)}$, then the solution is interior at $p_1^{K_4} = \frac{(\beta+\gamma\lambda)(1+\beta w_1)-2\beta\phi(1+\delta_H)}{2\beta(\gamma\lambda+\beta(1-\phi))}$, and if $w_1 \geq \omega_{b7}$, then the solution is at the right corner $p_1^{K_4} = \rho_b$.

Fifth, if $p_1 \in K_5$ (where $K_5 \equiv [\rho_b, \frac{1+\delta_H}{\beta}]$) and $p_1 \leq \frac{1}{\beta}$, then the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H)^2}{16\beta} + \frac{1-\phi}{16\beta}$; if $p_1 \in K_5$ and $p_1 > \frac{1}{\beta}$, then the retailer's total profit is: $\pi_R = E(\pi_{R2}^*) = \frac{\phi(1+\delta_H)^2}{16\beta} + \frac{1-\phi}{16\beta}$, in which case the first-period profit is zero. One can show that if $0 < w_1 < \omega_{b8} = \frac{\beta(2+6\delta_H)+2\gamma\lambda\delta_H+(1+\delta_H)\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{2\beta(2\beta+\gamma\lambda)}$, then the solution is at the left corner at $p_1^{K_5} = \rho_b$, and if $\omega_{b8} \leq w_1 < \frac{1}{\beta}$, then the solution is interior at $p_1^{K_5} = \frac{1+w_1\beta}{2\beta}$, and if $w_1 \geq \frac{1}{\beta}$, then the solution is $p_1^{K_5} = \frac{1}{\beta}$.

Comparing the profits corresponding to each of the prices $p_1^{K_i}$ for $i = 1, \dots, 5$, one can show that, when $0 < \delta_H \leq \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$, the retailer's first-period optimal price $p_1^*(w_1)$ is as follows.

(1) If $0 < \delta_H \leq \delta_{02}$, then

$$p_1^*(w_1) = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \leq w_1 < \omega_{b1} \\ \rho_a' & \text{if } \omega_{b1} \leq w_1 < \omega_{b2} \\ \frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\rho\beta^2+16\beta\gamma\lambda-\phi\,\gamma^2\lambda^2} & \text{if } \omega_{b2} \leq w_1 < \omega_{b3} \\ \rho_a & \text{if } \omega_{b3} \leq w_1 < \min\{\omega_{b4}, \omega_{b6}\} \text{ or } \omega_{b3} \leq w_1 < \omega_{x0} \\ \frac{w_1\beta^2+\gamma\lambda+\beta(w_1\gamma\lambda-1-2\delta_H\phi)}{2\beta\gamma\lambda} & \text{if } \omega_{b4} \leq w_1 < \omega_{b6} \\ \frac{(\beta+\gamma\lambda)(1+\beta w_1)-2\beta\phi(1+\delta_H)}{2\beta(\gamma\lambda+\beta(1-\phi))} & \text{if } \omega_{b6} \leq w_1 < \omega_{b7} \\ \rho_b & \text{if } \omega_{b7} \leq w_1 < \omega_{b8} \text{ or } \omega_{x0} \leq w_1 < \omega_{b8} \\ \frac{1+w_1\beta}{2\beta} & \text{if } \omega_{b8} \leq w_1 < \frac{1}{\beta} \\ \frac{1}{\beta} & \text{if } w_1 \geq \frac{1}{\beta} \end{cases}$$

(2) If $\delta_{02} < \delta_H \leq \frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda}$, then

$$p_1^*(w_1) = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \le w_1 < \omega_{b1} \\ \rho_a' & \text{if } \omega_{b1} \le w_1 < \omega_{b2} \text{ or } \omega_{b1} \le w_1 < \omega_{x1} \\ \frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\rho\beta^2+16\beta\gamma\lambda-\rho\,\gamma^2\lambda^2} & \text{if } \omega_{b2} \le w_1 < \min\{\omega_{b3}, \omega_y\} \\ \rho_b' & \text{if } \omega_{x1} \le w_1 < \omega_{b4}' \text{ or } \min\{\omega_{b3}, \omega_y\} \le w_1 < \omega_{b4}' \\ \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta_H\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2} & \text{if } \omega_{b4}' \le w_1 < \omega_{b5}' \\ \rho_a & \text{if } \omega_{b5}' \le w_1 < \omega_{b6} \text{ or } \omega_{b5}' \le w_1 < \omega_{x0} \\ \frac{(\beta+\gamma\lambda)(1+\beta w_1)-2\beta\phi(1+\delta_H)}{2\beta(\gamma\lambda+\beta(1-\phi))} & \text{if } \omega_{b6} \le w_1 < \omega_{b7} \\ \rho_b & \text{if } \omega_{b7} \le w_1 < \omega_{b8} \text{ or } \omega_{x0} \le w_1 < \omega_{b8} \\ \frac{1+w_1\beta}{2\beta} & \text{if } \omega_{b8} \le w_1 < \frac{1}{\beta} \\ \frac{1}{\beta} & \text{if } w_1 \ge \frac{1}{\beta} \end{cases}$$

where ω_{x0} satisfies $\pi_R^*|_{p_1^*=\rho_a} = \pi_R^*|_{p_1^*=\rho_b}$, ω_{x1} satisfies $\pi_R^*|_{p_1^*=\rho_a'} = \pi_R^*|_{p_1^*=\rho_b'}$ and ω_y satisfies $\pi_R^*|_{p_1^*=\frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\phi\beta^2+16\beta\gamma\lambda-\phi\,\gamma^2\lambda^2}} = \pi_R^*|_{p_1^*=\rho_b'}$. Since the mathematical expressions for ω_{x0}, ω_{x1} and ω_y are too cumbersome and too messy, due to space limitation, we will not list all the detailed expressions here. Plugging in p_1^* , we can get the retailer's profit $\pi_R^*(w_1)$ and the manufacturer's total profit $\pi_M^*(w_1)$.

Case 2: $\frac{1}{\beta} \in K_4$ (i.e., $\max\{\rho_a, \rho_b'\} \le \frac{1}{\beta} < \rho_b$). Note that $\frac{1}{\beta} \in K_4$ if and only if $\frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda} < \delta_H \le \frac{\beta+\gamma\lambda}{3\beta}$.

First, if $p_1 \in K_1$ (where $K_1 \equiv [0, \rho_a']$), the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1-\beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2+(1-\phi)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$. One can show that if $0 < w_1 < \omega_{b1} = \frac{16\beta^2+4\beta\gamma\lambda-12\gamma^2\lambda^2-4\beta\gamma\lambda\delta_H\phi-\gamma^2\lambda^2\delta_H\phi}{32\beta^3+40\beta^2\gamma\lambda+8\beta\gamma^2\lambda^2}$, the solution is interior at $p_1^{K_1} = \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}$; and if $w_1 \ge \omega_{b1} = \frac{16\beta^2+4\beta\gamma\lambda-12\gamma^2\lambda^2-4\beta\gamma\lambda\delta_H\phi-\gamma^2\lambda^2\delta_H\phi}{32\beta^3+40\beta^2\gamma\lambda+8\beta\gamma^2\lambda^2}$, the solution is at the right corner $p_1^{K_1} = \rho_a' = \frac{3}{4\beta+\gamma\lambda}$.

Second, if $p_1 \in K_2$ (where$K_2 \equiv [\rho_a', \min\{\rho_a, \rho_b'\}]$), the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda}$. If $0 < w_1 < \omega_{b2} = \frac{16\beta^2(2-\phi)-\gamma^2\lambda^2(8+\phi(4+\delta_H))+4\beta\gamma\lambda(6-\phi(5+\delta_H))}{8\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, then the solution can be shown to be at the left corner $p_1^{K_2} =$

ρ_a'; if $\omega_{b2} \le w_1 \le \omega_{b3} = \frac{(16\beta\gamma\lambda+16\phi\beta^2-\phi\gamma^2\lambda^2)\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}}{32\beta^2(2\beta+\gamma\lambda)} -$ $\frac{2(16\beta^3(\phi-2)+\beta\gamma^2\lambda^2\phi(5+2\delta_H)+\gamma^3\lambda^3\phi+4\beta^2\gamma\lambda(\phi(5+\delta_H)-8))}{32\beta^2(2\beta+\gamma\lambda)(\beta+\gamma\lambda)}$, then the solution is interior at $p_1^{K_2} =$ $\frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\phi\beta^2+16\beta\gamma\lambda-\phi\,\gamma^2\lambda^2}$; if $w_1 \ge \omega_{b3}$, then the solution is at the right corner $p_1^{K_2} =$ $\min\{\rho_a, \rho_b'\}$.

Third, if $p_1 \in K_{3b}$ (where $K_{3b} = [\rho_b', \rho_a]$) and $\delta > \delta_{02}$, the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta}$. If $0 < w_1 < \omega_{b4}'$, where $\omega_{b4}' = \frac{(16\beta\gamma\lambda+16\beta^2-\phi\gamma^2\lambda^2)\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}}{32\beta^2(2\beta+\gamma\lambda)} - \frac{32\beta^3-2\beta\phi\gamma^2\lambda^2(5+2\delta_H)-2\phi\gamma^3\lambda^3-8\beta^2\gamma\lambda(\phi-4+\delta_H\phi)}{32\beta^2(2\beta+\gamma\lambda)(\beta+\gamma\lambda)}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \rho_b'$, and if $\omega_{b4}' \le w_1 \le \omega_{b5}' = \frac{4\beta+12\beta\delta_H-(2+\phi+\delta_H\phi)\gamma\lambda}{8\beta^2+2\beta\gamma\lambda}$, then the solution is interior at $p_1^{K_{3b}} = \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta_H\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2}$, and if $w_1 \ge \omega_{b5}'$, then the solution is at the right corner $p_1^{K_{3b}} = \rho_a = \frac{3+3\delta_H}{4\beta+\gamma\lambda}$.

Fourth, if $p_1 \in K_4$ (where $K_4 \equiv [\max\{\rho_a, \rho_b'\}, \rho_b]$) and $p_1 \le \frac{1}{\beta}$, the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1 - \beta p_1)(p_1 - w_1) + \frac{\phi(1-p_1\beta+\delta_H)^2}{\beta+\gamma\lambda} + \frac{1-\phi}{16\beta}$; if $p_1 \in K_4$ and $p_1 > \frac{1}{\beta}$, then the retailer's total profit is: $\pi_R = E(\pi_{R2}^*) = \frac{\phi(1-p_1\beta+\delta_H)^2}{\beta+\gamma\lambda} + \frac{1-\phi}{16\beta}$, in which case the first-period profit is zero. If $0 < w_1 \le \omega_{b6}$, where $\omega_{b6} = \frac{2\beta\left(1+\phi+\delta_H(3+\phi)\right)-\gamma\lambda}{\beta(4\beta+\gamma\lambda)}$ then the solution can be shown to be at the left corner $p_1^{K_4} = \rho_a = \frac{3+3\delta_H}{4\beta+\gamma\lambda}$ and if $\omega_{b6} \le w_1 \le \omega_{v1} = \frac{\beta+\gamma\lambda+2\beta\delta_H\phi}{\beta^2+\beta\gamma\lambda}$, then the solution is interior at $p_1^{K_4} = \frac{(\beta+\gamma\lambda)(1+\beta w_1)-2\beta\phi(1+\delta_H)}{2\beta(\gamma\lambda+\beta(1-\phi))}$, and if $w_1 \ge \omega_{v1}$, then the solution is at the right corner $p_1^{K_4} = \frac{1}{\beta}$.

Fifth, if $p_1 \in K_5$ (where $K_5 \equiv [\rho_b, \frac{1+\delta_H}{\beta}]$) then the retailer's total profit is: $\pi_R = E(\pi_{R2}^*) = \frac{\phi(1+\delta_H)^2}{16\beta} + \frac{1-\phi}{16\beta}$, in which case the first-period profit is zero. Note that any price $p_1 \in K_5$ yields the same profit to the retailer, i.e., any $p_1 \in K_5$ is a solution to the retailer's maximization problem on the constraint set K_5, so we let $p_1^{K_5} = \rho_b$.

Comparing the profits corresponding to each of the prices $p_1^{K_i}$ for $i = 1, \dots, 5$, one can show that, when $\frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda} < \delta_H \leq \frac{\beta+\gamma\lambda}{3\beta}$, the retailer's first-period optimal price $p_1^*(w_1)$ is given as follows.

(3) If $\frac{3(\beta+\gamma\lambda)-2\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{9\beta+\gamma\lambda} < \delta_H \leq \frac{\beta+\gamma\lambda+4\sqrt{\beta(\beta+\gamma\lambda)}}{15\beta-\gamma\lambda}$, then

$$p_1^*(w_1) = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \leq w_1 < \omega_{b1} \\ \rho_a' & \text{if } \omega_{b1} \leq w_1 < \omega_{b2} \text{ or } \omega_{b1} \leq w_1 < \omega_{x1} \\ \rho_b' & \text{if } \omega_{b2} \leq w_1 \leq \omega_{b4}' \text{ or } \omega_{x1} \leq w_1 \leq \omega_{b4}' \\ \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2} & \text{if } \omega_{b4}' \leq w_1 \leq \omega_{b5}' \\ \rho_a & \text{if } \omega_{b5}' \leq w_1 < \omega_{b6} \text{ or } \omega_{b5}' \leq w_1 < \omega_{x0} \\ \frac{(\beta+\gamma\lambda)(1+\beta w_1)-2\beta\phi(1+\delta_H)}{2\beta(\gamma\lambda+\beta(1-\phi))} & \text{if } \omega_{b6} \leq w_1 < \omega_{b7} \\ \rho_b & \text{if } w_1 \geq \omega_{b7} \text{ or } w_1 \geq \omega_{x0} \end{cases}$$

where ω_{x0} satisfies $\pi_R^*|_{p_1^*=\rho_a} = \pi_R^*|_{p_1^*=\rho_b}$, ω_{x1} satisfies $\pi_R^*|_{p_1^*=\rho_a'} = \pi_R^*|_{p_1^*=\rho_b'}$.

(4) If $\frac{\beta+\gamma\lambda+4\sqrt{\beta(\beta+\gamma\lambda)}}{15\beta-\gamma\lambda} < \delta_H < \frac{\beta+\gamma\lambda}{3\beta}$, then

$$p_1^*(w_1) = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \leq w_1 < \omega_{b1} \\ \rho_a' & \text{if } \omega_{b1} \leq w_1 < \omega_{b2} \text{ or } \omega_{b1} \leq w_1 < \omega_{x1} \\ \rho_b' & \text{if } \omega_{b2} \leq w_1 < \omega_{b4}' \text{ or } \omega_{x1} \leq w_1 \leq \omega_{b4}' \\ \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2} & \text{if } \omega_{b4}' \leq w_1 < \omega_{b5}' \\ \rho_a & \text{if } \omega_{b5}' \leq w_1 < \omega_{b6} \\ \frac{(\beta+\gamma\lambda)(1+\beta w_1)-2\beta\phi(1+\delta_H)}{2\beta(\gamma\lambda+\beta(1-\phi))} & \text{if } \omega_{b6} \leq w_1 < \frac{\beta+2\beta\delta\rho+\gamma\lambda}{\beta^2+\beta\gamma\lambda} \\ \frac{1}{\beta} & \text{if } w_1 \geq \frac{\beta+2\beta\delta\rho+\gamma\lambda}{\beta^2+\beta\gamma\lambda} \end{cases}$$

Case 3: $\frac{1}{\beta} \in K_{3b}$, where $K_{3b} = [\rho_b', \rho_a]$ (i.e., $\rho_b' \leq \frac{1}{\beta} \leq \rho_a$). Note that $\frac{1}{\beta} \in K_{3b}$ if and only if $\delta_H \geq \frac{\beta+\gamma\lambda}{3\beta}$.

First, if $p_1 \in K_1$ (where $K_1 \equiv [0, \rho_a']$), the retailer's total profit is given by: $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1-\beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2+(1-\phi)(1+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)}$. One can show that if $0 < w_1 < \omega_{b1}$, where $\omega_{b1} = \frac{16\beta^2+4\beta\gamma\lambda-12\gamma^2\lambda^2-4\beta\gamma\lambda\delta_H\phi-\gamma^2\lambda^2\delta_H\phi}{32\beta^3+40\beta^2\gamma\lambda+8\beta\gamma^2\lambda^2}$, the solution is interior at $p_1^{K_1} = \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2}$; and if $w_1 \geq \omega_{b1}$, the solution is at the right corner $p_1^{K_1} = \rho_a'$.

Second, if $p_1 \in K_2$ (where$K_2 \equiv [\rho_a', \min\{\rho_a, \rho_b'\}]$), the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + \pi_{R2}^* = (1-\beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\phi)(1-p_1\beta)^2}{\beta+\gamma\lambda}$. If $0 < w_1 < \omega_{b2} = \frac{16\beta^2(2-\phi)-\gamma^2\lambda^2(8+\phi(4+\delta_H))+4\beta\gamma\lambda(6-\phi(5+\delta_H))}{8\beta(4\beta^2+5\beta\gamma\lambda+\gamma^2\lambda^2)}$, then the solution can be shown to be at the left corner $p_1^{K_2} = \rho_a'$; if $\omega_{b2} \leq w_1 \leq \omega_{b3} = \frac{(16\beta\gamma\lambda+16\phi\beta^2-\phi\gamma^2\lambda^2)\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}}{32\beta^2(2\beta+\gamma\lambda)} - \frac{2(16\beta^3(\phi-2)+\beta\gamma^2\lambda^2\phi(5+2\delta_H)+\gamma^3\lambda^3\phi+4\beta^2\gamma\lambda(\phi(5+\delta_H)-8))}{32\beta^2(2\beta+\gamma\lambda)(\beta+\gamma\lambda)}$, and the solution is interior at $p_1^{K_2} = \frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\phi\beta^2+16\beta\gamma\lambda-\phi\,\gamma^2\lambda^2}$; if $w_1 \geq \omega_{b3}$, then the solution is at the right corner $p_1^{K_2} = \rho_b'$.

Third, if $p_1 \in K_{3b}$ (where $K_{3b} = [\rho_b', \rho_a]$) and $p_1 \leq \frac{1}{\beta}$, the retailer's total profit is given by $\pi_R = D_1(p_1 - w_1) + E(\pi_{R2}^*) = (1-\beta p_1)(p_1 - w_1) + \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta}$; if $p_1 \in K_{3b}$ and $p_1 > \frac{1}{\beta}$, then the retailer's total profit is: $\pi_R = E(\pi_{R2}^*) = \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta}$, in which case the first-period profit is zero.

If $0 < w_1 < \omega_{b4}' = \frac{(16\beta\gamma\lambda+16\beta^2-\phi\gamma^2\lambda^2)\sqrt{\frac{2\gamma\lambda}{\beta+\gamma\lambda}}}{32\beta^2(2\beta+\gamma\lambda)} - \frac{32\beta^3-2\beta\phi\gamma^2\lambda^2(5+2\delta_H)-2\phi\gamma^3\lambda^3-8\beta^2\gamma\lambda(\phi-4+\delta_H\phi)}{32\beta^2(2\beta+\gamma\lambda)(\beta+\gamma\lambda)}$, then the solution can be shown to be at the left corner $p_1^{K_{3b}} = \rho_b'$, and if $\omega_{b4}' \leq w_1 \leq \omega_{v0} = \frac{8\beta^2+8\beta\gamma\lambda-\phi\beta\gamma\lambda-\phi\beta\gamma\lambda\delta_H-\phi\gamma^2\lambda^2}{8\beta^3+8\beta^2\gamma\lambda}$, then the solution is interior at $p_1^{K_{3b}} = \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta_H\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2}$, and if $w_1 \geq \omega_{v0}$, π_R increases in p_1, thus the solution is at the right corner $p_1^{K_{3b}} = \rho_a$ and $\pi_R(p_1^{K_{3b}}) = \frac{1-\phi}{16\beta} + \frac{\phi(\beta+\gamma\lambda)(1+\delta_H)^2}{(4\beta+\gamma\lambda)^2}$.

Fourth, if $p_1 \in K_4$ (where $K_4 \equiv [\max\{\rho_a, \rho_b'\}, \rho_b]$), the retailer's total profit is given by $\pi_R = E(\pi_{R2}^*) = \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta}$, in which case the first-period profit is zero. One can show that $\frac{\partial\pi_R^*}{\partial p_1} < 0$, i.e., π_R^* is decreasing in p_1, and hence, and the solution is at the left corner $p_1^{K_4} = \rho_a$.

Fifth, if $p_1 \in K_5$ (where $K_5 \equiv [\rho_b, \frac{1+\delta_H}{\beta}]$) then the retailer's total profit is: $\pi_R = E(\pi_{R2}^*) = \frac{\phi(1+\delta_H+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{1-\phi}{16\beta}$, in which case the first-period profit is zero, and any price $p_1 \in K_5$ yields the same profit to the retailer, i.e., any $p_1 \in K_5$ is a solution to the retailer's maximization problem on the constraint set K_5, so we let $p_1^{K_5} = \rho_b$.

Comparing the profits corresponding to each of the prices $p_1^{K_i}$ for $i = 1, \dots, 5$, one can show that the retailer's first-period optimal price $p_1^*(w_1)$ is given as follows.

(5) If $\delta_H \geq \frac{\beta+\gamma\lambda}{3\beta}$, then

$$p_1^*(w_1) = \begin{cases} \frac{8w_1\beta^2+\gamma\lambda(9+\delta_H\phi)+8\beta(1+w_1\gamma\lambda)}{16\beta^2+16\beta\gamma\lambda-\gamma^2\lambda^2} & \text{if } 0 \leq w_1 < \omega_{b1} \\ \rho_a' & \text{if } \omega_{b1} \leq w_1 < \omega_{b2} \text{ or } \omega_{b1} \leq w_1 < \omega_{x1} \\ \frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\phi\beta^2+16\beta\gamma\lambda-\phi\gamma^2\lambda^2} & \text{if } \omega_{b2} \leq w_1 < \min\{\omega_{b3}, \omega_{x2}\} \\ \rho_b' & \text{if } \omega_{x1} \leq w_1 < \omega_{b4} \text{ or } \min\{\omega_{b3}, \omega_{x2}\} \leq w_1 < \omega_{b4}' \\ \frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta_H\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2} & \text{if } \omega_{b4}' \leq w_1 < \min\{\omega_{v0}, \omega_{x3}\} \\ \rho_a & \text{if } w_1 > \min\{\omega_{v0}, \omega_{x3}\} \end{cases}$$

where ω_{x1} satisfies $\pi_R^*|_{p_1^*=\rho_a'} = \pi_R^*|_{p_1^*=\rho_b'}$; ω_{x2} satisfies $\pi_R^*|_{p_1^*=\frac{8w_1\beta^2+\gamma\lambda(8+\phi+\delta_H\phi)+8\beta(2\phi-1+w_1\gamma\lambda)}{16\phi\beta^2+16\beta\gamma\lambda-\phi\gamma^2\lambda^2}} = \pi_R^*|_{p_1^*=\rho_b'}$ and ω_{x3} satisfies $\pi_R^*|_{p_1^*=\frac{8(1+\beta w_1)(\beta+\gamma\lambda)+(1+\delta_H\gamma\lambda\phi)}{16\beta(\beta+\gamma\lambda)-\phi\gamma^2\lambda^2}} = \pi_R^*|_{p_1^*=\rho_a}$.

Finally, we proceed to finding the manufacturer's optimal price in the first period (i.e., w_1^*). Given p_1, the manufacturer chooses w_1 to maximize its profit. Unfortunately, the manufacture's profit function is non-quasi concave, and there are numerous local optima. Because of the algebraic complexity of profit expressions, comparing profits under different local optima turned out to be non-tractable. Thus, we have gone as far as possible in terms of finding closed-form solutions in terms of the underlying parameters. Fortunately, the model can be solved numerically under different parameter values. We have tried different numerical conditions, obtaining similar results. Below we provide one specific numerical example with $\beta = 1, \gamma = \frac{1}{5}, \lambda = \frac{1}{2}$ and $\delta_H = \frac{1}{4}$. Plugging these values into the manufacturer's profit function, we can readily obtain the manufacturer's optimal first-period wholesale price:

$$w_1^* = \begin{cases} \frac{94526960 + 59171200\sqrt{22} - 31679593\phi - 1869272\sqrt{22}\phi}{66675840 + 124259520\sqrt{22}} & \text{if } 0 \leq \phi \leq \phi_a \\ \frac{13728 - 7257\phi}{14432} & \text{if } \phi_a \leq \phi \leq \phi_b \\ \frac{-5913600 + 5721920\phi - 2948187\phi^2}{7040\,(-920 + 41\,\phi)} & \text{if } \phi_b < \phi \leq 1 \end{cases}.$$

where $\phi_a = \frac{-2827600 + 5366080\sqrt{22}}{167977 + 5510318\sqrt{22}} + \frac{5366080\sqrt{22}}{167977 + 5510318\sqrt{22}}$, and $\phi_b = \frac{63360}{71873}$. Plugging w_1^* into the expression for $p_1^*(w_1)$, we obtain the retailer's first-period price on the equilibrium path:

$$p_1^* = \begin{cases} \frac{30}{41} & \text{if } 0 \leq \phi \leq \phi_a \\ \frac{30}{41} & \text{if } \phi_a \leq \phi \leq \phi_b \\ \frac{3840 + 1513\,\phi}{7360 - 328\,\phi} & \text{if } \phi_b < \phi \leq 1 \end{cases}.$$

From expressions of w_1^* and p_1^*, we can obtain the firms' second-period equilibrium prices.

$$w_2^*|_{\delta=\delta_H} = \begin{cases} \frac{1085}{1804} & \text{if } 0 \leq \phi \leq \phi_b \\ \frac{95840 - 2587\phi}{161920 - 7216\phi} & \text{if } \phi_b < \phi \leq 1 \end{cases},$$

$$w_2^*|_{\delta=0} = \begin{cases} \frac{20}{41} & \text{if } 0 \leq \phi \leq \phi_b \\ \frac{7040 + 35053\,\phi}{80960 - 3608\,\phi} & \text{if } \phi_b < \phi \leq 1 \end{cases},$$

$$p_2^*|_{\delta=\delta_H} = \begin{cases} \frac{3255}{3608} & \text{if } 0 \leq \phi \leq \phi_b \\ \frac{3\,(-95840 + 2587\phi)}{352\,(-920 + 41\phi)} & \text{if } \phi_b < \phi \leq 1 \end{cases},$$

$$p_2^*|_{\delta=0} = \begin{cases} \frac{30}{41} & \text{if } 0 \leq \phi \leq \phi_b \\ \frac{3840 + 1513\,\phi}{7360 - 328\,\phi} & \text{if } \phi_b < \phi \leq 1 \end{cases}.$$

Using these prices, one can derive the manufacturer's and the retailer's equilibrium profits. The manufacturer's first-period equilibrium profit is given by

$$\pi_{M1}^* = \begin{cases} \frac{880\left(107417 + 67240\sqrt{22}\right) - 41\,(772673 + 45592\sqrt{22})\phi}{11296320\left(22 + 41\sqrt{22}\right)} & \text{if } 0 \leq \phi \leq \phi_a \\ \frac{429}{1681} - \frac{177\,\phi}{1312} & \text{if } \phi_a \leq \phi \leq \phi_b \\ \frac{(3520 - 1841\phi)\left(5913600 - 5721920\,\phi + 2948187\,\phi^2\right)}{56320\,(920 - 41\,\phi)^2} & \text{if } \phi_b < \phi \leq 1 \end{cases}.$$

The manufacturer's second-period expected equilibrium profit is given by

$$E(\pi_{M2}^*) = \begin{cases} \frac{235445\phi}{1183424} + \frac{220(1-\phi)}{1681} & \text{if } 0 \le \phi \le \phi_b \\ \frac{(95840 - 2587\phi)^2\phi}{56320\,(920 - 41\phi)^2} + \frac{(1-\phi)(3520 - 1841\,\phi)(7040 + 35053\phi)}{704\,(920 - 41\phi)^2} & \text{if } \phi_b < \phi \le 1 \end{cases}.$$

Hence, the manufacturer's total expected equilibrium profit is given by

$$E(\pi_M^*) = \begin{cases} \frac{9680\left(144377 + 136120\sqrt{22}\right) + 41\,(-3960253 + 7957813\,\sqrt{22}])\phi}{124259520\,(22 + 41\,\sqrt{22})} & \text{if } 0 \le \phi \le \phi_a \\ \frac{649}{1681} - \frac{1929\phi}{28864} & \text{if } \phi_a \le \phi \le \phi_b. \\ \frac{-12390400 + 7595200\,\phi - 3150169\,\phi^2}{28160\,(-920 + 41\phi)} & \text{if } \phi_b < \phi \le 1 \end{cases}$$

The retailer's first-period equilibrium profit is given by

$$\pi_{R1}^* = \begin{cases} \frac{880\,(-51977 + 36080\,\sqrt{22}) + 41\,(772673 + 45592\,\sqrt{22})\,\phi}{11296320\,(22 + 41\,\sqrt{22})} & \text{if } 0 \le \phi \le \phi_a \\ \frac{-99}{1681} + \frac{177\,\phi}{1312} & \text{if } \phi_a \le \phi \le \phi_b\,. \\ \frac{3(-3520 + 1841\,\phi)\left(844800 - 2351120\,\phi + 982729\,\phi^2\right)}{56320\,(920 - 41\,\phi)^2} & \text{if } \phi_b < \phi \le 1 \end{cases}$$

The retailer's second-period expected equilibrium profit is given by

$$E(\pi_{R2}^*) = \begin{cases} \frac{110}{1681} + \frac{1965\,\phi}{57728} & \text{if } 0 \le \phi \le \phi_a \\ \frac{235445\,\phi}{2366848} + \frac{110\,(1-\phi)}{1681} & \text{if } \phi_a \le \phi \le \phi_b. \\ \frac{(95840 - \phi)^2\phi}{112640\,(920 - 41\,\phi)^2} + \frac{5\,(3520 - 1841\phi)^2(1-\phi)}{352\,(920 - 41\,\phi)^2} & \text{if } \phi_b < \phi \le 1 \end{cases}$$

Hence, the retailer's total expected equilibrium profit is given by

$$E(\pi_R^*) = \begin{cases} \frac{832480\,(-19 + 40\,\sqrt{22}) + 41\left(525316 + 230789\,\sqrt{22}\right)\phi}{6061440\,(22 + 41\sqrt{22})} & \text{if } 0 \le \phi \le \phi_a \\ \frac{11}{1681} + \frac{9753\,\phi}{57728} & \text{if } \phi_a \le \phi \le \phi_b. \\ \frac{1982464000 + 27610956800\,\phi - 21061710560\phi^2 + 5439067503\,\phi^3}{112640\,(920 - 41\,\phi)^2} & \text{if } \phi_b < \phi \le 1 \end{cases}$$

Using the numerical values $\beta = 1$, $\gamma = \frac{1}{5}$, $\lambda = \frac{1}{2}$ and $\delta_H = \frac{1}{4}$ in the equilibrium price and profit expression from the benchmark without fairness concerns that we derived earlier (Part V-I-1), we find that the manufacturer's and the retailer's first-period wholesale and retail prices are $w_1^{*NF} = \frac{1}{2}$ and $p_1^{*NF} = \frac{3}{4}$, respectively. The firms' second-period prices are $w_2^{*NF}|_{\delta=\delta_H} = \frac{5}{8}$, $p_2^{*NF}|_{\delta=\delta_H} = \frac{15}{16}$ when $\delta = \delta_H$, and

$w_2^{*NF}|_{\delta=0} = \frac{1}{2}$, $p_2^{*NF}|_{\delta=0} = \frac{3}{4}$ when $\delta = 0$. The manufacturer's and the retailer's total expected equilibrium profits are given by $E(\pi_M^{*NF}) = \frac{1}{4} + \frac{9\,\phi}{128}$ and $E(\pi_R^{*NF}) = \frac{1}{8} + \frac{9\,\phi}{256}$, respectively.

Next, let us demonstrate that both the manufacturer and the retailer can benefit from consumers' fairness concerns even in the presence of demand uncertainty about δ. That is, we will show that in this setting with uncertainty, our main result about profit implications of fairness concerns is robust.

First, we show that the existence of consumers with fairness concerns can increase the manufacturer's profits in both periods. Let $\Delta\pi_{M1}^* = \pi_{M1}^* - \pi_{M1}^{*NF}$, $\Delta E(\pi_{M2}^*) = E(\pi_{M2}^*) - E(\pi_{M2}^{*NF})$ and $\Delta E(\pi_M^*) = E(\pi_M^*) - E(\pi_M^{*NF})$.

First, in each of the following three cases, we will show that $E(\pi_M^*) > 0$ and that we can have both $\Delta\pi_{M1}^* > 0$ and $\Delta E(\pi_{M2}^*) > 0$.

(1) if $0 \le \phi \le \phi_a$, $\Delta\pi_{M1}^* = \pi_{M1}^* - \pi_{M1}^{*NF} = \frac{880\left(107417 + 67240\sqrt{22}\right) - 41\left(772673 + 45592\sqrt{22}\right)\phi}{11296320\left(22 + 41\sqrt{22}\right)} - \frac{1}{8} > 0$, $\Delta E(\pi_{M2}^*) = E(\pi_{M2}^*) - E(\pi_{M2}^{*NF}) = \frac{235445\phi}{1183424} + \frac{220(1-\phi)}{1681} - (\frac{1-\phi}{8} + \frac{25\,\phi}{128}) > 0$ and the manufacturer's total equilibrium profit will increase, $\Delta E(\pi_M^*) = E(\pi_M^*) - E(\pi_M^{*NF}) > 0$. That is, $E(\pi_M^*) > E(\pi_M^{*NF})$.

(2) if $\phi_a \le \phi \le \phi_b$, $\Delta\pi_{M1}^* = \pi_{M1}^* - \pi_{M1}^{*NF} = \frac{429}{1681} - \frac{177\,\phi}{1312} - \frac{1}{8} > 0$, $\Delta E(\pi_{M2}^*) = E(\pi_{M2}^*) - E(\pi_{M2}^{*NF}) = \frac{235445\phi}{1183424} + \frac{220(1-\phi)}{1681} - (\frac{1-\phi}{8} + \frac{25\,\phi}{128}) > 0$ and the manufacturer's total equilibrium profit will increase, i.e., $\Delta E(\pi_M^*) = E(\pi_M^*) - E(\pi_M^{*NF}) > 0$.

(3) if $\phi_b < \phi \le 1$, $\Delta\pi_{M1}^* = \pi_{M1}^* - \pi_{M1}^{*NF} = \frac{(3520 - 1841\phi)(5913600 - 5721920\,\phi + 2948187\,\phi^2)}{56320\,(920 - 41\,\phi)^2} - \frac{1}{8}$. We can show that $\Delta\pi_{M1}^* > 0$ if $\phi_b < \phi < 0.96738$ and $\Delta\pi_{M1}^* < 0$ otherwise. $\Delta E(\pi_{M2}^*) = E(\pi_{M2}^*) - E(\pi_{M2}^{*NF}) = \frac{235445\phi}{1183424} + \frac{220(1-\phi)}{1681} - (\frac{1-\phi}{8} + \frac{25\,\phi}{128}) > 0$. The manufacturer's total equilibrium profit will increase, i.e., $\Delta E(\pi_M^*) = E(\pi_M^*) - E(\pi_M^{*NF}) > 0$.

In sum, we have $E(\pi_M^*) > E(\pi_M^{*NF})$ for all values of $\phi \in [0,1]$.

Second, we can show that the retailer can also become better off when some consumers have fairness concerns, leading to a win-win outcome for both the manufacturer and the retailer. Specifically, consider

$\phi_b < \phi < 1$. In this parameter range, we have $\Delta E(\pi_R^*) = E(\pi_R^*) - E(\pi_R^{*NF}) = \frac{1982464000 + 27610956800\,\phi - 21061710560\phi^2 + 5439067503\phi^3}{112640\,(920-41\phi)^2} - (\frac{1}{8} + \frac{9\,\phi}{256})$. One can show that if $0.923 < \phi < 1$, then $\Delta E(\pi_R^*) > 0$. Hence, consumers' fairness concerns can benefit both the manufacturer and the retailer even in the presence of demand uncertainty.

Part V-II. Analysis of demand uncertainty when demand may decrease

In this subsection, we explore the scenario where second-period demand will increase by $\delta > 0$ with probability ψ and may decrease by δ with probability $1-\psi$.[11] Other aspects of the model are the same as in the first scenario that we described above.

V-II (1). Benchmark with No Fairness Concerns

Let us start the analysis with a benchmark model without fairness concerns. In the first period, the aggregate market demand is given by $D_1^{NF} = 1 - \beta p_1$; in the second period, the potential demand change can be either positive or negative. With probability $\psi \in [0,1]$ the second-period demand will increase by δ, and the market demand will be given by $D_2^{NF}|_\delta = 1 + \delta - \beta p_2$, where $\delta > 0$ represents a positive demand change in the second period. Similarly, with probability $1-\psi$, the second-period demand will decrease, in which case market demand will be given by $D_2^{NF}|_{-\delta} = 1 - \delta - \beta p_2$. The manufacturer's and the retailer's total expected profits can be written as $E(\pi_M^{NF}) = E(\pi_{M1}^{NF} + \pi_{M2}^{NF})$ and $E(\pi_R^{NF}) = E(\pi_{R1}^{NF} + \pi_{R2}^{NF})$. Note that π_{Mi}^{NF} and π_{Ri}^{NF} $(i \in \{1,2\})$ are the same as previously defined in Section 3 of the main paper, and the manufacturer's and the retailer's second-period profit π_{M2}^{NF} and π_{R2}^{NF} depend on the state of the second-period demand increase. More specifically, the expressions for $E(\pi_M^{NF})$ and $E(\pi_R^{NF})$ are given by: $E(\pi_M^{NF}) = \pi_{M1}^{NF} + \psi\ \cdot \pi_{M2}^{NF}|_\delta + (1-\psi)\cdot\pi_{M2}^{NF}|_{-\delta}$ and $E(\pi_R^{NF}) = \pi_{R1}^{NF} + \psi\ \cdot \pi_{R2}^{NF}|_\delta + (1-\psi)\cdot\pi_{R2}^{NF}|_{-\delta}$, respectively.

[11] In this model, we assume $\delta < 1$, so that if the marked demand in the second period is in low state, the demand intercept is still positive in the second period.

One can readily find the equilibrium outcome using backward induction. Since the analysis is very similar to that in our core model, we omit the technical details. We find that, in equilibrium, the manufacturer's and the retailer's first-period wholesale and retail prices are given by $w_1^{*NF} = \frac{1}{2\beta}$ and $p_1^{*NF} = \frac{3}{4\beta}$, respectively. Their second-period prices are $w_2^{*NF}|_\delta = \frac{1+\delta}{2\beta}$, $p_2^{*NF}|_\delta = \frac{3(1+\delta)}{4\beta}$ when the second-period demand increases, and $w_2^{*NF}|_{-\delta} = \frac{1-\delta}{2\beta}$, $p_2^{*NF}|_{-\delta} = \frac{3(1-\delta)}{4\beta}$ when the second-period demand decreases. The manufacturer's and the retailer's total expected equilibrium profits are given by $E(\pi_M^{*NF}) = \frac{2+\delta(\delta-2+4\psi)}{8\beta}$ and $E(\pi_R^{*NF}) = \frac{2+\delta(\delta-2+4\psi)}{16\beta}$, respectively.

V-II (2). Analysis with Fairness Concerns

We solve the game by backward induction. That is, we will first find the second-period subgame equilibrium outcome. Then, we will solve for the first-period equilibrium.

Derivation of the second-period subgame perfect equilibrium.

In the second period, given the first-period retail price p_1 and the second-period demand realization (i.e., δ or $-\delta$), the manufacturer chooses its second-period wholesale price w_2, followed by the retailer choosing its second-period retail price p_2. Since the firms observe δ or $-\delta$ when making their second-period decisions, the second-period analysis is identical to the one in our core model. Specifically, one can show that when the second-period demand increases, the manufacturer's and the retailer's second-period subgame equilibrium prices are given by:

$$w_2^*(p_1)|_\delta = \begin{cases} \frac{1+\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ \frac{2p_1\beta-1-\delta+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \rho_a \le p_1 \le \rho_b, \\ \frac{1+\delta}{2\beta} & \text{if } p_1 > \rho_b \end{cases}$$

$$p_2^*(p_1)|_\delta = \begin{cases} \frac{3(1+\delta+p_1\gamma\lambda)}{4(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ p_1 & \text{if } \rho_a \le p_1 \le \rho_b, \\ \frac{3(1+\delta)}{4\beta} & \text{if } p_1 > \rho_b \end{cases}$$

where $\rho_a = \frac{3+3\delta}{4\beta+\gamma\lambda}$ and $\rho_b = \frac{(1+\delta)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. Using $w_2^*(p_1)|_\delta$ and $p_2^*(p_1)|_\delta$, we readily find the manufacturer's and retailer's second-period subgame equilibrium profits when the second-period demand increases,

$$\pi_{M2}^*|_\delta = \begin{cases} \frac{(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ \frac{(1-p_1\beta+\delta)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } \rho_a \le p_1 \le \rho_b, \\ \frac{(1+\delta)^2}{8\beta} & \text{if } p_1 > \rho_b \end{cases}$$

$$\pi_{R2}^*|_\delta = \begin{cases} \frac{(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a \\ \frac{(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda} & \text{if } \rho_a \le p_1 \le \rho_b\ . \\ \frac{(1+\delta)^2}{16\beta} & \text{if } p_1 > \rho_b \end{cases}$$

Similarly, if the second-period demand decreases (in "$-\delta$" case), then one can show that the manufacturer's and retailer's second-period subgame equilibrium prices are as follows.

$$w_2^*(p_1)|_{-\delta} = \begin{cases} \frac{1-\delta+p_1\gamma\lambda}{2(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a' \\ \frac{2p_1\beta-(1-\delta)+p_1\gamma\lambda}{\beta+\gamma\lambda} & \text{if } \rho_a' \le p_1 \le \rho_b', \\ \frac{1-\delta}{2\beta} & \text{if } p_1 > \rho_b' \end{cases}$$

$$p_2^*(p_1)|_{-\delta} = \begin{cases} \frac{3(1-\delta+p_1\gamma\lambda)}{4(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a' \\ p_1 & \text{if } \rho_a' \le p_1 \le \rho_b', \\ \frac{3(1-\delta)}{4\beta} & \text{if } p_1 > \rho_b' \end{cases}$$

where $\rho_a' = \frac{3(1-\delta)}{4\beta+\gamma\lambda}$ and $\rho_b' = \frac{(1-\delta)(\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}+6\beta+2\gamma\lambda)}{4\beta(2\beta+\gamma\lambda)}$. Using $w_2^*(p_1)|_{-\delta}$ and $p_2^*(p_1)|_{-\delta}$, we readily find the manufacturer's and retailer's second-period subgame equilibrium profits when the second-period demand decreases,

$$\pi_{M2}^*|_{-\delta} = \begin{cases} \frac{(1-\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a' \\ \frac{(1-\delta-p_1\beta)(2p_1\beta-1+\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } \rho_a' \le p_1 \le \rho_b', \\ \frac{(1-\delta)^2}{8\beta} & \text{if } p_1 > \rho_b' \end{cases}$$

$$\pi_{R2}^*|_{-\delta} = \begin{cases} \frac{(1-\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } 0 \le p_1 \le \rho_a' \\ \frac{(1-p_1\beta-\delta)^2}{\beta+\gamma\lambda} & \text{if } \rho_a' \le p_1 \le \rho_b' \\ \frac{(1-\delta)^2}{16\beta} & \text{if } p_1 > \rho_b' \end{cases}.$$

For future reference, note that the manufacturer's wholesale price w_2^* jumps down at ρ_b' . The manufacturer's second-period profit is continuous at ρ_b' and the retailer's profit jumps up at ρ_b'.

Derivation of the first-period equilibrium.

We proceed to solving for the first-period equilibrium. In the first period, the manufacturer and the retailer do not know the realization of the second-period market demand change, δ or $-\delta$, but they know its probability distribution. Hence, the manufacturer and the retailer make their first-period pricing decisions to maximize the sum of their first-period and expected second-period profits. The manufacturer's and retailer's second-period expected profits are given by $E(\pi_{M2}) = \psi\, \pi_{M2}^*|_\delta + (1-\psi)\, \pi_{M2}^*|_{-\delta}$ and $E(\pi_{R2}) = \psi\, \pi_{R2}^*|_\delta + (1-\psi)\, \pi_{R2}^*|_{-\delta}$, respectively.

Define $\delta_{01} = \frac{\gamma^3\lambda^3+16\beta^2(\gamma\lambda+3\sqrt{2\gamma\lambda(\beta+\gamma\lambda)})+\beta\gamma\lambda(17\gamma\lambda+12)\sqrt{2\gamma\lambda(\beta+\gamma\lambda)}}{576\beta^3+328\beta^2\gamma\lambda+41\beta\gamma^2\lambda^2+\gamma^3\lambda^3}$. Then, $E(\pi_{M2})$ and $E(\pi_{R2})$ are as follows.

(1) if $\delta \le \delta_{01}$, then

$$E(\pi_{M2}) = \begin{cases} \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho_a'] \\ \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta-p_1\beta)(2p_1\beta-1+\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a', \rho_a] \\ \frac{\psi(1-p_1\beta+\delta)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} + \frac{(1-\psi)(1-\delta-p_1\beta)(2p_1\beta-1+\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a, \rho_b'] \\ \frac{\psi(1-p_1\beta+\delta)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} + \frac{(1-\psi)(1-\delta)^2}{8\beta} & \text{if } p_1 \in [\rho_b', \rho_b] \\ \frac{\psi(1+\delta)^2}{8\beta} + \frac{(1-\psi)(1-\delta)^2}{8\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}] \end{cases},$$

$$E(\pi_{R2}) = \begin{cases} \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho'_a] \\ \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-p_1\beta-\delta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho'_a, \rho_a] \\ \frac{\psi(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda} + \frac{(1-\psi)(1-p_1\beta-\delta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho_a, \rho'_b] \\ \frac{\psi(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda} + \frac{(1-\psi)(1-\delta)^2}{16\beta} & \text{if } p_1 \in [\rho'_b, \rho_b] \\ \frac{\psi(1+\delta)^2}{16\beta} + \frac{(1-\psi)(1-\delta)^2}{16\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}]. \end{cases} .$$

(2) if $\delta > \delta_{01}$, then

$$E(\pi_{M2}) = \begin{cases} \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho'_a] \\ \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta-p_1\beta)(2p_1\beta-1+\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho'_a, \rho'_b] \\ \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{8(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta)^2}{8\beta} & \text{if } p_1 \in [\rho'_b, \rho_a] \\ \frac{\psi(1-p_1\beta+\delta)(2p_1\beta-1-\delta+p_1\gamma\lambda)}{\beta+\gamma\lambda} + \frac{(1-\psi)(1-\delta)^2}{8\beta} & \text{if } p_1 \in [\rho_a, \rho_b] \\ \frac{\psi(1+\delta)^2}{8\beta} + \frac{(1-\psi)(1-\delta)^2}{8\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}] \end{cases} ,$$

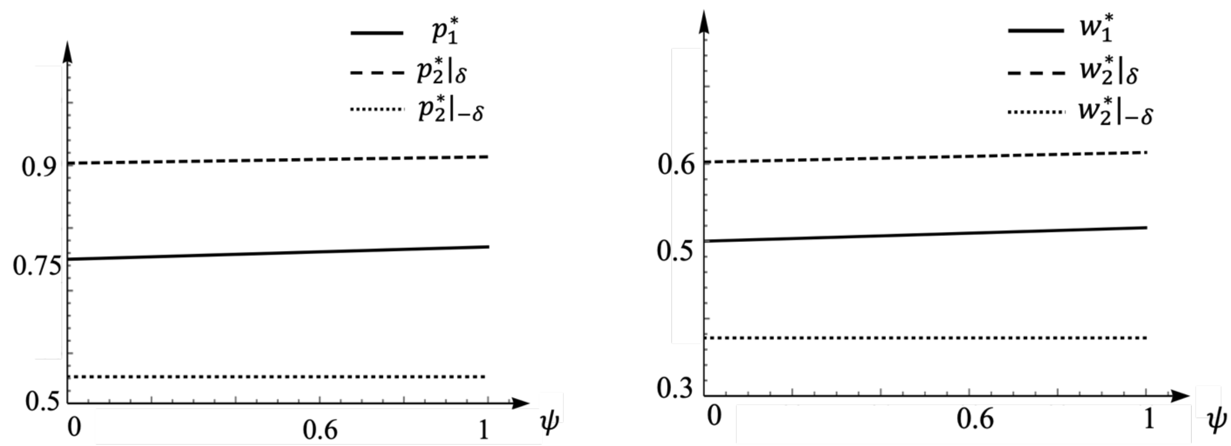
$$E(\pi_{R2}) = \begin{cases} \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} & \text{if } p_1 \in [0, \rho'_a] \\ \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-p_1\beta-\delta)^2}{\beta+\gamma\lambda} & \text{if } p_1 \in [\rho'_a, \rho'_b] \\ \frac{\psi(1+\delta+p_1\gamma\lambda)^2}{16(\beta+\gamma\lambda)} + \frac{(1-\psi)(1-\delta)^2}{16\beta} & \text{if } p_1 \in [\rho'_b, \rho_a] \\ \frac{\psi(1-p_1\beta+\delta)^2}{\beta+\gamma\lambda} + \frac{(1-\psi)(1-\delta)^2}{16\beta} & \text{if } p_1 \in [\rho_a, \rho_b] \\ \frac{\psi(1+\delta)^2}{16\beta} + \frac{(1-\psi)(1-\delta)^2}{16\beta} & \text{if } p_1 \in (\rho_b, \frac{1+\delta}{\beta}] \end{cases} .$$

Given w_1, the retailer chooses its first-period price p_1 to maximize its total expected profit $\pi_R = (p_1 - w_1)\, D_1 + E(\pi_{R2})$, where $D_1 = \begin{cases} 1 - \beta p_1 & \text{if } 0 < p_1 < \frac{1}{\beta} \\ 0 & \text{if } p_1 \geq \frac{1}{\beta}. \end{cases}$

Let p_1^* denote the retailer's best-response price, i.e., $p_1^* \equiv \underset{p_1}{\operatorname{argmax}}\, \pi_R$. We will obtain the optimal first-period retail prices within each of the following intervals $K_1 \equiv [0, \rho'_a]$, $K_2 \equiv [\rho'_a, \min\{\rho_a, \rho'_b\}]$, $K_{3a} \equiv [\rho_a, \rho'_b]$, $K_{3b} \equiv [\rho'_b, \rho_a]$, $K_4 \equiv [\max\{\rho_a, \rho'_b\}, \rho_b]$ and $K_5 \equiv [\rho_b, \frac{1+\delta}{\beta}]$. Note that, when $\delta \leq \delta_{01}$, we have $\rho_a \leq \rho'_b$, and $\rho_a > \rho'_b$ otherwise. After finding the locally optimal prices in each of these intervals, the price that yields the highest total profit is the retailer's globally optimal first-period retail price in response to w_1. Denote the retailer's optimal first-period price within the interval K_i by $p_1^{K_i}$, $i \in \{1, \dots, 5\}$. Note that if $p_1 \geq \frac{1}{\beta}$, then there will be no sales in the first period (i.e., $D_1 = 0$), so the retailer's total profit π_R equals to its second-period expected profit $E(\pi_{R2}^*)$. Also note that $E(\pi_{R2}^*)$ is piecewise concave on each interval K_i. Hence, the local maximizer in the interval K_i is either in the interior of K_i (and hence satisfies the first-order condition) or it is at one of the endpoints of K_i.

The analysis of this model turned out to be algebraically much more difficult. Specifically, while we can obtain closed-form solutions for local optima, finding the globally optimal price is algebraically non-tractable due to cumbersome profit expressions corresponding to some local optima. Hence, in this section we provide a numerical example to show that our main results will qualitatively hold. Suppose that $\beta = 1$, $\gamma = \frac{1}{5}$, $\lambda = \frac{1}{2}$ and $\delta = \frac{1}{4}$. Then, we find that the manufacturer's and the retailer's first-period equilibrium prices are given by: $w_1^* = \frac{-3097600 - 42240\,\psi - 27\,\psi^2}{7040\,(-880 + \psi)}$ and $p_1^* = \frac{5280 + 73\,\psi}{7040 - 8\,\psi}$. One can show that, in this special case, both w_1^* and p_1^* increase with the probability ψ. From expressions of w_1^* and p_1^*, we can obtain the firms' second-period equilibrium prices. When the second-period demand increases, $w_2^*|_\delta = \frac{-93280 + 27\,\psi}{176\,(-880 + \psi)}$ and $p_2^*|_\delta = \frac{3\,(-93280 + 27\,\psi)}{352\,(-880 + \psi)}$; when the second-period demand decreases: $w_2^*|_{-\delta} = \frac{3}{8}$ and $p_2^*|_{-\delta} = \frac{9}{16}$. Using these prices, one can derive the manufacturer's and the retailer's equilibrium profits. The firms' first-period equilibrium profits are given by: $\pi_{M1}^* = \frac{(1760 - 81\,\psi)(3097600 + 42240\,\psi + 27\,\psi^2)}{56320\,(-880 + \psi)^2}$, $\pi_{R1}^* = \frac{(1760 + 27\,\psi)(1760 - 81\,\psi)}{56320\,(880 - \psi)}$, respectively. The manufacturer's and the retailer's second-period expected equilibrium profits are given by: $E(\pi_{M2}^*) = \frac{3066624000 + 5627564800\,\psi + 1936440\,\psi^2 - 3231\,\psi^3}{56320\,(-880 + \psi)^2}$, $E(\pi_{R2}^*) = \frac{3066624000 + 5627564800\,\psi + 1936440\,\psi^2 - 3231\,\psi^3}{112640\,(-880 + \psi)^2}$, respectively. Their total expected equilibrium profits are given by $E(\pi_M^*) = \frac{4840000 + 3102660\,\psi + 2709\,\psi^2}{28160\,(880 - \psi)}$, $E(\pi_R^*) = \frac{8518400000 + 5454099200\,\psi - 1722600\,\psi^2 + 1143\,\psi^3}{112640\,(-880 + \psi)^2}$, respectively. Figure AV-1 illustrates the effect of ψ on equilibrium retail and wholesale prices. We show that the manufacturer's and retailer's prices tend to increase in ψ.

Figure AV-1 Effect of Uncertainty on Wholesale and Retail Prices[12]



a. Effect of ψ on retail prices

b. Effect of ψ on wholesale prices

Next, we demonstrate that our main qualitative results about profit implications of fairness concerns continue to hold even in the presence of demand uncertainty about δ.

In the benchmark case without consumer fairness concerns, the manufacturer's and the retailer's first-period wholesale and retail prices are $w_1^{*NF} = \frac{1}{2}$ and $p_1^{*NF} = \frac{3}{4}$, respectively. The firms' second-period prices are $w_2^{*NF}|_\delta = \frac{5}{8}$, $p_2^{*NF}|_\delta = \frac{15}{16}$ when the second-period demand increases, and $w_2^{*NF}|_{-\delta} = \frac{3}{8}$, $p_2^{*NF}|_{-\delta} = \frac{9}{16}$ when the second-period demand decreases. The manufacturer's and the retailer's total expected equilibrium profits are given by $E(\pi_M^{*NF}) = \frac{25}{128} + \frac{\psi}{8}$ and $E(\pi_R^{*NF}) = \frac{25 + 16\,\psi}{256}$, respectively.

First, we show that the existence of consumers with fairness concerns can increase the manufacturer's total profits. Let $\Delta E(\pi_M^*) = E(\pi_M^*) - E(\pi_M^{*NF})$. One can show that, $\Delta E(\pi_M^*) = E(\pi_M^*) - E(\pi_M^{*NF}) = \frac{4840000 + 3102660\,\psi + 2709\,\psi^2}{28160\,(880 - \psi)} - (\frac{25}{128} + \frac{\psi}{8}) \geq 0$, with strict inequality when $\psi > 0$. That it, $E(\pi_M^*) > E(\pi_M^{*NF})$. Second, we show that the retailer can also become better off when some consumers have fairness concerns, leading to a win-win outcome for both the manufacturer and the retailer. Let $\Delta E(\pi_R^*) = E(\pi_R^*) -$

[12] Figure AV-1 is illustrated using $\beta = 1$, $\gamma = \frac{1}{5}$, $\lambda = \frac{1}{2}$ and $\delta = \frac{1}{4}$.

$E(\pi_R^{*NF})$. One can show that, $\Delta E(\pi_R^*) = E(\pi_R^*) - E(\pi_R^{*NF}) = \frac{8518400000 + 5454099200\,\psi - 1722600\,\psi^2 + 1143\,\psi^3}{112640\,(-880+\psi)^2} - \frac{25+16\,\psi}{256} \geq 0$, with strict inequality when $\psi > 0$. That is, $E(\pi_R^*) > E(\pi_R^{*NF})$. Hence, our numerical analysis shows that as long as there is some positive chance that demand will increase in the second period, consumers' fairness concerns can make both the manufacturer and the retailer better off.