Mechanics 1 - Summary notes

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Chapter 1 - Mathematical models in mechanics

Mechanics has some very specific terminology:

- **Particle** A body that is treated as a single point. Has a mass but no dimensions.
- **Bead** A particle with a hole in the middle so it can be threaded onto a string or wire.
- **Lamina** A flat object with an area, but no height.
- **Uniform lamina** The equal areas of the lamina have equal masses. The center of mass of the lamina is the center of the area.
- **Rigid body** An object that does not change it's shape when it collides with other objects.
- **Wire** A smooth or rough thread that beads can be threaded onto.
- **Rod** An object with no width of breadth, only length. It's mass is concentrated along it's length.
- **Uniform rod** A rod in which equal lengths have equal masses.
- **Light object** An object with no mass. Can be a rod, string etc.
- **Inextensible string** A string that will not stretch under tension and so is always the same length.
- **Smooth surface** A surface that has no frictional resistance.
- **Rough surface** A surface that offers frictional resistance to objects that are moving upon it.
- **Smooth pulley** A pulley without any friction in it's bearings.
- **Peg** A support from which another object may be hung or rest upon. Can be treated as smooth or rough. If the peg is smooth, then only contact

forces will be acting on it, but if it is rough, then frictional forces will be present too.

- **Plane surface** A surface that is completely flat.
- **Earth's surface** Is treated as a plane surface.

Chapter 2 - Vectors and their application in mechanics

Scalar quantities are completely specified by their magnitude, vector quantities require a direction too.

The notation for a vector quantity is either to write it in bold font ($\mathbf{v}$), or to put an arrow over it ($\overrightarrow{AB}$).

The **i** and **j** notation can be used to represent vectors, where **i** is the horizontal unit and **j** is the vertical unit. The magnitude of a vector written in **i** and **j** notation is found using $\sqrt{i^2 + j^2}$.

In order to split a vector into **i** and **j** components, you must resolve it.

In order to resolve a vector $\mathbf{v}$, the **i** component will be $cos(\theta)\mathbf{v}$ and the **j** component will be $sin(\theta)\mathbf{v}$, where θ is the angle betwen the $\mathbf{v}$ and the horizontal. In other words:

> The component of a force is the product of the magnitude of the force and the cosine of the angle between the force and the required direction.

A position vector describes the position of an object relative to an other vector. Velocities can be relative to another object too. If particle A has a position/velocity **a** and particle B has a position/velocity **b** then the position vector of B relative to A/velocity of B relative to A is **b-a**.

The velocity of an object can be described as a vector. The magnitude of that vector is the speed of the particle.

The velocity of an object is the differential of the displacement of the object ($\frac{dr}{dt} = v$). If the velocity is constant, then $displacement = velocity \times time$.

The acceleration of an object is the differential of the velocity of the object ($\frac{dv}{dt} = a$). If the acceleration is constant, then $a = \frac{\Delta v}{t}$.

Chapter 3 - Kinematics of a particle

The four *suvat* equations are:

$$v = u + at$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Where:

u = initial velocity
v = final velocity
a = acceleration
t = time taken
s = distance

$\frac{v+u}{2}$ will give the average velocity.

Retardation is negative acceleration. Particles that are moving freely upwards against gravity are retarding (since gravity is acting down, meaning the acceleration is negative).

Speed time graphs have the speed of an object on the y-axis and the time on the x-axis. This means that:

- The gradient of the line is the acceleration ($a = \frac{dv}{dt}$).
- The area under the graph is the distance travelled by the particle ($s = \frac{1}{2}(u+v)t$).

Often useful when working out the area under a graph, the area of a trapezium is $\frac{1}{2}h(a+b)$ where a is the length of the top edge, b is the length of the bottom edge and h is the height of the trapezium.

Chapter 4 - Statics of a particle

The resultant of two forces is the sum of the two forces. If you construct a vector shape with $n+1$ sides using the magnitudes and directions of the forces you are trying to find the resultant of, then the line required to complete the shape will be the direction and magnitude of the resultant force.

You can also use to cosine rule:

$$C^2 = A^2 + B^2 - (2 \times A \times B \times cos(c))$$

And the sine rule:

$$\frac{A}{sin(a)} = \frac{B}{sin(b)}$$

In order to find the length and angles in any shape you have constructed from these forces.

Another way to find the resultant of a number of forces is to resolve all the forces, and add up their components to get the resultant force.

If all of the forces acting on a particle are acting in the same plane, they are said to be *coplanar*.

A system of forces is said to be in equilibrium if all of their lines of action pass through a single point and their resultant is the zero vector.

A 'normal' force is produced when an object is resting on another object (Newton's third law).

Friction always acts opposite to motion, and μ is always between 0 and 1. The force of friction is sufficient to prevent motion until the $F = \mu R$, which is said to be the limiting value of friction.

Chapter 5 - Dynamics of a particle moving in a straight line or plane

Newton's three laws are as follows:

1. A particle will only accelerate if it is acted on by a resultant force.
2. The force F applied to a particle is proportional to the mass m of the particle and the acceleration produced.

$$F = ma$$

3. The forces between two bodies in contact are equal in magnitude, but opposite in direction.

3. AKA: Every action has an equal and opposite reaction.

The of gravity acting on an object is it's weight, which is equal to the product of the objects mass and the gravitational acceleration ($9.8ms^{-2}$).

$$weight = mg$$

If two particles are connected by a string, then the tension in the string is equal on both particles, and acts towards the center of the string. Pulleys work in the same way, but the tension acts towards the pulley.

The momentum of a particle of mass m that is moving with a velocity v is mv.

If a constant force F acts on a particle for a time t then the impulse will be equal to Ft.

The impulse on a particle is also equal to the change in momentum:

$$I = mv - mu$$

Impulse is measured in Newton seconds (Ns^{-1}).

Momentum is conserved for two particles experiencing a collision or jerk (providing no external forces are involved). For a collision between two particles with masses of m_1 and m_2 respectively:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

In other words, the momentum before the impact is equal to the momentum after the impact.

Chapter 6 - Moments

The moment of the force F about a point P is equal to the product of the magnitude of F and the perpendicular distance from the point P to the line of action of the force F.

If F is not acting perpendicular to the line between F and the pivot, then the component of F that is perpendicular to the line must be found (usually using $sin(\theta)F$).

The unit of moments is the Newton meter (Nm).

For a lamina to be in equlibrium, the following must apply:

- The resultant force in any direction must be zero.
- The algebraic sum of the moments about any point must be zero.

For any lamina in equlibrium, you can take moments about it in order to find missing masses or lengths.

If a lamina or rod is said to be uniform if it's mass is evenly distributed. The center of mass of such an object is it's midpoint.

If a lamina or rod is non-uniform, then it's center of mass is not situated at it's midpoint.

If an object is supported by one or more support, then you will be able to resolve vertically using both the weight of the object (and any objects resting on it) and the reaction from the pivots.

An object will begin tilting when the reaction force on a support has a zero magnitude.