Momentum and Newton's laws

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December 9, 2012

Newton's three laws

Learn these, sometimes you've got to state them on the exam.

Newton's first law

Definition: *An object will remain at rest or keep travelling at a constant velocity unless it is acted on by an external force.*
Another way of saying this is that the momentum of an object is constant unless an external force acts upon it.

Newton's second law

Definition: *The net force acting on an object is directly proportional to the rate of change of the linear momentum of that object. The net force and the change in momentum are in the same direction.*
This means that

$$net\ force \propto rate\ of\ change\ of\ momentum$$

Which can also be written as:

$$F = \frac{\Delta p}{\Delta t}$$

N.b. the Newton is defined so that there doesn't need to be a constant to turn the proportional sign into an equals sign.
Where:

Quantity	Abbreviation	Unit
Change in momentum	Δp	Kilogram meter per second ($kgms^{-1}$)
Change in time	Δt	Second (s)
Force	F	Newton (N)

This can also be defined as:
Alternate definition: *The net force on an object is equal to the rate of change of it's momentum. The next force and the change in momentum are in the same direction*

An extension of the second law

We can change Δp into $mv - mu$. Since m is constant, we can take it out as a factor like so:

$$F = m\frac{v-m}{\Delta t}$$

You may have noticed that we can then substitute the second half of the equation for acceleration (since acceleration is the rate of change of the speed):

$$F = ma$$

Newton's third law

This is the easy one! It's definition is as follows: *When two bodies interact, the forces that they exert on each other are equal and opposite.*
These two forces are always:
However, most people know it by this definition: **Alternate definition:** *Every action has an equal and opposite reaction.*

- Of the same type (e.g. magnetic)
- Are equal in magnitude
- Are opposite in direction
- Act on different bodies (usually the two bodies that are creating the force)

Impulse

Impulse is defined as the *rate of change of momentum* or $Impulse = F \times t$. The unit of impulse is the *Newton second (Ns)*.
You can find the impulse by working out the area of a force time graph (useful if the force isn't constant). Here's a force time graph with the area highlighted:

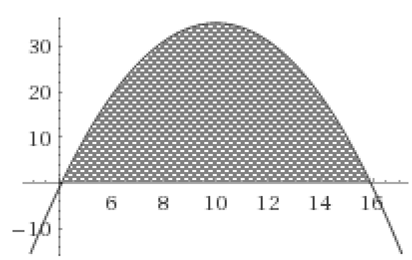

Figure 1: The area under a Force Time graph is the impulse

Derivation of the law of conservation of momentum from Newton's second and third laws

If we imagine two trolleys colliding. Both have the same speed and mass but are moving towards each other in opposite directions.

- The total momentum is therefore zero.
- When they collide, they decelerate at the same rate (but in opposite directions) and so there is two forces that are equal and opposite.
- The time of the collision is the same for each trolley, so the impulse for each trolley ($F\Delta t$) is equal in magnitude, but acts in opposite directions.
- The impulse in each trolley is the same as it's change in momentum.

We can conclude that if one trolley gains momentum in one direction, then the other gains an equal amount in an opposite direction and the total momentum in the system is the same.

Questions to practice

Question 1
A 1000kg spaceship is stationary in space. It's engines are activated for two minutes and provide a continuous force of 20N for this time, after which they deactivate.

1. First, find the velocity of the spaceship after it's engines have stopped.
2. Now calculate the acceleration of the spaceship and work done by the engines.

Answers

1.
$$F = \frac{\Delta p}{t}$$

So we can use this to find the momentum, and sub in $\Delta p = mv - mu$ to find the velocity.

$$20 = \frac{1000v - 0}{120}$$
$$\frac{20 \times 120}{1000} = v$$
$$v = \underline{2.4ms^{-1}}$$

2. Finding the acceleration is easy, just use $F = ma$:

$$20 = 1000a$$

$$a = \underline{0.020ms^{-2}}$$

The work done is equal to the change in kinetic energy. Since the kinetic energy was zero to begin with, we just need to find the final kinetic energy of the spaceship:

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2} \times 1000 \times 2.4^2$$

$$KE = \underline{2880Joules}$$