Circular motion

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What is circular motion?

Circular motion is a movement of an object along the circumference of a circle or rotation along a circular path. Some examples of circular motion include:

- Wheels of a car.
- Orbit of planets around the Sun.
- The hands of a clock.

Equations to remember

The equations in this topic aren't very hard to remember and there isn't many of them. However, the concepts required to understand them are sometimes more difficult to grasp.

- To convert between radians and degrees, use the following:

$$radians = \frac{2\pi}{360} \times degrees$$

- The time taken for an object to go around a circle once is called the time period. It's relation to the frequency of the motion is defined by:

$$f = \frac{1}{T}$$

- The centripetal force acting on an object of a given mass and tangential velocity moving in a circular motion with a given radius can be found using the equation:

$$F = \frac{mv^2}{r}$$

- The centripetal acceleration of an object with a given tangential velocity and moving in a circular motion with a given radius can be calculated by using:
$$a = \frac{v^2}{r}$$
N.b. This can also be used to find the force of gravity on an object (since you can find the acceleration due to gravity).

- To find the speed of an object travelling around a circle:
$$v = \frac{2\pi r}{t}$$
You might have noticed, this is just a slight variation of speed equals distance over time.

- The angular velocity of an object is equal to the angle rotated divided by time:
$$\omega = \frac{\theta}{t}$$

The quantities and their respective units are shown in this table:

Quantity	Abbreviation	Unit
Time period	T	Second (s)
Frequency	f	Hertz (Hz)
Force	F	Newton (N)
Mass	m	Kilogram (kg)
Radius	r	Metre (m)
Acceleration	a	Metres per second squared (ms^{-2})
Tangential velocity	v	Metres per second (ms^{-1})
Time	t	Seconds(s)
Angular velocity	ω	Radians per second(rad/s)
Angle	θ	Radians(rad)

Points to remember

- Objects moving in a circle aren't in equilibrium. They need an external force to act on them to keep them moving in a circle. (Think Newton's first law - an object remains in equilibrium (i.e. moving at a constant speed in a straight line) unless acted upon by another force).

- The centripetal force is an imaginary force we use to describe all the forces that add up together to make an object move in a circle. It's **not** an actual force.

- The centripetal force always acts towards the centre of the circle, and since the velocity of the object is a tangent on the circular path of the object, the centripetal force is also perpendicular to the velocity of the object.

- Objects travelling in a circular motion may (not always) have a steady speed, but their velocity is constantly changing. This is because velocity is a vector quantity - it has a magnitude and a direction while speed is a scalar quantity, having only a magnitude.
- The centripetal force (like all forces) has both a horizontal and vertical component. You can use this to explain situations where objects 'defy' gravity. For example, if you whirl a conker around your head, the vertical component of the tension in the string is equal to the weight of the conker.

Questions to practice

Question 1
Calculate the magnitude of the centripetal force that keeps the Earth in orbit around the Sun. Where:

Mass of the Earth	$6.0 \times 10^{24} kg$
Speed of the Earth in orbit	$30000 ms^{-1}$
Radius of the orbit of the Earth around the Sun	$1.5 \times 10^{11} m$

Answer
We can use the centripetal force equation to find the force like so:

$$F = \frac{6.0 \times 10^{24} \times 30000^2}{1.5 \times 10^{11}}$$

$$F = \underline{3.6 \times 10^{22} N}$$

Question 2
What provides the centripetal force that keeps the Earth in orbit?

Answer
The gravity of the Sun provides the centripetal force keeping Earth orbiting it.

Question 3
If you were to swing a bucket of water in vertical circle with a radius of 1m what is the minimum time the bucket could take to complete the circle without spilling water?

Answer
First, we must find the minimum velocity that the bucket can travel without having water fall out, then we can find how long it'll take for the bucket to travel the circumference of the circle at this speed.

We can use $F = \frac{mv^2}{r}$ along with$F = ma$ to find the minimum velocity that we can have the bucket travel at without spillage. Combining the two equations gives us:

$$\frac{mv^2}{r} = ma$$

The masses cancel out and we can sub in the the value for gravity as acceleration ($9.8ms^{-2}$) and the radius ($1ms^{-1}$) so $\frac{v^2}{1} = 9.8$.

Therefore

$$v = 3.13ms^{-1}$$

Now we have to find the time it takes for the the bucket to travel around the circumference of the circle, so we must first find it's circumference: $radius \times 2\pi$ Therefore $radius = 2\pi$

To find the minimum time, we must divide the circumference by the velocity:

$$\frac{2\pi}{3.13} = \underline{2.01s}$$

Question 4
Explain why an aeroplane will lose altitude when banking unless the pilot increases the throttle.

Answer
When the aeroplane is banking, the upwards lift = $\cos\theta\times$total lift. When $0 < \theta < 90$ (i.e. when the plane is banking) $cos\theta < 1$ so the pilot must increase the total lift to keep the same vertical lift.

Question 5
A lawnmower rotates at $3500rpm$. The blade has a radius of $0.23m$. Find the velocity and acceleration at the tip of the blade.

Answer
To solve this question we must first convert rpm into rps:

$$3500rpm = \frac{3500}{60}rps = 58.3rps$$

Then we can work out the time period of the blade.

$$t = \frac{1}{f} = \frac{1}{58.3} = 0.01714...s$$

Then we can sub it into the equation to find the speed of an object travelling around a circle:

$$v = \frac{2\pi r}{t} = \frac{2 \times \pi \times 0.23}{0.01714} = \underline{84.3m/s}$$

Now we can work out the acceleration:

$$a = \frac{v^2}{r} = \frac{84.3^2}{0.23} = \underline{30897ms^{-2}}$$

Question 6
A pendulum is swinging. It swings up to 90 degrees and back again every time. Find the tension in the string when the pendulum is at the bottom of it's swing. The length of the string is h.

Answer
The gravitational potential energy is equal to the kinetic energy, so we can find v in terms of the length of the string (since the height of the swing is the radius of the swing (since it swings to 90 degrees)):

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

Now we need to find the tension.

$$F = \frac{mv^2}{r}$$

Since the centripetal force at the bottom of the swing is equal to the tension minus the weight:

$$T - mg = \frac{mv^2}{h}$$

Now we can sub in out value for v:

$$T - mg = \frac{m(\sqrt{2gh})^2}{h} = \frac{2mgh}{h} = 2mg$$

$$T = \underline{3mg}$$