The set $\mathbb{N}$ *contains?*

1

The set $\mathbb{R}$ *contains?*

2

The set $\mathbb{Z}$ *contains?*

3

The set $\mathbb{Q}$ *contains?*

4

What is this?
$\emptyset$

5

What is this?
$\mathbb{S}$

6

What does $X \subseteq Y$ *mean?*

7

What does $'$ *mean after a set (or* ^{c}*)?*

8

What does $x \in X$ *mean?*

9

What does $x \notin X$ *mean?*

10

For each a *in* X, $a \in X \iff a \in Y$.
How is this represented?

11

How else could we express:
$X \subseteq Y \iff Y \subseteq X$

12

The set of real numbers (all finite and infinite decimal numbers).

2

The set of natural numbers (all non-negative integers).

1

The set of rational numbers.
Contains all m/n for $m, n \in \mathbb{Z}$

4

The set of integers.

3

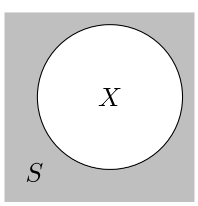
The universal set, containing all possible elements.

6

The null set.

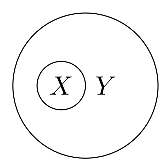
5

The complement of the set. E.g. X':



8

X is a subset of Y
Y is a superset of X
X is included in Y
Y includes X



7

x is not a member of X

10

x is contained in / is a member of X

9

$X = Y$

12

$X = Y$

11

What does $X \cup Y$ mean?

13

What does $X \cap Y$ mean?

14

The truth table for the `and` function is:

Input 1	*Input 2*	*Input 1 `and` Input 2*
T	*T*	
T	*F*	
F	*T*	
F	*F*	

15

The truth table for the `or` function is:

Input 1	*Input 2*	*Input 1 `or` Input 2*
T	*T*	
T	*F*	
F	*T*	
F	*F*	

16

The truth table for the `implies` function is:

Input 1	*Input 2*	*Input 1 `implies` Input 2*
T	*T*	
T	*F*	
F	*T*	
F	*F*	

17

An operation is *if:*

$$a1 \circledast a2 = a2 \circledast a1$$

18

An operation is *if:*

$$(a1 \circledast a2) \circledast a3 = a1 \circledast (a2 \circledast a3)$$

19

Is $(v + w + x)$ a valid expression in the formal language?

20

Is $(x + 4)$ a valid expression in the formal language?

21

Is $((x \times 0) + (y + z)))$ a valid expression in the formal language?

22

What expression does this parse tree represent?

$$\dfrac{x \quad \dfrac{y \quad z}{\cdot}\ (\times)}{\cdot}\ (-)$$

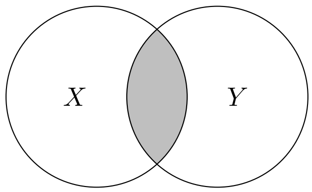
23

Evaluate the following parse tree

$$\dfrac{140 \quad \dfrac{10 \quad \dfrac{3 \quad 1}{\cdot}\ (\times)}{\cdot}\ (-)}{\cdot}\ (\div)$$

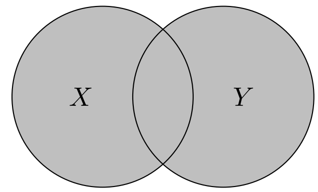
24

*The **intersection** of the sets X and Y.*



14

*The **union** of the sets X and Y.*



13

The truth table for the `or` function is:

Input 1	*Input 2*	*Input 1 `or` Input 2*
T	*T*	*T*
T	*F*	*T*
F	*T*	*T*
F	*F*	*F*

16

The truth table for the `and` function is:

Input 1	*Input 2*	*Input 1 `and` Input 2*
T	*T*	*T*
T	*F*	*F*
F	*T*	*F*
F	*F*	*F*

15

An operation is commutative if:

$$a1 \circledast a2 = a2 \circledast a1$$

18

The truth table for the `implies` function is:

Input 1	*Input 2*	*Input 1 `implies` Input 2*
T	*T*	*T*
T	*F*	*F*
F	*T*	*T*
F	*F*	*T*

17

No, there aren't enough brackets.
$((v + w) + x)$ would be valid though!

20

An operation is associative if:

$$(a1 \circledast a2) \circledast a3 = a1 \circledast (a2 \circledast a3)$$

19

No, since there are too many brackets.
$((x \times 0) + (y + z))$ would be valid though!

22

No, since 4 isn't an allowable atom.
$(x + 0)$ would be valid though!

21

$$(140 \div (10 - (3 \times 1))) = 20$$

$$\cfrac{140 \qquad \cfrac{10 \qquad \cfrac{3 \quad 1}{3}\ (\times)}{7}\ (-)}{20}\ (\div)$$

24

$$(x - (y \times z))$$

23

Use the fact that $\cup$ is associative to re-arrange:
$X \cup (Y \cup Z)$

25

Use the fact that $\cap$ is associative to re-arrange:
$X \cap (Y \cap Z)$

26

Use the distributive law on:
$X \cup (Y \cap Z)$

27

Use the distributive law on:
$X \cap (Y \cup Z)$

28

Use absorbsion on:
$X \cup (X \cap Y)$

29

Use absorbsion on:
$X \cap (X \cup Y)$

30

What three things happen when De Morgan's law is applied to an expression?

31

What does involution mean?

32

What is the symbol for logical negation?

33

What is the symbol for conjunction?

34

What is the symbol for disjunction?

35

What is the symbol for logical and?

36

$Y \cap (X \cap Z)$
or
$Z \cap (X \cap Y)$

26

$Y \cup (X \cup Z)$
or
$Z \cup (X \cup Y)$

25

$(X \cap Y) \cup (X \cap Z)$

28

$(X \cup Y) \cap (X \cup Z)$

27

X

30

X

29

If an expression is negated twice, they cancel each other out.
$X'' = X$

32

1. *The expression is negated (involution is applied if it's already negated)*
2. *Each sub expression is negated (again, applying involution)*
3. *Each union inside the expression is turned into an intersection and vice versa*

31

$\wedge$

34

$\neg$

33

$\wedge$

36

$\vee$

35

What is the symbol for logical or?

37

What is the symbol for implication?

38

What is the symbol for bi-implication?

39

The truth table for the `bi-implication` *function is:*

Input 1	*Input 2*	*Input 1* $\iff$ *Input 2*
T	T	
T	F	
F	T	
F	F	

40

An expression is a when all of its possible outcomes are true

41

An expression is when at least one of its possible outcomes are true

42

An expression is a when none of its possible outcomes are true

43

What is the notation to say A is a tautology?

44

What is the notation to say A is satisfiable?

45

What is the notation to say A is a contradiction?

46

Use the fact that $\vee$ is associative to re-arrange:
$X \vee (Y \vee Z)$

47

Use the fact that $\wedge$ is associative to re-arrange:
$X \wedge (Y \wedge Z)$

48

$\implies$

38

$\vee$

37

The truth table for the `bi-implication` *function is:*

Input 1	*Input 2*	*Input 1* $\iff$ *Input 2*
T	*T*	*T*
T	*F*	*F*
F	*T*	*F*
F	*F*	*T*

40

$\iff$

39

An expression is satisfiable when at least one of its possible outcomes are true

42

An expression is a tautology when all of its possible outcomes are true

41

$\models A$

44

An expression is a contradiction when none of its possible outcomes are true

43

$\not\models A$

46

$\not\models \neg A$

45

$Y \wedge (X \wedge Z)$
or
$Z \wedge (X \wedge Y)$

48

$Y \vee (X \vee Z)$
or
$Z \vee (X \vee Y)$

47

What are the two possible rearrangements of $A \implies B$*?* 49	*What is the rearrangement of* $\neg(A \implies B)$ 50
What is the rearrangement of $A \implies \neg B$? 51	*Rearrange* $A \iff B$ 52
Rearrange $A \iff B$ 53	*Rearrange* $A \iff B$ 54
Rearrange $\neg(A \iff B)$ 55	*Rearrange* $\neg(A \iff B)$ 56
What two conditions are there for Negation Normal Form? 57	*What two steps do we do to get an expression into Negation Normal Form?* 58
What three conditions are there for Conjunctive Normal Form? 59	*What two steps do we do to get an expression into Conjunctive Normal Form?* 60

$$A \wedge \neg B$$

50

$$\neg A \vee B$$
$$\neg B \implies \neg A$$

49

$$(A \implies B) \wedge (B \implies A)$$

52

$$B \implies \neg A$$

51

$$(A \wedge B) \vee (\neg A \wedge \neg B)$$

54

$$(\neg A \vee B) \wedge (\neg B \vee A)$$

53

$$\neg(A \wedge B) \wedge (A \vee B)$$

56

$$(A \wedge \neg B) \vee (B \wedge \neg A)$$

55

1. *Remove all implication and bi-implication operations by applying the logical identities*
2. *Apply De Morgan's laws to any expressions that are negated*

58

1. *The expression is build up of literals using only conjunction and disjunction*
2. *Negation can be used, but only on literals, not expressions*

N.b. a literal is a formula that is either atomic or the negation of an atomic formula (i.e. x or $\neg x$)

57

1. *Get rid of nested brackets using identities*
2. *Use the distributive identities to bring all the disjunctions inside the conjunctions.*

60

1. *The formula must be in NNF already*
2. *There must be no nested brackets*
3. *Conjunction must be used outside of the brackets, and disjunction inside the brackets*

59

What is the CNF test for tautologies? 61	*What three conditions are there for Disjunctive Normal Form?* 62
What is the DNF test for contradictions? 63	*What is the universal quantifier?* 64
What is the existential quantifier? 65	*What can we do to a universal quantifier with a negation such as this:* $\neg\forall x P(x)$ 66
What can we do to an existential quantifier with a negation such as this: $\neg\exists x P(x)$ 67	*What is the arity of a unary symbol?* 68
Is disjunction inclusive or exclusive? 69	*What does 'iff' mean?* 70
What does 'PL' stand for? 71	*What is a truth valuation?* 72

1. *The formula must be in NNF already*
2. *There must be no nested brackets*
3. *Disjunction must be used outside of the brackets, and conjunction inside the brackets*

62

Each expression in the formula must have both a literal and the negation of the literal.

E.g. $(p_1 \vee p_2 \vee \neg p_1) \wedge (p_3 \vee \neg p_2 \vee p_2)$

61

$\forall$

64

Each expression in the formula must have both a literal and the negation of the literal.

E.g. $(p_1 \wedge p_2 \wedge \neg p_1) \vee (p_3 \wedge \neg p_2 \wedge p_2)$

63

We can turn it into an existential quantifier, such as:

$$\exists x \neg P(x)$$

66

$\exists$

65

1

Arity - the number of arguments that a function can take

68

We can turn it into a universal quantifier, such as:

$$\forall x \neg P(x)$$

67

If and only if.

70

Inclusive.

69

A truth valuation is a list of values define the input values for an expression. E.g.:
$(x = T, y = F)$

72

Propositional Logic

71

If $A \iff B$ is a tautology, what does that mean?

73

How can we show that $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$?

74

How does $\mathbb{P}(A^c)$ relate to $\mathbb{P}(A)$?

75

If $B \subseteq A$ what is the probability of the set difference between A and B?

76

$\mathbb{P}(A \cup B) =$

77

$\mathbb{P}(A^c \cap B) =$

78

$\mathbb{P}(A \cap B^c) =$

79

Define a probability measure

80

What condition must be satisfied for two events to be disjoint?

81

What two conditions must be satisfied for a mapping to be a probability measure?

82

How do we find the probability of an event?

83

What does an indicator function do?

84

$$
\begin{aligned}
\mathbb{P}(A \cup B) &= \sum_{i=1}^{n} 1_{A \cup B}(\omega_i) p_i \\
&= \sum_{i=1}^{n} (1_A(\omega_i) + 1_B(\omega_i)) p_i \\
&= \sum_{i=1}^{n} (1_A(\omega_i)) p_i + \sum_{i=1}^{n} (1_A(\omega_i)) p_i \\
&= \mathbb{P}(A) + \mathbb{P}(B)
\end{aligned}
$$

74

A and B are logically equivalent.

73

$$\mathbb{P}(A\ B) = \mathbb{P}(A) - \mathbb{P}(B)$$

Also, $\mathbb{P}(B) \geq \mathbb{P}(A)$

76

$$\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$$

75

$$\mathbb{P}(A^c \cap B) = \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

78

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

77

A probability measure is a mapping between a collection of events and the probability that each event will occur.

80

$$\mathbb{P}(A \cap B^c) = \mathbb{P}(A) - \mathbb{P}(A \cap B)$$

79

The probability measure of the event Ω must equal 1. $\mathbb{P}(\Omega) = 1$
If A and B are disjoint, then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

82

The events must have no intersection: $A \cap B = \emptyset$

81

If an element is inside a set, then the indicator function returns 1, otherwise, it returns 0. The notation is $1_{SET}(ELEMENT)$.

84

Add up all the probability measures inside the event:

$$\sum_{i=1}^{n} 1_A(\omega_i) p_i$$

Where n is the number of elements inside Ω.

83

$\mathbb{P}(A^c \cap B) =$

85

How do we find the probability of a set in a sample space where all the outcomes are equally likely?

86

How do we find the binomial coefficient $\binom{n}{k}$*?*

87

How do we find the number of selections possible in a set when order is important and items can be replaced?

88

How do we find the number of selections possible in a set when order is important and items cannot be replaced?

89

What is the probability that an event A will occur if we know that an event B will occur?

90

How do we represent the probability of one event A occurring given that another event B will occur?

91

What is the definition of the independence of two events?

92

What are the two crucial properties of partitions?

93

If n events are mutually disjoint, then what is the sum of their probabilities?

94

What is the condition for the law of total probability to be true?

95

What is the formula for the law of total probability?

96

$$\mathbb{P}(A) = \frac{\#A}{\#\Omega}$$

86

$$\mathbb{P}(A^c \cap B) = \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

85

$$n^k$$

88

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

87

$$\frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

90

$$\frac{n!}{(n-k)!}$$

89

$$\mathbb{P}(A)\mathbb{P}(B) = \mathbb{P}(A \cap B)$$

92

$$\mathbb{P}(A|B) \left(= \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \right)$$

91

$$\mathbb{P}(A_0 \cup \cdots \cup A_n)$$

94

The pieces do not overlap and together they make up the whole sample space.

93

$$\mathbb{P}(A) = \mathbb{P}(A|E_1)\mathbb{P}(E_1) + \cdots + \mathbb{P}(A|E_n)\mathbb{P}(E_n)$$

96

The events must be mutually disjoint. (They often form a partition of the sample space)

95

Describe the law of total probability in English.

97

What is the definition of conditional probability?

98

What is the definition of the multiplicative law?

99

Define Bayes' theorem.

100

Define the term 'Random Variable'.

101

In order to construct a new random variable from an old one, what do you do?

102

Define the pmf function.

103

Define the cdf function.

104

What's the difference between a cdf and a pmf?

105

What does an open dot mean on a cmf graph? What does a filled dot mean?

106

How do we find the average value of a random variable?

107

How do we find the mean of a random variable that has been constructed from another random variable using a function f?

108

$$\mathbb{P}(E_i|A) = \frac{\mathbb{P}(E_i \cap A)}{\mathbb{P}(A)}$$

98

Take a partition of a sample space consisting of the events E_1 to E_n. For any event A, the probability of A is equal to the sum of the probability of A choose E_m multiplied by the probability of E_m for each m from 1 to n.

97

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

100

$$\mathbb{P}(E_i \cap A) = \mathbb{P}(A|E_i)\mathbb{P}(E_i)$$

99

Apply a function to the old random variable:

$$Y(\omega) = f(X(\omega))$$

102

A mapping from the sample space to real numbers.

101

$$F(x) = \mathbb{P}(X \leq x)$$

104

$$p(r_i) := \mathbb{P}(X = r_i)$$

103

An open circle shows where a value is **not**. *A filled dot shows what the value is at that point on the graph.*

106

The cdf is cumulative, and shows the total probability of the outcomes up to and including the outcome it takes. The pmf shows only the probability of that outcome.

105

$$\mathbb{E}(X) = \mathbb{P}(X = r_1)f(r_1) + \cdots + \mathbb{P}(X = r_n)f(r_n)$$

108

$$\mathbb{E}(X) = \mathbb{P}(X = r_1)r_1 + \cdots + \mathbb{P}(X = r_n)r_n$$

107

If we take the mean of a random variable in the form $\mathbb{E}(aX+b)$, then how can we re-arrange that? 109	*What can you do with $Var(aX+b)$?* 110
How do you find the standard deviation of a random variable? 111	*What is the variance of a random variable that contains one element?* 112
$Var(X) = ?$ 113	*How many outcomes does the Bernoulli distribution have, what is its range and its pmf?* 114
What is the mean and the variance of a function that has the Bernoulli distribution? 115	*What is the range of the binomial distribution? What is its pmf?* 116
What is the notation to say that a random variable X has a specific distribution? 117	*What is the pmf of a Poisson distribution?* 118
What is the mean and variance of a Poisson distribution? 119	*What is the pmf of the geometric distribution?* 120

$$Var(aX+b) = a^2 Var(X)$$

110

$$a\mathbb{E}(X)+b$$

109

0

112

$$SD(X) = \sqrt{Var(X)}$$

111

The Bernoulli distribution has two outcomes. Its range is $R = \{0,1\}$ and its pmf is given by $p_x(1) = p, p_x(0) = 1-p$.

114

$$Var(X) = \mathbb{E}(X-\mu)^2 = \sum_{i=1}^{m} (r_i - \mu)^2 \mathbb{P}(X = r_i)$$

113

The range is $R = \{0, 1, \ldots, n\}$. The pmf is:

$$p_x(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

116

$$\mathbb{E}(X) = p$$

$$Var(X) = p(1-p)$$

115

$$p_x(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

118

$$X\ Distribution(n_1, n_2, \ldots, n_i)$$

117

$$p_x(k) = (1-p)^{k-1} p$$

120

$$\mathbb{E}(X) = Var(X) = \lambda$$

119

What is the mean and the variance of the geometric distribution?

121

What is the formula for a geometric series?

122

What is the formula for the chain rule?

123

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

122

$$\mathbb{E}(X) = \frac{1}{p}$$

$$Var(X) = \frac{1-p}{p^2}$$

121

$$y = f(x)^n$$

$$\frac{\delta y}{\delta x} = n \cdot f(x)^{n-1} \cdot f'(x)$$

123