

Lagrange Multipliers: Removing some confusions related to some geometric interpretations.

Todian Mishtaku

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1 Introduction

Sometimes, when we deal with a complex task we have to recall some simple concepts, for which we may have the assumption that we knew everything about them until we come across some of their applications not necessarily complex but different from situations we have met priorly; and then surprisingly, what was easy and completely understandable by us starts to be associated with doubts.

The aim of this article is not to repeat lessons about Lagrange Multipliers, but to remove confusion caused by a missed step or wrongly illustrated concept, which I have come across often in many tutorials of both types of video and text, that creates an obstacle for people like me who do not get some conclusions for granted without having the proof or/and feeling its correctness.

As our process of understanding usually involves confusion, we do not have to blame ourselves for why we do not understand clearly the explanation. The lack of explanation clarity may be often caused because the authors assume acknowledgment of the problematic step for themselves and skip it while providing their explanation or because the authors themselves reproduced unconsciously that segment of the article from other sources.

As a result of such deficiencies in the explanation, some problems look extremely difficult before knowing how to tackle them. Fortunately, they look extremely easy after providing an understandable explanation. I hope this article will clear up some of the geometric-related confusion while grasping Lagrange Multipliers.

2 Some examples to get started

2.1 What is wrongly illustrated when explaining Lagrange Multipliers?

Please have a look at the following picture. While the authors mention the gradient, they associate the explanation with a hill and an arrow like in the picture below. In fact, this arrow shows the direction of the object/man walking up the hill to reach the top. But caution! This arrow does not correspond with the gradient which is in fact a 2-dimensional vector and not a 3-dimensional vector. We will show later what the gradient looks like.

The following illustrations are showing tangent lines (when we take derivatives of $z=f(x,y)$ with respect to y with fixed x) to the location represented by the blue point.

While the real vectors representing gradients for the corresponding situations are presented below. The gradient of a function of two arguments is a 2-dimensional vector parallel with xOy plane with the start at our location on the hill represented by the blue dot.

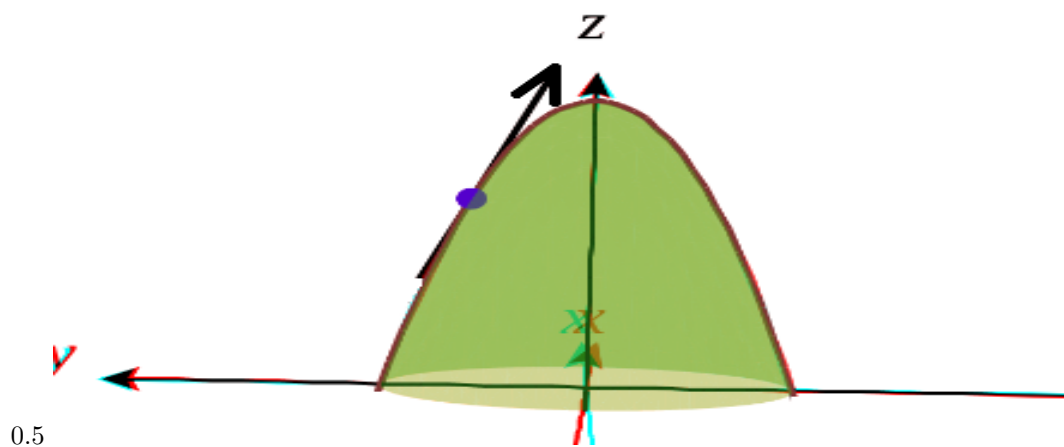


Figure 1: Caption1

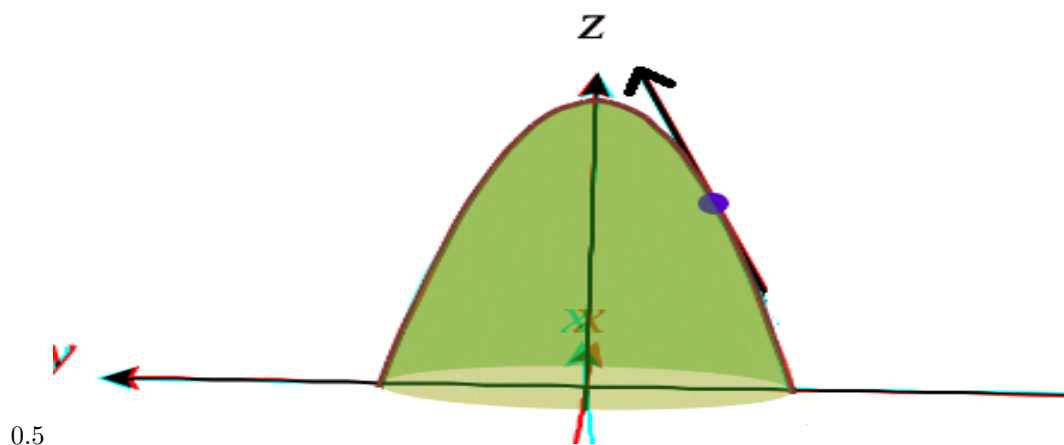


Figure 2: Caption 2

Figure 3: The black arrow is not the gradient. It just shows the direction to be followed while walking over a physical object (hill) in order to reach the top.

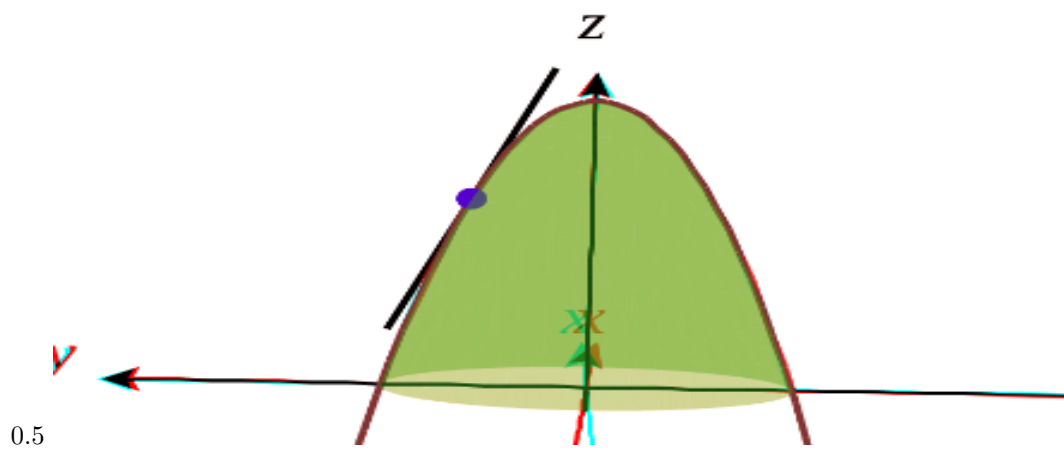


Figure 4: Caption1

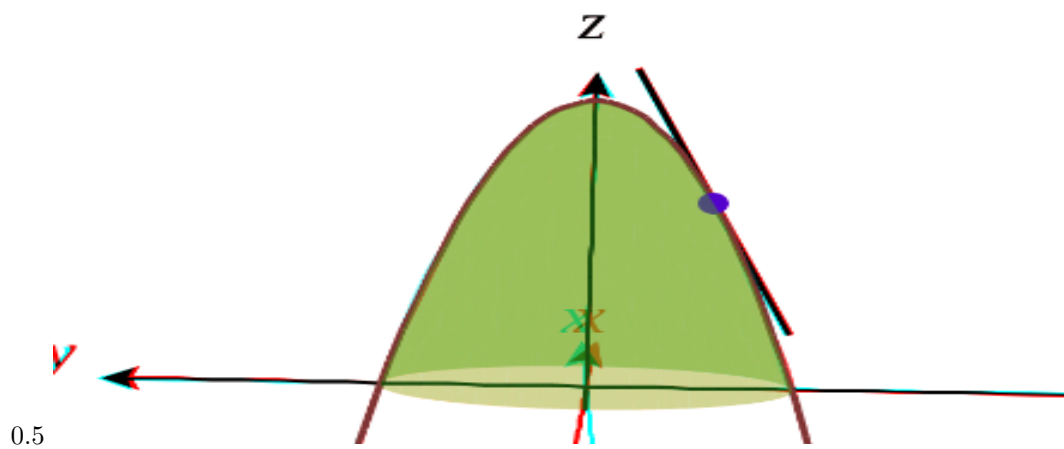


Figure 5: Caption 2

Figure 6: The following illustrations are showing tangent lines (when we take derivatives of $z=f(x,y)$ with respect to y with fixed x) to the location represented by the blue point.

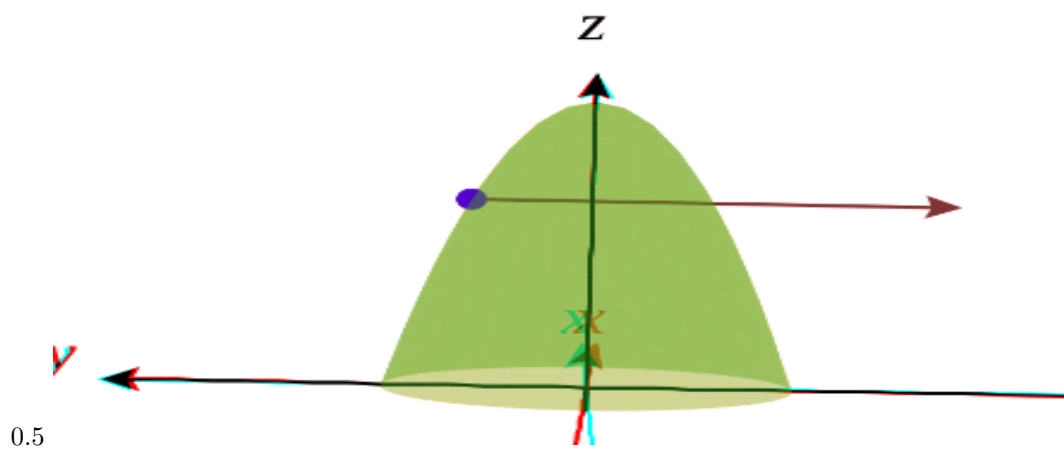


Figure 7: Caption1

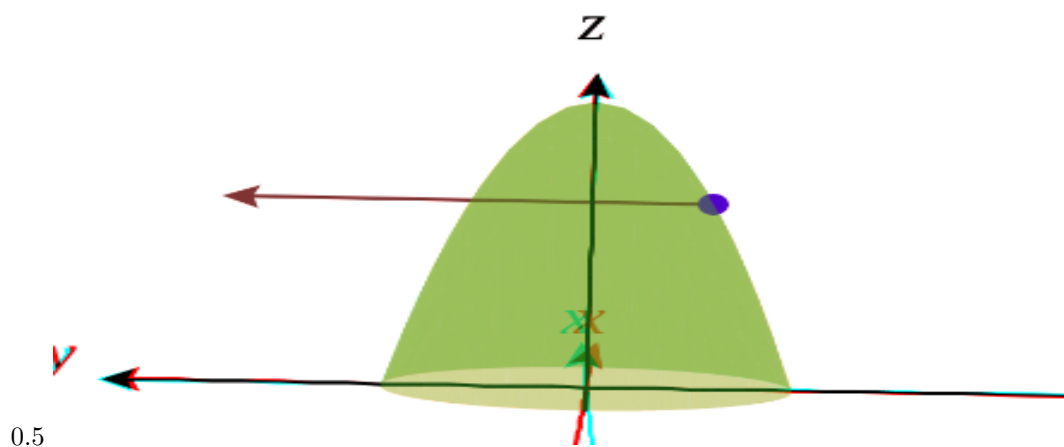


Figure 8: Caption 2

Figure 9: The black arrow represents the real gradient which is a 2-dimensional vector parallel with xOy plane with the start at our location on the hill represented by the blue dot.