(1) $\rho(x_m, x_{m+p}) \leq \rho(x_m, x_{m+1}) + \rho(x_{m+1}, x_{m+2}) + \dots + \rho(x_{m+p-2}, x_{m+p-1}) + \rho(x_{m+p-1}, x_{m+p})$

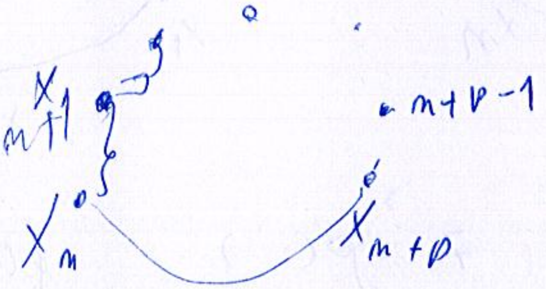
$$\left.\begin{aligned}
\rho(x_m, x_{m+1}) &\leq \alpha^{m-1}(x_1, x_2)\\
\rho(x_{m+1}, x_{m+2}) &\leq \alpha^{m}(x_1, x_2)\\
\rho(x_{m+2}, x_{m+3}) &\leq \alpha^{m+1}(x_1, x_2)\\
\rho(x_{m+p-2}, x_{m+p-1}) &\leq \alpha^{m+p-3}(x_1, x_2)\\
\rho(x_{m+p-1}, x_{m+p}) &\leq \alpha^{m+p-2}(x_1, x_2)
\end{aligned}\right\}$$

$\rho(x_{m+1}, x_{m+2}) \leq \alpha\, \rho(x_m, x_{m+1})$

$\underline{m-1} \longrightarrow \underline{m+p-2}$

$\cancel{(m+p-2)-(m-2)+1 =}$
$\cancel{m+p-3-m+2-1} =$
$= \underline{p-2}$

$m+p-2-(m-1)+1 =$
$m+p-2-m+\cancel{1}-\cancel{1} = \underline{\underline{(p-2)}}$



$\rho(x_m, x_{m-1}) \leq \rho(Tx_{m-1}, Tx_{m-2}) \leq \alpha\, \rho(x_{m-1}, x_{m-2})$

$\boxed{\rho(x_m, x_{m-1}) \overset{(1)}{\leq} \alpha\, \rho(x_{m-1}, x_{m-2})} \overset{(1)}{\leq} \alpha \cdot \alpha\, \rho(x_{m-2}, x_{m-3})$

$\leq \alpha \cdot \alpha \cdot \alpha\, \rho(x_{m-3}, x_{m-4})$

$\rho(x_m, x_{m-1}) \leq \alpha\, \rho(x_{m-1}, x_{m-2}) \leq \alpha^2 \rho(x_{m-2}, x_{m-3}) \leq$

$\alpha^3 \rho(x_{m-3}, x_{m-4})$

$$\boxed{\rho(x_m, x_{m-1}) \leq \alpha^{m-2} \rho(x_2, x_1)}$$

$$\boxed{\rho(x_{m+1}, x_m) \leq \alpha^{m-1} \rho(x_2, x_1)}$$

$(1) \Rightarrow \rho(x_m, x_{m+p}) \leq \alpha^{m-1}(x_2, x_1) + \alpha^{m}(x_2, x_1) + \dots + \alpha^{m+p-2} \rho(x_2, x_1)$

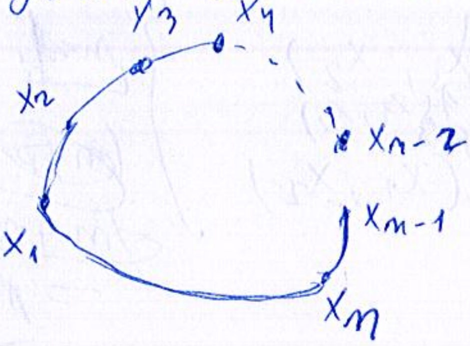
$\Rightarrow \alpha^{m-1}\underbrace{(1 + \alpha + \alpha^2 + \dots + \alpha^{p-1})}\, \rho(x_2, x_1)$

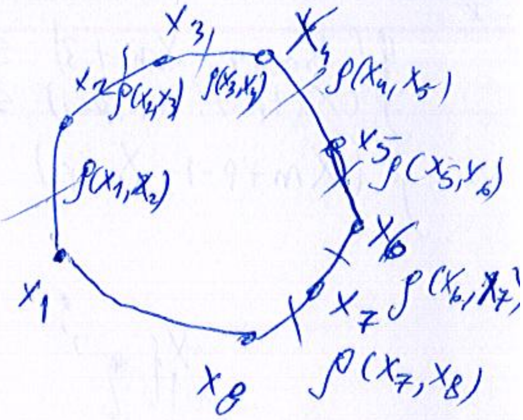
$= \alpha^{m-1}\left(1 \cdot \frac{\alpha^p - 1}{\alpha - 1}\right) \cdot \rho(x_2, x_1) \leq \alpha^{m-1} \frac{1-\alpha^p}{1-\alpha} \rho(x_2, x_1)$; $0 < \alpha < 1$; $1 - \alpha^p \leq 1$

$\Rightarrow \rho(x_m, x_{m+p}) \leq \frac{\alpha^{m-1}}{1-\alpha} \rho(x_2, x_1)$ $\quad m \to \infty$ $\left(\frac{\alpha^{m-1}}{1-\alpha} \rho(x_2, x_1) \to 0\right.$

$\rho(x_1, x_n) \leq \rho(x_1, x_2) + \rho(x_2, x_3) + \dots + \rho(x_{n-2}, x_{n-1}) + \rho(x_{n-1}, x_n)$

$\rho(x_n, x_1) \leq \rho(x_n, x_{n-1}) + \rho(x_{n-1}, x_{n-2}) + \rho(x_{n-2}, x_{n-3}) + \dots + \rho(x_3, x_2) + \rho(x_2, x_1)$





$\underline{n = 8}$

$\rho(x_1, x_8) \leq 7 \times \rho(\ \ )$

$\rho(x_{n+1}, x_{n+2}) \leq \alpha\rho(x_n, x_{n+1})$

$\rho(x_{n+1}, x_{n+2}) \leq \alpha^2(x_{n-1}, x_n)$

$\rho(x_{n+1}, x_{n+2}) \leq \alpha^n(x_1, x_2)$

$\rho(x_n, x_{n+1}) \leq \alpha^{n-1}\rho(x_1, x_2)$

$\rho(x_{n-1}, x_n) \leq \alpha^{n-2}\rho(x_1, x_2)$

$\rho(x_{n+p}, x_n) = \rho(x_n, x_{n+p})$

$\rho(x_n, x_{n+p}) \leq \rho(x_n, x_{n+1}) + \rho(x_{n+1}, x_{n+2}) + \dots + \rho(x_{n+p-2}, x_{n+p-1}) + \rho(x_{n+p-1}, x_{n+p})$

$$\boxed{\rho(x_n, x_{n+p}) \leq \rho(x_n, x_{n+1}) + \rho(x_{n+1}, x_{n+2}) + \rho(x_{n+2}, x_{n+3}) + \dots + \rho(x_{n+p-2}, x_{n+p-1}) + \rho(x_{n+p-1}, x_{n+p})}$$

~~Def~~ Ban $T: E \longrightarrow E$

T zwężający $\iff \exists \alpha \in (0,1)\ (\forall x, y \in E)\ \rho(T(x), T(y)) \leq \alpha\rho(x, y)$

$x_1,\ x_2 = Tx_1,\ x_3 = Tx_2 \qquad x_n = Tx_{n-1}$ —

enough $\forall n$

$\boxed{\rho(x_{n+1}, x_{n+2}) = \rho(Tx_n, Tx_{n+1}) \leq \alpha\rho(x_n, x_{n+1})} = \alpha(\rho(Tx_{n-1}, Tx_n)) \leq$

$\leq \alpha^2\rho(x_{n-1}, x_n)$

$\rho(x_{n+1}, x_{n+2}) \leq \alpha(x_n, x_{n+1}) \leq \alpha^2(x_{n-1}, x_{n-2}) \leq \alpha^3(x_{n-2}, x_{n-3})$

$\rho(x_n, x_{n+1}) \leq \alpha^2(x_{n-2}, x_{n-3}) \leq \alpha^{n-2}(x_2, x_1)$

$$\boxed{\rho(x_n, x_{n+1}) \leq \alpha^{n-2}(x_1, x_2)}$$

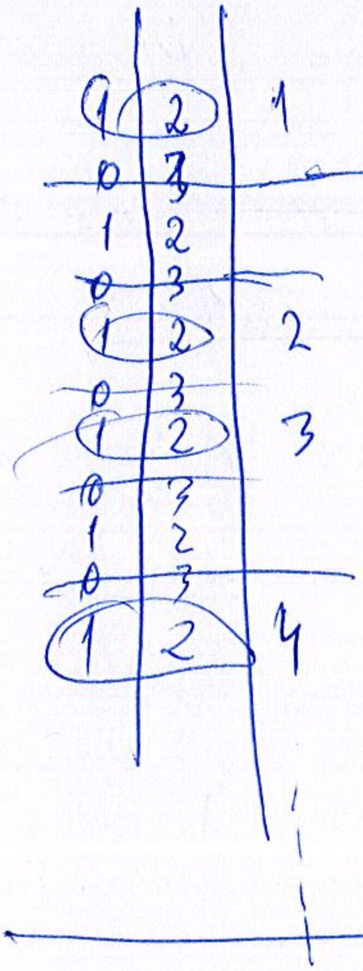
$\rho(x_n, x$

Bolzan – Weirstrass

1
2
1
2
1
2
1
2
1

1
1
1
1
1
1

1
1
1



delete

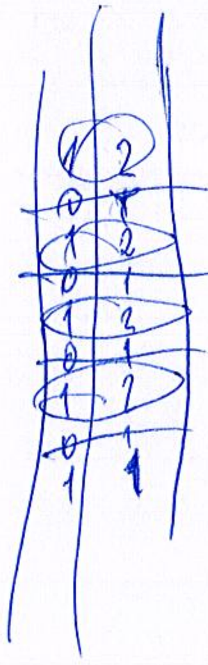
subsequence 1 is ∞, we remove

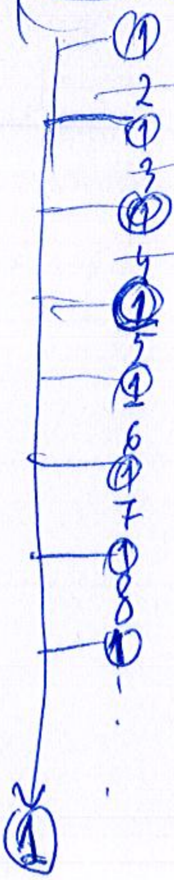
$10^{20}+1$

k=1 k=2 in k steps

∃ α × 1

we remove





∞

31 / 54 / 41 / 66 / 62 / 89 / 80 / 26 / 6 / 34 / 70 / 51 / 61

57 / 9 / 74 / 43 / 48 / 45 / 25 / 32 / 16 / 13 / 71 / 82 / 15

58 / 38 / 88 / 65 / 49 / 77 / 35 / 23 / 7 / 85 / 52 / 67 / 75

59 / 11 / 44 / 2

X K = 1.

m = 1	1	7	5	4	3	2	9
m = 2	0	6					
m = 3	1	6					
	0	7					
	1	6					
	0	6					
	1	7	~~4~~				
	0	6					
	4	6					
	0	7					
	1	6					
	0	6					
	1	7	5				

1 7 5

$x_{m,k}$

$x_{m,1}$ 1 -2 1 2 1 -2 1 2 1 -2 1 2 1 -2 1 2 1 -2 1 — — → $k=1$

$x_{m,2}$ -2 3 2 3 -2 3 2 3 -2 3 2 3 -2 3 2 3 -2 3 2 $k=2$

$x_{m,3}$ 5 -5 5 -5 5 -5 5 -5 -5 5 -5 5 -5 5 -5 $k=3$

4 2 4 3 4 3 5 $k=4$

$x_{1,1}$ $x_{2,1}$

K = 1, - - m

m = 1: 1 -2 1 2 1 -2 1 2 1 -2 1 2 1 -2 1 2

m = 2: -2 3 4 3 -2 3 2 3 -2 3 4 3 -2 3 2 3

1 1 4 1 1 1 7 1 1 1 4 1 1 1 7

5 6 7 8

$\binom{n}{0} = 1$

$\binom{n}{1} = n$

$\binom{n}{n} = 1$

$$\boxed{\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$$

$\sqrt[n]{n} \le \frac{n-1}{n} + 1$

$\sqrt[n]{n} \le 1 - \frac{1}{n} + 1$

$\sqrt[n]{n} \le 2 - \frac{1}{n}$

$\sqrt[n]{n} - 1 \le 1 - \frac{1}{n}$

$\binom{0}{0}$

$\binom{1}{0} \quad \binom{1}{1}$

$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$

$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$

$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$

$\binom{5}{5} = \cancel{\binom{4}{5}} + \binom{4}{4}$

k = 1   k = 2   k = 3

n = 0   1

n = 1   1   1

n = 2   1   2   1   k = 2

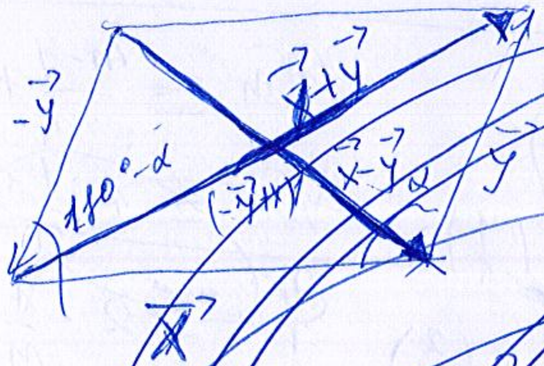
n = 3   1   3   3   1   k = 3

n = 4   1   4   6   4   1

n = 5   1   5   10   10   5   1

Reminder $n \geq n(\sqrt[n]{n}-1)+1 \iff n-1 \geq n(\sqrt[n]{n}-1)$ $\quad \frac{n-1}{n}+1 \geq \sqrt[n]{n}$

$$|\vec{x}+\vec{y}|^2 = |\vec{x}|^2+|\vec{y}|^2-2|\vec{x}||\vec{y}|\cos\alpha$$



$$|\vec{x}-\vec{y}|^2 = |\vec{x}|^2+|\vec{y}|^2-2|\vec{x}||\vec{y}|\cos(180-\alpha)^2$$
$$= |\vec{x}|^2+|\vec{y}|^2+2|\vec{x}||\vec{y}|\cos\alpha$$
$$\cos(180-\alpha) = -\cos\alpha$$

$$|\vec{x}+\vec{y}|^2+|\vec{x}-\vec{y}|^2 = 2|\vec{x}|^2+2|\vec{y}|^2 \to \text{Parallelogram Law}$$

$\boxed{\text{Theorem 1}}$

If parallelogram law holds, then $\|\cdot\|$ is generated by an inner product i.e. $x, y \in E$

$$\|x+y\|^2+\|x-y\|^2 = 2\|x\|^2+2\|y\|^2 \Rightarrow \|\cdot\| = (\,,\,)$$

$\boxed{\text{Theorem 2}}$ $(\alpha \Rightarrow \beta) \iff \sim\beta \Rightarrow \sim\alpha$ $\qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$\boxed{\lim_{n\to\infty}\sqrt[n]{n} = 1}$$

$$\sqrt[n]{n} = 1+(\sqrt[n]{n}-1) \qquad \binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$$

$$(\sqrt[n]{n})^n = [1+(\sqrt[n]{n}-1)]^n$$

$$n = \sum_{k=0}^{n}\binom{n}{k}1^{n-k}(\sqrt[n]{n}-1)^k = 1^n(\sqrt[n]{n}-1)^0+\binom{n}{1}1(\sqrt[n]{n}-1)^1$$
$$+\binom{n}{2}1^{n-2}(\sqrt[n]{n}-1)^2+\binom{n}{3}1(\sqrt[n]{n}-1)^2+\ldots(\sqrt[n]{n}-1)^n$$

$$= 1+n(\sqrt[n]{n}-1)+\frac{n(n-1)}{2}(\sqrt[n]{n}-1)+\ldots(\sqrt[n]{n}-1)^n$$

$$n \geq 1 \Rightarrow \sqrt[n]{n} \geq \sqrt[n]{1} \Rightarrow (\sqrt[n]{n}-1) \geq 0 \Rightarrow (\sqrt[n]{n}-1)^{2k+1} \geq 0,\ k \geq 0$$

$$\Rightarrow \cancel{n} > n(\sqrt[n]{n}-1)\ \cancel{n}\frac{(n-1)}{2}(\sqrt[n]{n}-1)$$

$$\frac{2}{n-1} \geq (\sqrt[n]{n}-1)^2 \qquad \sqrt[n]{n}-1 \leq \sqrt{\frac{2}{n-1}} \qquad |\sqrt[n]{n}-1| \leq \sqrt{\frac{2}{n-1}}$$

Th 2.1 Compact set is bounded !

Proof

A is a compact set

Let $A \subset \bigcup_{k=1}^{n} B(x, r_k)$

Let $A \subset \bigcup_{x \in A} B(x, 1)$ → Always true

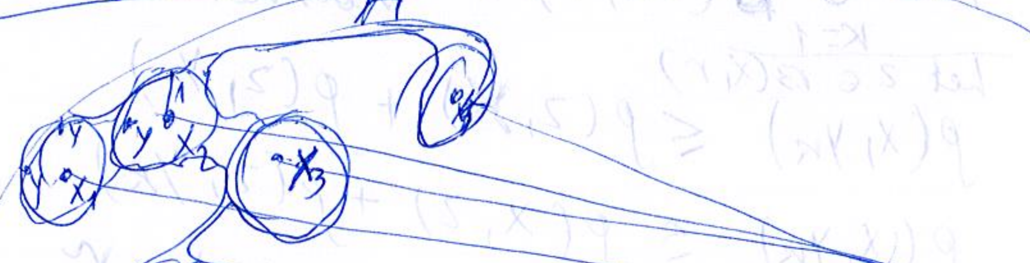
A is bounded $\Leftrightarrow$ $(\exists x_0 \in E, \exists r > 0)$ $A \subset B(x_0, r)$

Let's show that

$A \subset \bigcup_{x \in A} B(x, 1) \xrightarrow{A \text{ is compact}} A \subset \bigcup_{k=1}^{n} B(x_k, 1)$

Let $x_0 \in E$ (any point of E)

$r = \max(\rho(x_0, x_1), \rho(x_0, x_2), \rho(x_0, x_3 \ldots, \rho(x_0, x_n)) + 1$



$y \in A \Rightarrow y \in B(x_k, 1) \Leftrightarrow \rho(x_k, y) < 1$

What about $y \in B(x_0, r) \Leftrightarrow \rho(x_0, y)$?

$\rho(x_0, y) \le \underbrace{\rho(x_0, x_k)}_{\le r-1} + \underbrace{\rho(x_k, y)}_{<1} < r-1+1$

$\rho(x_0, y) < r \Rightarrow y \in B(x_0, r)$

Th1
Compact Set is Closed

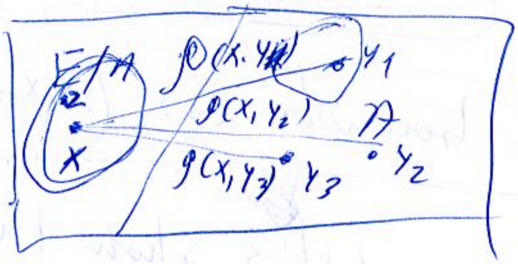
A is a compact set
Let's prove A is closed. That is $\iff E \setminus A$ is open.

Let's show $E \setminus A$ is open

$\forall\ x \in E \setminus A$ Let's find a $r > 0$ such that:

$K(x, r) \subset E \setminus A.$ $(\iff K(x,r) \not\subset A)$

Let $\underset{(\text{any})}{y} \in A$



$A \subset \bigcup\limits_{k=1}^{m} B(y, r_y)$

$\boxed{r_y = \frac{1}{2}\rho(x, y)}$

A is compact $\Rightarrow \exists\ k_1 \dots k_m$ such that $A \subset \bigcup\limits_{k=1}^{m} B(y_k, r_k)$

~~z ∈ B(x, r)~~ $\quad r = \min(r_1, \dots r_k) \Rightarrow r \leq r_k \ \forall k$

$A \subset \bigcup\limits_{k=1} B(y_k, r)$. Let's show that z which $\in B(x, r)$ does not lay in any of $B(y_k, r_k)$

Let $z \in B(x, r)$

$\rho(x, y_k) \leq \rho(z, x) + \rho(z, y_k)$

$\rho(x, y_k) \leq \rho(x, z) + \rho(z, y_k)$

$z \in B(x, r) \Rightarrow \rho(x, z) < r$

$\boxed{2r_k} = \rho(x, y_k) \leq r + \rho(z, y_k) \leq \boxed{r_k} + \rho(z, y_k) \quad \forall k$

$2r_k \leq r_k + \rho(z, y_k) \Rightarrow \boxed{\rho(z, y_k) > r_k}$

$\Rightarrow z \notin B(y_k, r_k)$

i.e. $z \in B(x, r) \wedge \boxed{z \notin B(y_k, r_k),\ \forall k}$ $\quad$ $E \setminus A$ is open

$\iff z \notin \bigcup B(y_k, r_k) \supset A$

$\Rightarrow z \notin A \Rightarrow z \notin E \setminus A$

$\Rightarrow B(x, r) \subset E \setminus$

$d_1' = OD$

$$\frac{d_1}{d_1'} = \frac{r-y}{\cancel{r}} = \frac{r-y}{r}$$

$$\frac{d}{d_1'} = \frac{x}{x'} = \frac{z}{z'}$$

solution

$$\frac{x}{x'} = \frac{z}{z'} = \frac{r-y}{y} = c \Rightarrow x+y+z=r$$

$x'+z'=r$

$x = c \cdot x'$

$z = c \cdot z'$

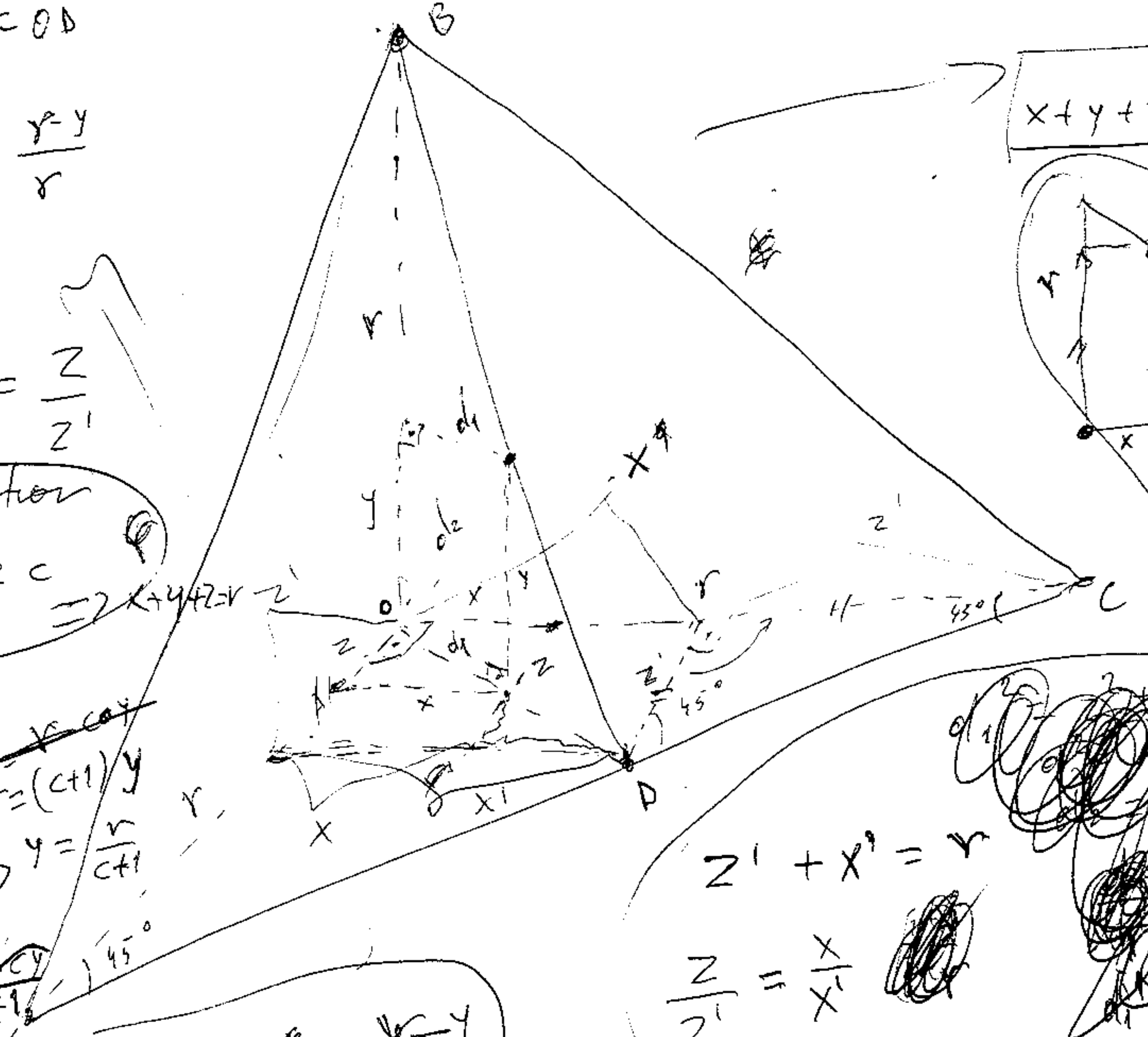
$r-y = c \cdot y \Leftrightarrow$ ~~$y = r - cy$~~ $r = (c+1)y$ $y = \frac{r}{c+1}$

$y = r - c \cdot y \Rightarrow$

$$x+z+y = c \cdot x' + cz' + \frac{r}{c+1}$$

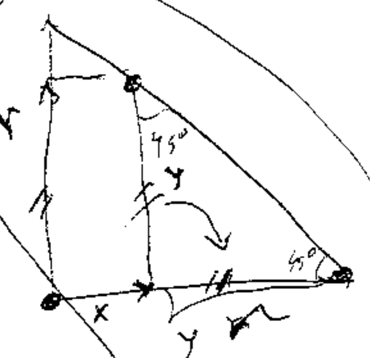
$$= c(x'+z') + \frac{r}{c+1} =$$

$$= c \cdot r + \frac{r}{c+1} = r\frac{c+1}{c+1} = r$$



$$\boxed{\frac{z}{z'} = \frac{x}{x'} = \frac{r-y}{r}}$$

$\boxed{x+y+z \overset{?}{=} r}$



$$z'+x' = r$$

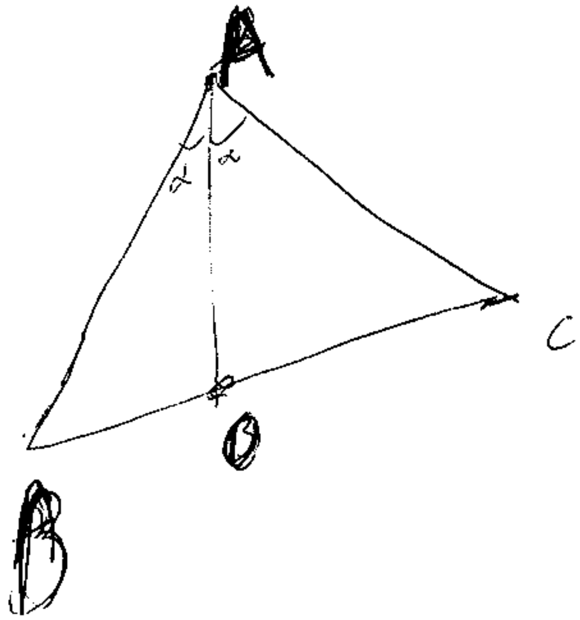
$$\frac{z}{z'} = \frac{x}{x'}$$

$$x'+z' = r$$

$z^2+x^2+y^2$

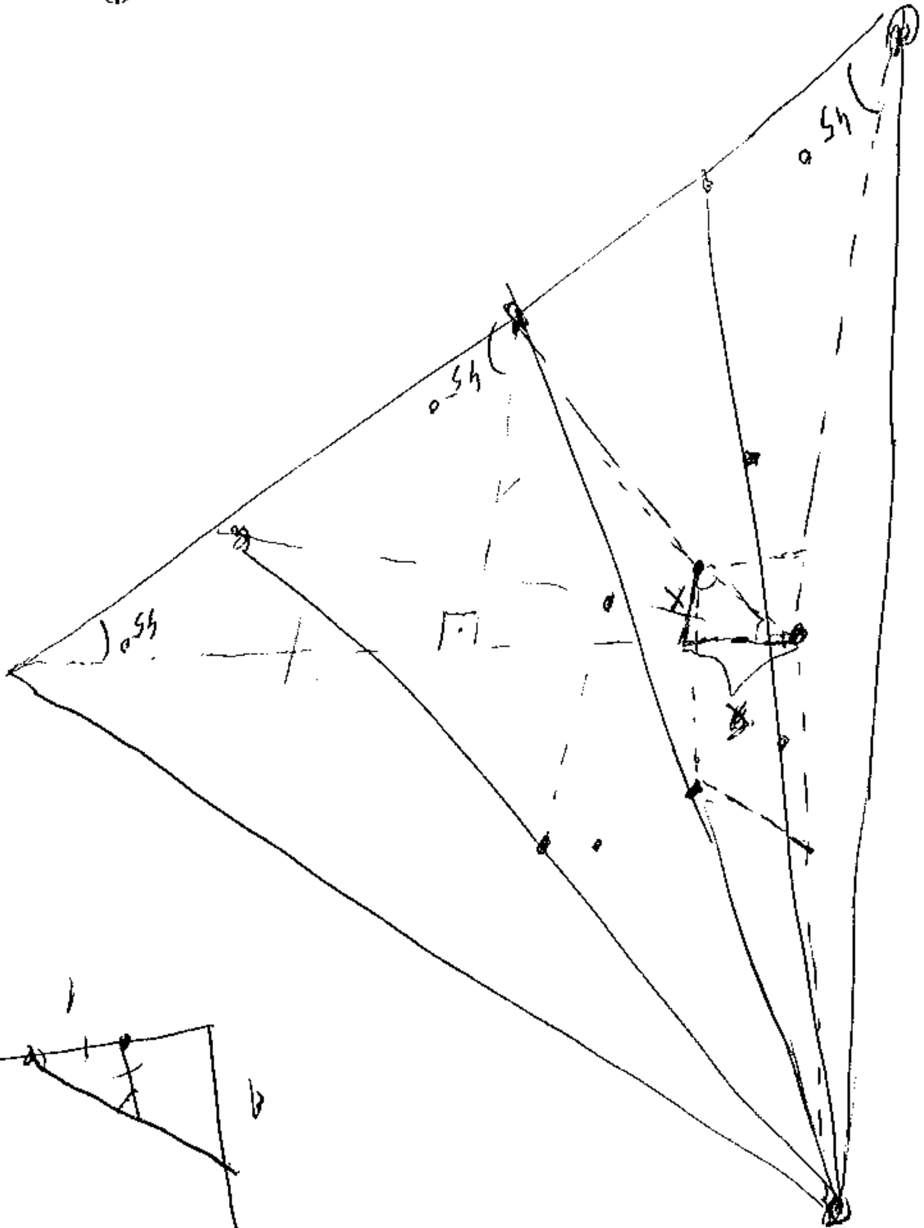
$\Rightarrow x+y \; (x'+y'+r)$

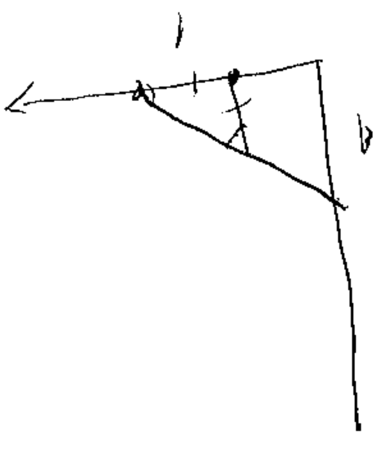
$y = c\,r$

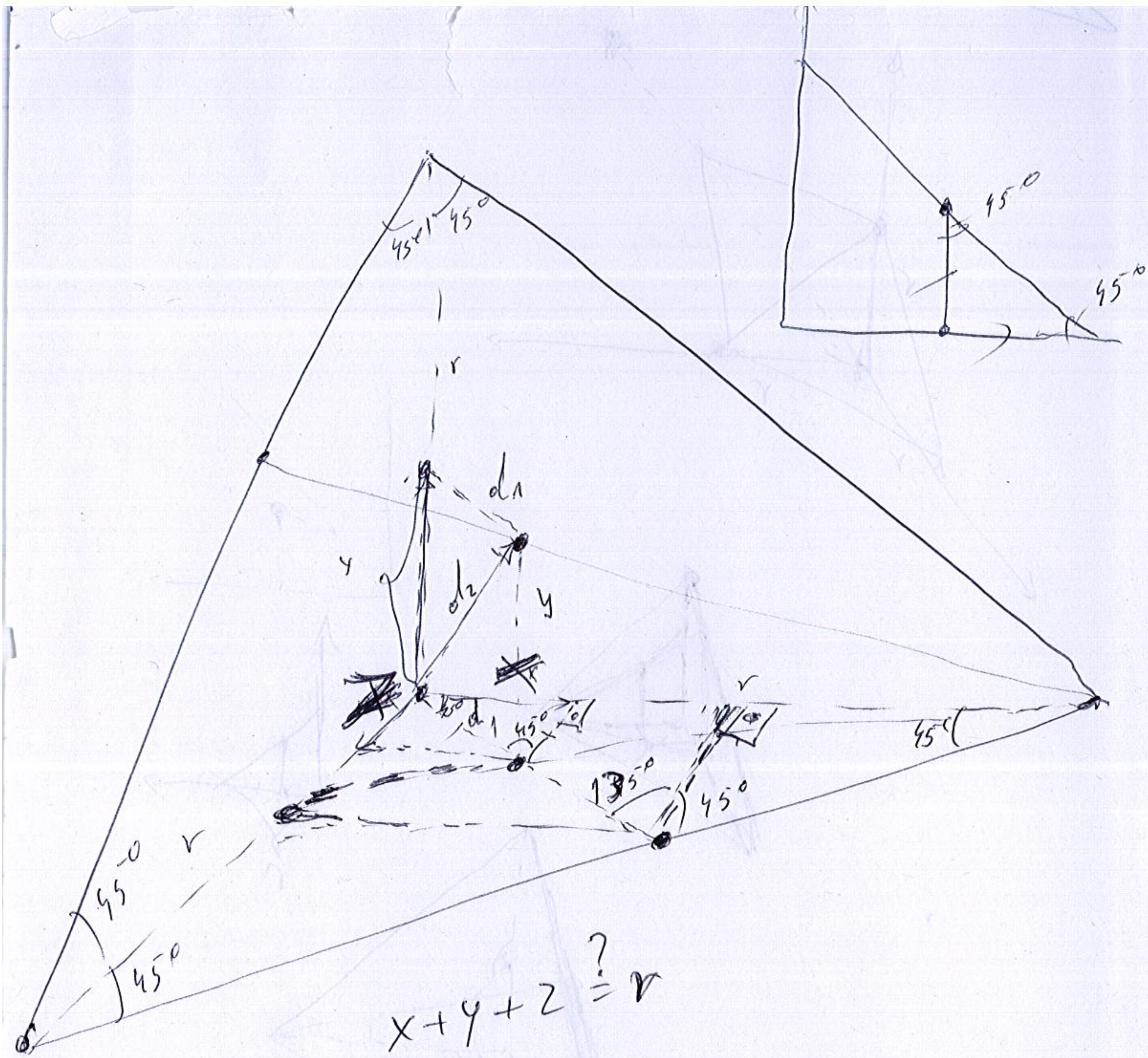


$$\frac{OB}{OC} = \frac{AB}{AC}$$

$$\frac{OB}{AB} = \frac{OC}{AC}$$







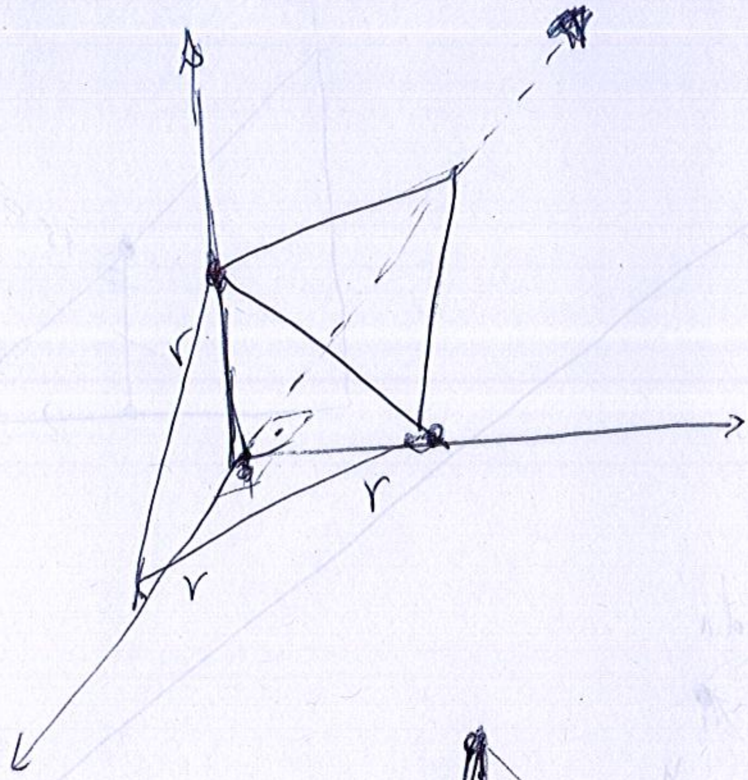
$$x + y + z \overset{?}{=} r$$

$$d_1^2 = z^2 + x^2$$

$$d_1 = z$$

$$d_2^2 = d_1^2 + y^2$$

$$\boxed{d_2^2 = z^2 + x^2 + y^2}$$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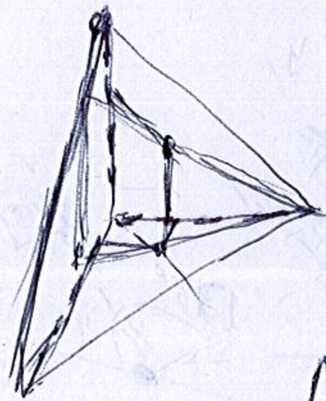

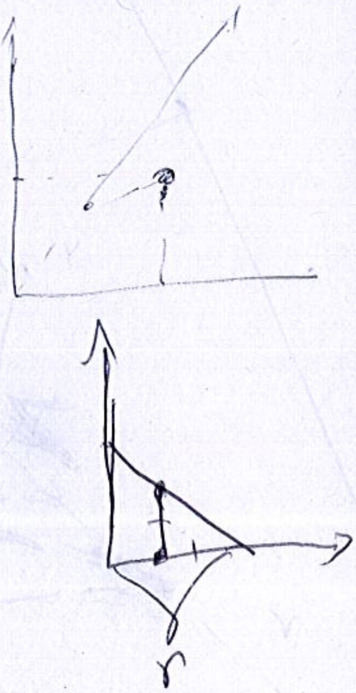
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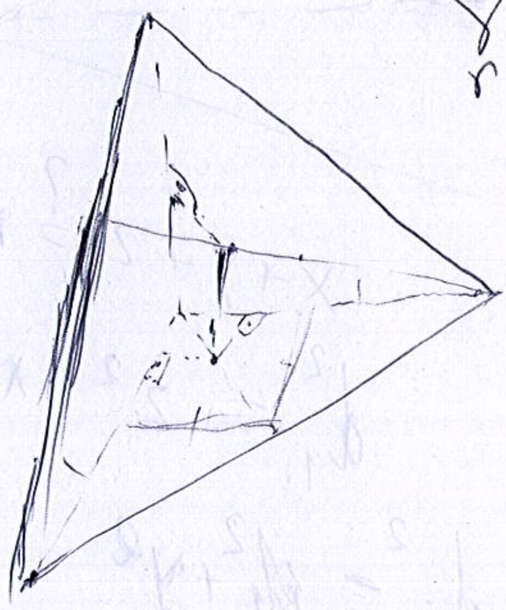
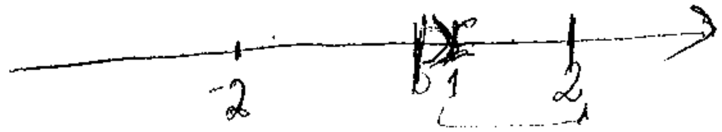




$$A = (-2,1) \cup [1,2)$$

$$(-2,1) = (-2,1) \cap A$$

$$[1,2) = (0,2) \cap A$$

$$(0,2) \cap A = (0,2) \cap \big((-2,1) \cup [1,2)\big)$$

$$= \big((0,2) \cap (-2,1)\big) \cup \big((0,2) \cap [1,2)\big)$$

$$[0,1) \cup [1,2)$$

$\forall x \in R$

$\exists x_n \in Q$

$x_n \nearrow$

$\exists y_n \in R \setminus Q$

$y_n \searrow$

Proof

$x \in Q$

$Q \ni x_n = x \searrow$

$Q \ni y_n = x \nearrow$

$R \setminus Q \ni x_n = x - \frac{\sqrt{2}}{n} \nearrow$

$R \setminus Q \ni y_n = x + \frac{\sqrt{2}}{n} \searrow$

$x \in Q$

a) $x_n = x \in Q$

b) $y_n = x$ $\quad x_n, y_n \in Q$

$x_n, y_n \in R \setminus Q$

$x_n = x - \frac{\sqrt{2}}{n}$

$y_n = x + \frac{\sqrt{2}}{n}$

x_n jest ($x_n = x$ a) malejący, $y_n = x$ b) rosnący) $\in R \setminus Q$

$x \in R \setminus Q$ $\quad x_n = x \in R \setminus Q$

x $\quad$ ~~$x_n = \alpha_0, \alpha_1 \ldots \alpha_n$~~

$x \in R \setminus Q$

a) $x_n = \alpha_0, \alpha_1 \ldots \alpha_n \in Q \quad \forall n \in N$

b) $y_n = x_n + (0,1)^n$ $\qquad 0,1^n(0,1-1)$

$$y_{n+1} - y_n = \left(\alpha_0 + \frac{\alpha_1}{10^1} + \frac{\alpha_2}{10^2} + \ldots + \frac{\alpha_n}{10^n} + \frac{\alpha_{n+1}}{10^{n+1}} + (0,1)^{n+1}\right) \quad 0,99\,(0,1)^n$$

$$- \left(\alpha_0 + \frac{\alpha_1}{10} + \frac{\alpha_2}{10^2} + \ldots \frac{\alpha_n}{10^n} + 0,1^n\right) = \frac{\alpha_{n+1}}{10^{n+1}} + (0,1)^{n+1} - (0,1)^n = \frac{\alpha_{n+1}}{10^{n+1}} + 0,99(0,1)^n$$

$(0,1) \overset{?}{\subset} \bigcup_{n\geq 2}^{\infty} [\frac{1}{n}, 1-\frac{1}{n}]$

Proof: $0 < x < 1$

$\exists n$? such that $x \in [\frac{1}{n}, 1-\frac{1}{n}]$

$x = 0 + a$
$x = 1 - b$

$0 + a = x = 1 - b$
$1 - b = a$
$b = 1 - a$

$a \leq x \leq 1 - a$

$\frac{1}{n} < x < 1 - \frac{1}{n}$

$\frac{1}{x} < n \Rightarrow n > x$

$x < 1 - \frac{1}{n} \Rightarrow \frac{1}{n} < 1 - x$

$n > \frac{1}{1-x}$

$\begin{cases} n > x \\ n > \frac{1}{1-x} = \cdot \end{cases}$

$\frac{1}{n} < x < 1 - \frac{1}{n}$

$\frac{1}{n} < x$

$0 \leq \varepsilon < x < 1 - \varepsilon < 1$

$0 < x < 1$

$\Updownarrow$

$\varepsilon \leq x \leq 1 - \varepsilon$

$\varepsilon = \frac{1}{n}$

$\frac{1}{n} < x < 1 - \frac{1}{n}$

$\varepsilon = \frac{1}{n}$ we can find such n ($n = \frac{1}{\varepsilon}$)

i.e. $\exists n = \frac{1}{\varepsilon} : \frac{1}{n} < x < 1 - \frac{1}{n}$

$\varepsilon = \frac{1}{n} = \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$

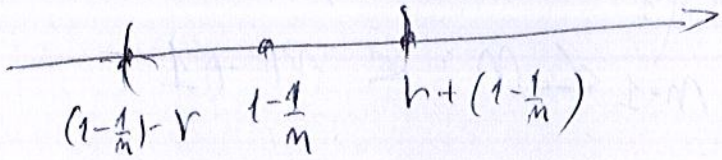
$\forall x \in (0,1) \; \exists n \geq 2 : x \in [\frac{1}{n}, 1-\frac{1}{n}]$

$\Downarrow$

$x \in \bigcup [\frac{1}{n}, 1-\frac{1}{n}]$

$\exists r > 0 \ \cancel{\exists} n \ \forall n \quad r := \text{funct}(n)$ such that

$$1 - \frac{1}{n} \notin B\left(1 - \frac{1}{n}, r\right)$$



$$n - 1 < n < n + 1 \Rightarrow \frac{1}{n+1} < \frac{1}{n} < \frac{1}{n-1}$$

$$\cancel{1 + \frac{1}{n+1} < 1 + \frac{1}{n} < 1 + \frac{1}{n-1}}$$

$$-\frac{1}{n-1} < -\frac{1}{n} < -\frac{1}{n+1} \Rightarrow 1 - \frac{1}{n-1} < 1 - \frac{1}{n} < 1 - \frac{1}{n+1}$$

$f(n-1) \quad f(n) \quad f(n+1)$

$$f(n-1) < f(n) < f(n+1)$$

$$\frac{2n-2}{2} < \frac{2n-1}{2} < 1 - \frac{1}{n} < \frac{2n+1}{2} < \frac{2n+2}{2}$$

$$r \leq \left|\frac{m-n}{mn}\right| \leq \left|\frac{m}{mn}\right| \leq \frac{1}{n}$$

$$n + \frac{1}{2} - 1 + \frac{1}{n} = \frac{3}{2}n - \frac{1}{2}$$

$$\boxed{r \leq \frac{1}{n} \Leftrightarrow \left|\left(1 - \frac{1}{n}\right) - \left(1 - \frac{1}{m}\right)\right| > r}$$

$n - \frac{1}{2} - 1 + \frac{1}{n} < 0$

$\frac{3n}{2} - \frac{3}{2} < 0$

$\frac{3}{2}(n-1) < 0$

$n - 1 < 0$

$\boxed{n > 1}$

$\frac{3n}{2} > \frac{3}{2}$

$n \neq 1$

$n = 2 \quad r \leq \frac{1}{2}$

$r \leq \frac{1}{3}$

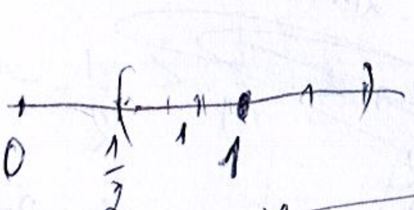
$x_1 = 1 - \frac{1}{1} = 1$

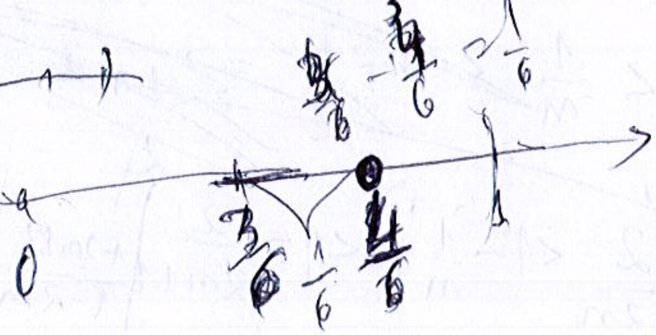
$x_2 = 1 - \frac{1}{2} = \frac{1}{2}$

$x_3 = 1 - \frac{1}{3} = \frac{2}{3}$

$x_4 = 1 - \frac{1}{4} = \frac{3}{4}$

$x_5 = 1 - \frac{1}{5} = \frac{4}{5}$





$\forall f_n = 1 - \frac{1}{n}$

$\exists \varepsilon > 0$

$\forall n' \neq n \quad V_n = f(n)$

$f_{n'} \notin B(\ )$

$n - 1 = \frac{2n-2}{2}$

$f_{n-1} \left(\quad f_n \quad \right) f_{n+1}$

$n-1 < n < n+1$

$\frac{1}{n+1} < \frac{1}{n} < \frac{1}{n-1}$

$\frac{2n-2}{2} < n < \frac{2n+2}{2}$

$\frac{2n-2}{2} < \frac{2n-1}{2} < n$

$\frac{1}{n - \frac{1}{2}}$

$\frac{3-2n}{1-2n} < 1 - \frac{1}{n} < \frac{2n-1}{2n}$

$n-1 < n < n+1$

$n - \frac{1}{2} < n < n + \frac{1}{2} \iff \frac{2n-1}{2} < n < \frac{2n+1}{2}$

$\frac{2}{2n+1} < \frac{1}{n} < \frac{2}{2n-1}$

$n + \frac{1}{2} < \frac{1}{n} \cancel{< n - \frac{1}{2}} \Rightarrow \cancel{\frac{1}{n} < \frac{1}{2}} \quad \frac{1}{n} < \frac{-1}{2} - n$

$1 + \frac{2}{1-2n} < 1 - \frac{1}{n} < 1 - \frac{2}{2n+1} \quad \frac{1-2n+2}{1-2n} \leq 1 - \frac{1}{n} \leq \frac{2n+1-2}{2n+1}$

$S = \{ 1 - \frac{1}{n} \}$

$\forall s \in S : \exists r > 0 \quad B(s, r) \cap S \setminus \{s\} = \emptyset$

$\exists r > 0$

$x_0 = 1 - \frac{1}{n} \quad \boxed{B(1 - \frac{1}{n}, r) \ni 1 - \frac{1}{n} \text{ always}}$

Let's show $\forall x_m \neq 1 - \frac{1}{n} = x_0 \wedge x \in S \quad x \notin B(x_0, r)$

$\Downarrow$

$\forall m \neq n \quad 1 - \frac{1}{m} \notin (1 - \frac{1}{n}, r)$

$x_0 - r \quad x_0 \quad r \quad x = 1 - \frac{1}{m}$

$r + x$

$1 - \frac{1}{n}$

$\sim \boxed{(1 - \frac{1}{n} - r) \leq 1 - \frac{1}{m} \leq 1 - \frac{1}{n} + r} \Leftrightarrow x_m \in B(x_0, r)$

$\boxed{\forall m, x_m = 1 - \frac{1}{m} \geq |x_0 - r|}$

$\cancel{\frac{1}{m} - 1 < \frac{1}{m} - \frac{1}{m} < \frac{1}{n}}$

$\begin{cases} 1 - \frac{1}{m} > 1 - \frac{1}{n} + r \\ 1 - \frac{1}{m} < 1 - \frac{1}{n} - r \end{cases}$

$\cancel{|1 - \frac{1}{m}| \geq |(1 - \frac{1}{n}) - r|} \quad x_0 - r \leq 1 - \frac{1}{m} \leq x_0 + r$

$r < \frac{1}{m} - \frac{1}{n}$

$r < \frac{1}{n} - \frac{1}{m}$

$m < n \Rightarrow \frac{1}{m} > \frac{1}{n} \quad \Leftarrow 1 - \frac{1}{m} > 1 - \frac{1}{n} \Rightarrow x_n > x_m$

$m > n \Rightarrow \frac{1}{m} < \frac{1}{n} \quad 1 - \frac{1}{n} < \frac{1}{m} - \frac{1}{n}$

$r < \frac{m - n}{mn}$

$r < \frac{n - m}{mn}$

$1 - \frac{1}{m} \geq 1 - \frac{1}{n} - r_0 \Rightarrow -\frac{1}{m} \geq -(\frac{1}{n} + r_0)$

$(=) \quad \frac{1}{m} \leq \frac{1}{n} + r \quad \Rightarrow r \geq \frac{1}{m} - \frac{1}{n} = \frac{n - m}{mn}$

$r \geq \frac{n - m}{mn} \Rightarrow x_m \geq |x_0 - r| \Rightarrow x_m \notin B(x_0, r)$

$S = \{1 - \frac{1}{n}\}$

is isolated

$\exists r > 0 \quad s \in S \quad \forall y \in S, y \neq s : y \notin B(s, r)$

$\Leftrightarrow \forall y \notin B(s,r) = \{ y' : |s - y'| < r \}$, $y \neq s$

$\Leftrightarrow \forall y \in \{ y' : |s - y'| \geq r \}$, $y \neq s$

$\exists r > 0 \quad 1 - \frac{1}{m} \in S \overset{m \neq n}{\Rightarrow} 1 - \frac{1}{n} \in \{ y : |s - y'| \geq r \}$

$1 - \frac{1}{n} \in S \Rightarrow 1 - \frac{1}{m} \in \{ |(1 - \frac{1}{n}) - (1 - \frac{1}{m})| \geq r \}$

$r = ?$ $\quad \boxed{r = funct(n)}$ $\quad r = funct(n)$, $r \leq \frac{|m-n|}{m \cdot n}$

$\exists r \, \forall m$

$r \leq \frac{|m-n|}{mn}$ $\quad |1 - \frac{1}{n} - 1 + \frac{1}{m}| \geq r \Leftrightarrow \left|\frac{1}{m} - \frac{1}{n}\right| \geq r \Rightarrow r \leq \frac{|m-n|}{m \cdot n}$

$r \leq \left|\frac{1}{m} - \frac{1}{n}\right| = \left|\frac{n-m}{mn}\right| = \left|\frac{m-n}{mn}\right|$

$r \leq \frac{|m-n|}{mn} \leq \frac{m}{mn}$

$r > 0$

$\boxed{r \leq \frac{1}{n}}$

$r \quad \frac{m}{mn} \leq r \leq \frac{|m-n|}{mn}$

$m > n$

$0 < r \leq \frac{m-n}{mn}$

$r \leq \frac{1}{n} - \frac{1}{m} \leq \frac{1}{m} \geq r + \frac{1}{m}$

$0 < r \leq \frac{n-m}{mn} \quad n > m$

$r \leq \frac{1}{n} - \frac{1}{m} \geq \frac{1}{m} \leq r + \frac{1}{m}$

for such r any $s_m \neq s_n$ $\quad s_m \in B($