①

PRECISION / RECALL

		Actual	
		1	0
Predicted	1	True positives	False Positives
	0	False Negatives	True Negatives

Logistic regression: $0 \leq h_\theta(x) \leq 1$

$$\left.\begin{aligned} P(y=1 \mid x;\theta) &= h_\theta(x) \\ P(y=0 \mid x;\theta) &= 1-h_\theta(x) \end{aligned}\right\}$$

Starting threshold 0.5 $P(y=1)=h_\theta(x) \geq 0,5$

(1) i.e for all x that $h_\theta(x) \geq 0,5$ we predict positivity $y=1$
• for all x that $h_\theta(x) < 0,5$ we predict negativity $y=0$

0,7 $P(y=1 \mid x,\theta) = h_\theta(x) \geq 0,7$

(2) we predict positive when $h_\theta(x) \geq 0,7$
we predict negative when $h_\theta(x) < 0,7$

As $h_\theta(x)$: $0,5 \longrightarrow 0,7$ $\Longrightarrow$

[. . . 0,5]
[. . . 0,7]

1) True positives may decrease because we do not capture positive cases which may fall in $h_\theta(x) \in (0,5; 0,7)$, i.e we may lose some of them.
In other words, True positives does not increase with the increase of $h_\theta(x)$
2) False positives may ↘, because we do not include positives cases from (0,5; 0,7) (True either false) and surely does not increase

①

② $h_\theta(x) \nearrow$ from 0,5 to 0,7 $\quad P(y=1|x;\theta) \nearrow$ $\; = h_\theta(x)$

~~$P(y=0|x;\theta) = 1 - P(y=1|x;\theta) = 1 - [h_\theta(x) \geq 0,7]$~~

		Actual 1	Actual 0
Predicted	1	TP	FP
	0	FN (↑)	TN (↑)

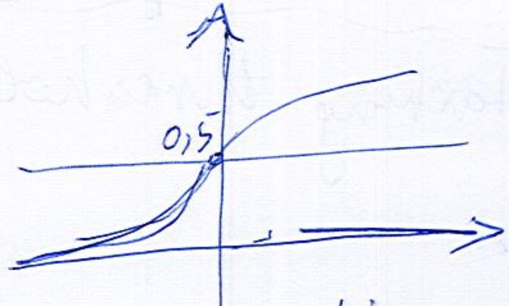
When $h_\theta(x) \nearrow$

If $h_\theta(X) < 0,7$ then we consider as "predicted negative $Y = 0$

$P(y=0|x;\theta)$ ~~when $h_\theta(x) < 0,7$~~

$-h_\theta(x) \geq -0,7 \Rightarrow 1 - h_\theta(x) \geq 0,3$

previously $1 - h_\theta(x) \geq 0,5$



We moved from $h_\theta(x) < 0,5$ to $h_\theta(x) < 0,7$

So we expect to have possibly only more negative predictions

since FN may ↑ and TN may ↑ and sure that they do not decrease

Threshold increases ⇒ horizontal border may lift up (and surely not fall down).

③

$h_\theta(x)$ changes from 0,7 to 0,5

$0{,}7 \xrightarrow{h_\theta(x)} 0{,}5$ (↓)

Pred. $y = 1$

$\boxed{h_\theta(x) \geq 0{,}7 \Rightarrow h_\theta(x) \geq 0{,}5}$

⇓

- We will have possibly only more positive predictions (both TP, FP) and not less.

		Actual 1	Actual 0
Predicted	1	TP	FP
	0	FN	TN

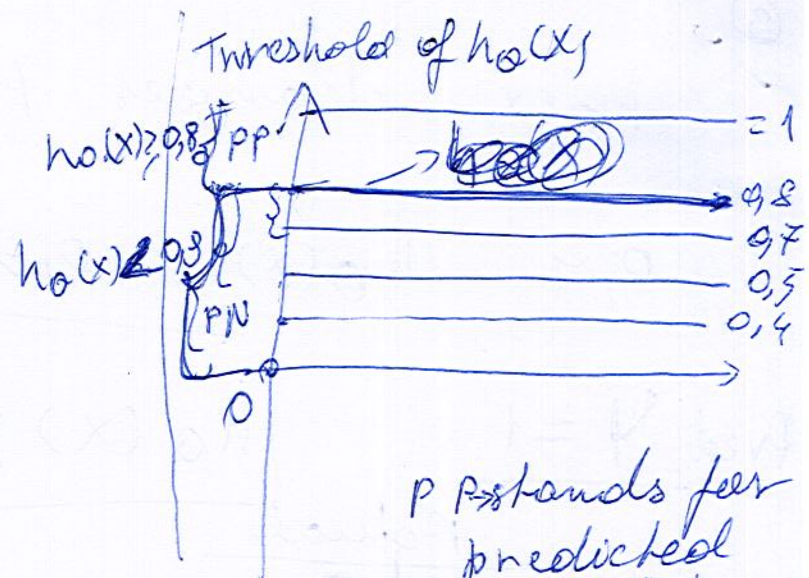
↓ as $h_\theta(x)$ ↓

$\boxed{h_\theta(x) < 0{,}7) \Rightarrow h_\theta(x) < 0{,}5}$

- We whill have possibly only less negative predictions and not more (because possibly we are omitting cases falling in (0,5; 0,7)).

4) Precision $= \dfrac{TP}{PP} = \dfrac{TP}{TP+FP}$ (a)

Recall $= \dfrac{TP}{\text{Actual Posit}} = \dfrac{TP}{TP+FN}$ (b)



P stands for predicted positive

PN -> predicted negative

$h_\theta(x) \nearrow \Longrightarrow \begin{cases} TP \searrow (1) \\ FP \searrow (2) \\ FN \nearrow (3) \\ TN \nearrow (4) \end{cases}$

$\xrightarrow{\text{because of (2)}}$ Precision $\nearrow$

$\xrightarrow{\text{because FN (3)}}$ Recall $\searrow$

$h_\theta(x) \searrow \begin{cases} TP \nearrow (5) \\ FP \nearrow (6) \\ FN \searrow (7) \\ TN \searrow (8) \end{cases}$

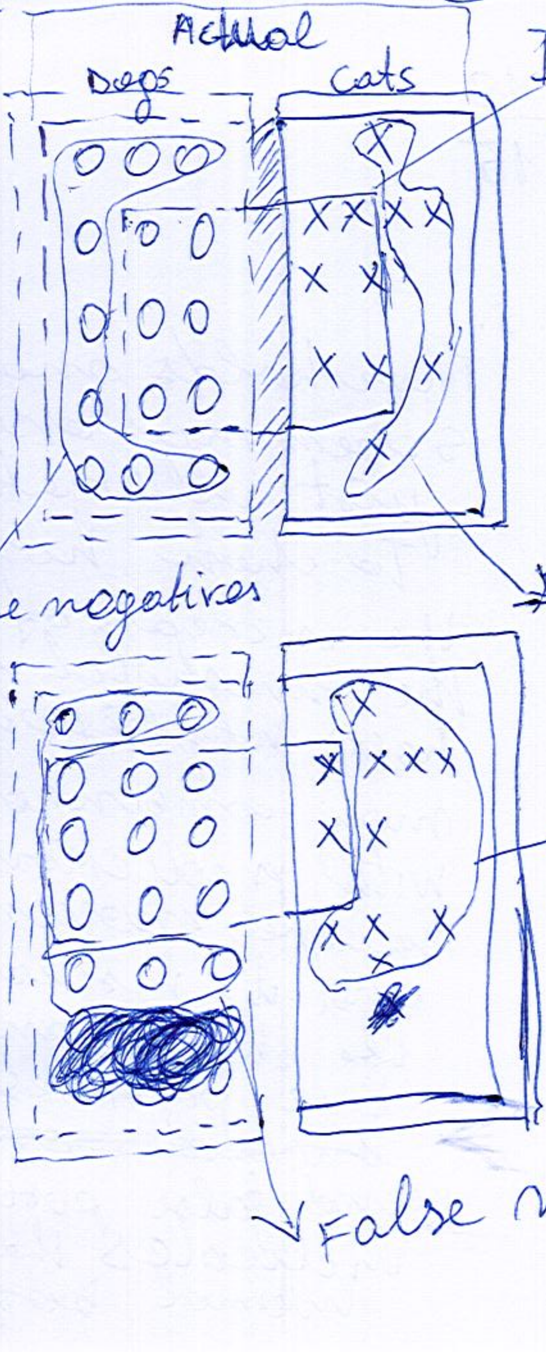
$\xrightarrow{(6)}$ Precision falls $\searrow$

$\xrightarrow{(7)}$ Recall up $\nearrow$

5

$$\frac{a}{a+b} = \frac{1}{\frac{a+b}{a}} = \frac{1}{1+\frac{b}{a}}$$

$h_\theta(x) \geq 0,5$



In> Internal ~~square~~ rectangular is what we predict as positive

Clasifier should returns $y=1$ for dogs

so aim of of classification is to detect dogs.

$$\text{PRECISION} = \frac{TP}{TP+FP}$$

TP + FP -> is what is inside small ~~square~~ rectangular

i.e.

Precision = —— = ——

Recall = ——

$\boxed{\text{Case 1}}$

$$\text{Precision} = \frac{3}{3+3} = \frac{1}{2}$$

$$\text{Recall} = \frac{6}{15}$$

$\boxed{\text{Case 2}}$

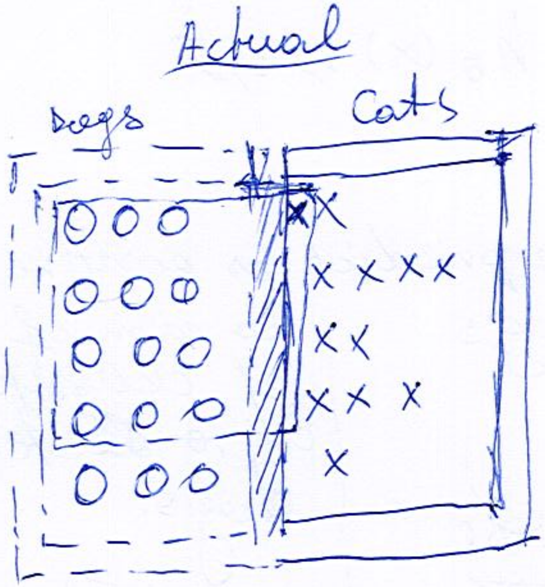
$$Pr = \frac{9}{9+2} = \frac{9}{11}$$

$$\text{Recall} = \frac{9}{15}$$

Let's assume $h_\theta(x) \geq 0,5$ for case 1 and $h_\theta(x) \geq 0,7$ assign to case 2.

In case 1 classifier misses incorrectly dogs and incorrectly includes cat

(6)



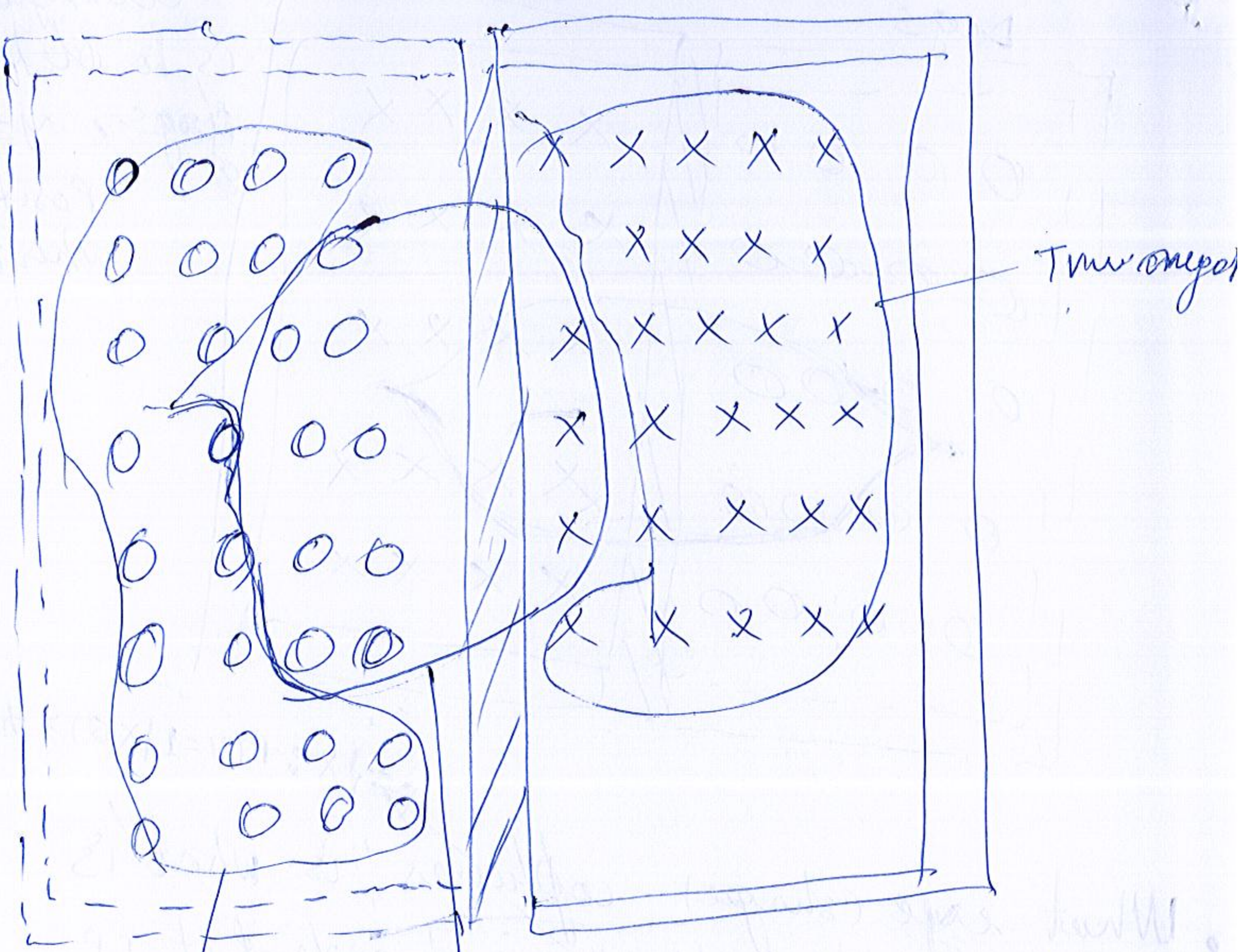
$h_\theta(x) \geq 0{,}9$

$$\text{Precision} = \frac{12}{12+1} = \frac{12}{13}$$

$$\text{Recall} = \frac{12}{12+3} = \frac{12}{15}$$

P. S.
Thresholds and schemmas are just illustratives to show how the increase of the threshold for $h_\theta(x)$ ~~re...~~ may combined with prediction square's movements and its resizing i.e. how many true positives ~~includes and~~ and false positive includes the internal box

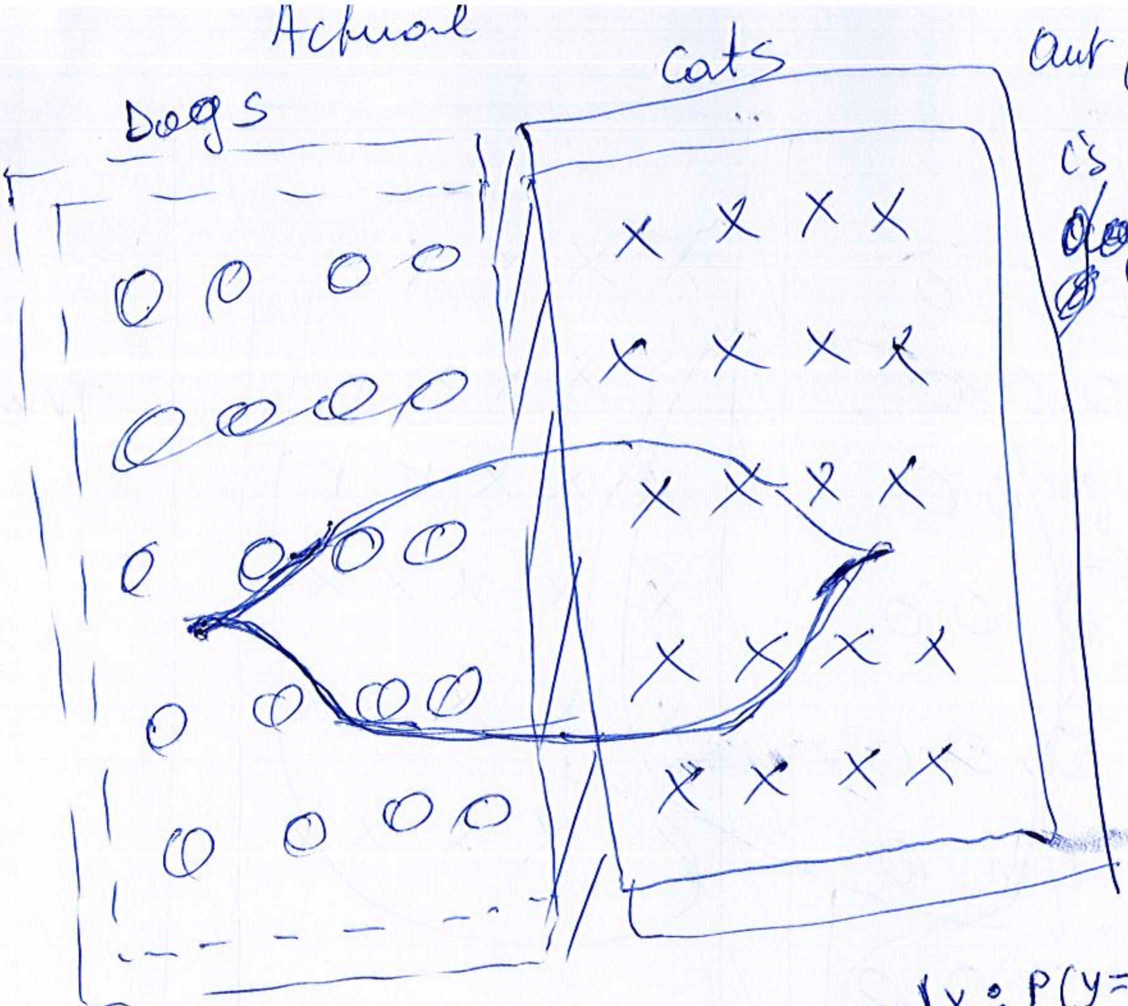
(7)



Predicted as ~~positive~~ positive (TP + FP)

It remains to be considered as predicted ~~predicted~~ "False negatives" because had not been captured as predicted positive

(8)



Our Classifier is to detect dogs. $y=1$ Positive when "dog"

$= \{x : P(y=1|x;\theta) \geq th\}$

- What eye (shape) captures is what "is predicted "positive". Left side of it TP and right side FP.
- What remains out of eye is predicted as "Negative" $= \{x : P(y=1|x;\theta) < th\}$

BTW. $P(y=1|x;\theta) = h_\theta(x)$. All this divided into:

left side (without eye) is FN while

Right side (without eye) is TN