2020

H2.(1)

a) $\langle X-2Y, 2X+Y\rangle = {}^t(X-2Y)S(2X+Y)$

$= ({}^tX - 2{}^tY)S(2X+Y)$

$= 2{}^tXSX + {}^tXSY - 4{}^tYSX - 2{}^tYSY$

$= 2\langle X, X\rangle + \langle X, Y\rangle - 4\langle Y, X\rangle - 2\langle Y, Y\rangle$

$= 8$

b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

c) ${}^tXSX = (x\ \ y)\begin{pmatrix} a & b \\ b & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$= ax^2 + 2bxy + dy^2$

$= a(x+\frac{b}{a}y)^2 + (-\frac{b^2}{a} + d)y^2$

c-1 : $x + \frac{b}{a}y^2$

c-2 : $-\frac{b^2}{a} + d$

c-3 : $-\frac{b^2}{a} + d > 0$

2)

$\{ {}^t(x, y, z) \in R^3 : 2x+y-z=0 \}$ 中的向量可表示为:

$$\begin{pmatrix} x \\ y \\ 2x+y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} y ,$$

基底为 $\vec{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

将 $\vec{u}_2$ 正规化: $\vec{\hat{u}}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$,

与 $\vec{u}_2$ 直交的向量 $\vec{v}_1$ 为:

$$\vec{v}_1 = \vec{u}_1 - ({}^t\vec{u}_1 \, \vec{\hat{u}}_2) \, \hat{u}_2$$

$$= \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

正规化, 得 $\vec{v}_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

综上, $\vec{u}_2$ 与 $\vec{v}_1$ 即为所求基底

3)

a) $${}^tF(A) = {}^t\left(\frac{A + {}^tA}{2}\right)$$
$$= \frac{{}^tA + A}{2}$$
$$= F(A)$$

$\Rightarrow F(A)$ 为对称行列

b) $$G(A) = A - F(A)$$
$$= A - \frac{A + {}^tA}{2}$$
$$= \frac{A - {}^tA}{2}$$

$${}^tG(A) = \frac{{}^tA - A}{2}$$
$$= -G(A)$$

$$F(G(A)) = \frac{G(A) + {}^tG(A)}{2}$$
$$= O_{n\times n}$$