3.

(1)

$$\mathrm{grad}\,\phi = (z e^{xz}\sin y + e^{x}\cos y)\vec{i} + (e^{xz}\cos y - e^{x}\sin y)\vec{j} + x e^{xz}\sin y\,\vec{k}$$

在点 (1, 0, 1) 处

$\mathrm{grad}\,\phi = e\vec{i} + e\vec{j}$

$A = \vec{i} - 2\vec{j} + 2\vec{k}$

$|A| = 3$

A的方向为:

$$\mathrm{grad}\,\phi \cdot \frac{A}{|A|} = -\frac{e}{3}$$

(2)

S上点的坐标向量为:

$$\vec{r} = x\vec{i} + y\vec{j} + (-6x-3y+3)\vec{k}$$

则.

$$\frac{\partial \vec{r}}{\partial x} = \vec{i} - 6\vec{k}$$

$$\frac{\partial \vec{r}}{\partial y} = \vec{j} - 3\vec{k}$$

$$\frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y} = 6\vec{i} + 3\vec{j} + \vec{k}$$

在S上, $A = (-6x-3y+3)\vec{i} - 3\vec{j} + 4xy\vec{k}$.

$$\Rightarrow A \cdot \left(\frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y}\right) = 4xy - 36x - 18y + 9$$

点 $(x, y, -6x-3y+3)$ 在S上時, $0 \le x \le \frac{1}{2}$ 且 $0 \le y \le -2x+1$.

所求面積分为:

$$\int_0^{1/2} dx \int_0^{-2x+1} dy \,(4xy - 36x - 18y + 9)$$

$$= 4\int_0^{1/2} dx\,(2x^3 + 7x^2 - 4x)$$

$$= -\frac{17}{24}$$

4.

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow \begin{cases} \dfrac{\partial x}{\partial r} = \cos\theta \\ \dfrac{\partial y}{\partial r} = \sin\theta \end{cases} \cdot \begin{cases} \dfrac{\partial x}{\partial \theta} = -r\sin\theta \\ \dfrac{\partial y}{\partial \theta} = r\cos\theta \end{cases}$$

$$\frac{\partial u}{\partial r} = \frac{\partial x}{\partial r}\frac{\partial u}{\partial x} + \frac{\partial y}{\partial r}\frac{\partial u}{\partial y} = \cos\theta\frac{\partial u}{\partial x} + \sin\theta\frac{\partial u}{\partial y},$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial x}{\partial \theta}\frac{\partial u}{\partial x} + \frac{\partial y}{\partial \theta}\frac{\partial u}{\partial y} = -r\sin\theta\frac{\partial u}{\partial x} + r\cos\theta\frac{\partial u}{\partial y},$$

同理，$\dfrac{\partial v}{\partial r} = \cos\theta\dfrac{\partial v}{\partial x} + \sin\theta\dfrac{\partial v}{\partial y}$,

$$\frac{\partial v}{\partial \theta} = -r\sin\theta\frac{\partial v}{\partial x} + r\cos\theta\frac{\partial v}{\partial y}$$

柯西·黎曼方程式为：

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

代入得：

$$\begin{aligned}\frac{\partial u}{\partial r} &= \cos\theta\frac{\partial u}{\partial x} + \sin\theta\frac{\partial u}{\partial y} \\ &= \cos\theta\frac{\partial v}{\partial y} - \sin\theta\frac{\partial v}{\partial x} \\ &= \frac{1}{r}\frac{\partial v}{\partial \theta}\end{aligned}$$

$$\begin{aligned}\frac{\partial v}{\partial r} &= \cos\theta\frac{\partial v}{\partial x} + \sin\theta\frac{\partial v}{\partial y} \\ &= -\cos\theta\frac{\partial u}{\partial y} + \sin\theta\frac{\partial u}{\partial x} \\ &= -\frac{1}{r}\frac{\partial u}{\partial \theta}\end{aligned}$$

5.

(1)

$$1 = \iint_{\Omega} f(x,y)\,dxdy$$
$$= \frac{1}{c}\left\{\int_0^1 e^{-x}dx\int_0^1 dy + \int_0^1 dx\int_0^1 e^{-y}dy\right\}$$
$$= \frac{1}{c}\,\frac{2(e-1)}{e}$$
$$\Rightarrow C = \frac{2(e-1)}{e}$$

(2)

$$f_X(x) = \int_0^1 f(x,y)\,dy$$
$$= \frac{1}{c}\int_0^1 (e^{-x}+e^{-y})\,dy$$
$$= \frac{e}{2(e-1)}e^{-x} + \frac{1}{2}$$

同理：

$$f_Y(y) = \frac{e}{2(e-1)}e^{-y} + \frac{1}{2}$$

$$f(x,y) \neq f_X(x)f_Y(y)$$

$\Rightarrow$ X、Y 不相互独立.

(3)

$$f_{X|Y=0}(x) = \frac{f(x,0)}{f_Y(0)}$$
$$= \frac{e(e^{-x}+1)}{2e-1}$$

$$E(X|Y=0) = \int_0^1 x f_{X|Y=0}(x)\,dx$$
$$= \frac{e}{2e-1}\int_0^1 x(e^{-x}+1)\,dx$$
$$= \frac{3e-4}{2(2e-1)}$$