F1-2

(1)

A)

$$y = \frac{1}{1+e^{-x}} \Rightarrow e^{-x} = \frac{1-y}{y}$$

$$\frac{dy}{dx} = \frac{d}{dx}\frac{1}{1+e^{-x}}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} \quad \cdots ①$$

$$= y(1-y)$$

$$= -y^2 + y$$

由①可得，对于任意实数 x，均有 $\frac{dy}{dx} > 0$

由 $\lim_{x\to-\infty} y = 0$，$\lim_{x\to\infty} y = 1 \Rightarrow y \in (0, 1)$

且 $\frac{dy}{dx} = -y^2 + y$

$$= -(y-\tfrac{1}{2})^2 + \tfrac{1}{4}$$

綜上，$\frac{dy}{dx} \in (0, \frac{1}{4}]$

B)

$$\frac{dy}{dx} = \frac{d}{dx}\,\frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$= \frac{4}{(e^x + e^{-x})^2} \quad \cdots \quad ②$$

$$= -y^2 + 1$$

由②可得，对于任意实数 x，均有 $\frac{dy}{dx} > 0$

由 $\lim\limits_{x \to -\infty} y = -1$，$\lim\limits_{x \to \infty} = 1 \Rightarrow y \in (-1, 1)$

且 $\frac{dy}{dx} = -y^2 + 1 \leq 1$

综上，$\frac{dy}{dx} \in (0, 1]$

(2)

設 $\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$ ，$\theta \in [0, \frac{\pi}{2}]$

$g(\theta) = \cos^3\theta + 2\sin^3\theta$

$g'(\theta) = 3\sin\theta\cos\theta(-\cos\theta + 2\sin\theta)$

$g'(\theta) = 0 \Rightarrow \theta = \arctan\frac{1}{2} \;/\; 0 \;/\; \frac{\pi}{2}$

θ	$\cdots$	0	$\cdots$	$\arctan\frac{1}{2}$	$\cdots$	$\frac{\pi}{2}$	$\cdots$
$g'(\theta)$	$+$	0	$-$	0	$+$	0	$-$
$g(\theta)$	$\nearrow$	1	$\searrow$	$\frac{2}{\sqrt{5}}$	$\nearrow$	2	$\searrow$

綜上，極大值为 1、2

極小值为 $\frac{2}{\sqrt{5}}$