Appendix: PyPSA Unit Commitment and Economic Dispatch Model

A Model

A.1 Indices and Sets

$a = (i,j) \in \mathcal{A} \subset \mathcal{N} \times \mathcal{N}$	Set of directed transmission lines (arcs)
$b \in \mathcal{B}$	Set of time blocks (e.g., days, weeks, seasons)
$f \in \mathcal{F}$	Set of fuel types (e.g., gas, coal, uranium)
$g \in \mathcal{G}$	Set of all generators
$g \in \mathcal{G}_n$	Set of generators at node n
$g \in \mathcal{G}^{\text{Committable}}$	Set of committable generators
$g \in \mathcal{G}^{\text{Thermal}}$	Set of thermal generators
$g \in \mathcal{G}^{\text{Renew}}$	Set of renewable generators
$\{i,j\} \in \mathcal{L} \subset \mathcal{N} \times \mathcal{N}$	Set of undirected transmission lines
$\{i,j\} \in \mathcal{L}_n$	Set of undirected transmission lines incident to node n
$n \in \mathcal{N}_l$	Set of nodes that are endpoints of line l
$n \in \mathcal{N}$	Set of transmission network nodes
$s \in \mathcal{S}$	Set of energy storage units
$s \in \mathcal{S}_n$	Set of energy storage units at node n
$t \in \mathcal{T}$	Set of time periods
$t \in \mathcal{T}_{-k}$	Set of time periods excluding time period k
$n_g \in \mathcal{N}$	Node at which generator g is located

The notation $\{i,j\}$ means that transmission line $\{i,j\}$ is undirected, whereas the (i,j) means directional flow from i to j. One can think of the set $\mathcal{L}$ as an upper triangular $N \times N$ binary matrix $\mathbf{L}$ having positive entries $l_{i,j}$ when transmission line $\{i,j\}$ exists for $i < j$.

A.2 Deterministic Parameters

A.2.1 Deterministic Cost Parameters

C_g^{VOM}	Variable O&M cost (\$/MWh) for generator type g
C_g^{Startup}	Start up cost (\$/start up) for generator type g
C_g^{Shutdown}	Shut down cost (\$/shut down) for generator type g

A.2.2 Operational/Technological Parameters

$F_{g,b,t}^{\text{CapFactor}}$	Capacity factor (fraction) for generator type g in time block b and in time period t
D_t	Load (MW) in time period t
$D_{n,b,t}$	Load (MW) at node n in time block b and in time period t
F_g^{Res}	Fraction of committable generator g's capacity that can be allocated to reserves
H_g	Heat rate (MMBtu/MWh) of generator type g
R_g^{Down}	Ramp down limit (MW per time step) for generator type g
R_g^{Up}	Ramp up limit (MW per time step) for generator type g
$R_g^{\text{Down,Off}}$	Ramp down limit (MW per time step) for generator type g shutting down ($R_g^{\text{Down,Off}} = \max\{R_g^{\text{Down}}, P_g^{\text{min}}\}$)
$R_g^{\text{Up,Start}}$	Ramp up limit (MW per time step) for generator type g when starting up
$R_{b,t}^{\text{Res}}$	Minimum reserves required in time period t
P_g^{max}	Maximum power (MW) from generator type g
P_g^{min}	Minimum power (MW) from generator type g
$P_l^{\text{Trans,max}}$	Maximum power or capacity (MW) that can be transmitted on line $l = \{i, j\}$
$Q_s^{\text{max,Discharge}}$	Maximum power (MW) that can be discharged from storage unit s
T^{Step}	Time step, e.g., one hour

A.3 Decision Variables

Below “in (b, t)” means “in time block b and in time period t.”

A.3.1 Continuous decision variables

Standard UCED decision variables

$p^{\text{Gen}}_{g,b,t}$	Power from generator type g in (b,t)	[MW]
$p^{\text{Res}}_{g,b,t}$	Power from generator type g allocated to reserves in (b,t)	[MW]
$p^{\text{Trans}}_{l,b,t}$	Power transmitted on line $l = \{i,j\}$ in (b,t)	[MW]
$p^{\text{Trans}}_{i,j,b,t}$	Power transmitted on arc (i,j) in (b,t)	[MW]
$p^{\text{Excess}}_{n,b,t}$	Excess power or curtailment at node n in (b,t)	[MW]
$p^{\text{Unmet}}_{n,b,t}$	Unmet demand/power (unserved load) at node n in (b,t)	[MW]

Storage decision variables

$q^{\text{Charge}}_{s,b,t}$	Power charged into storage unit s in (b,t)	[MW]
$q^{\text{Discharge}}_{s,b,t}$	Power discharged from storage unit s in (b,t)	[MW]
$q^{\text{Res}}_{s,b,t}$	Power allocated to reserves from storage unit s in (b,t)	[MW]
$q^{\text{SOC}}_{s,b,t}$	Energy in storage unit s at the end of (b,t)	[MWh]

Demand response decision variables

$d^{\text{TotalDR}}_{n,b,t}$	Total amount of DR fulfilled at node n in (b,t)	[MW]
$d^{\text{DR}}_{k,b,t}$	Amount of DR bidder k's load that is fulfilled in (b,t)	[MW]

A.3.2 Integer decision variables

$u_{g,b,t} \in \{0,1\}$	Takes value 1 if generator g is turned on (started up) in (b,t); 0 otherwise
$v_{g,b,t} \in \{0,1\}$	Takes value 1 if generator g is turned off (shut down) in (b,t); 0 otherwise
$w^{\text{On}}_{g,b,t} \in \{0,1\}$	Takes value 1 if generator g is on in (b,t); 0 otherwise

A.4 Simple day-ahead unit commitment and economic dispatch model without demand response

This optimization model is a mixed-integer linear program (MILP). It allows one to model generators as integer clusters, while still preserving unit commitment constraints.

$$\begin{aligned}
\min \quad & \sum_{g\in\mathcal{G}^{\text{Thermal}}} \sum_{b\in\mathcal{B}} \sum_{t\in\mathcal{T}} C^{\text{VOM}}_{g,t} T^{\text{Step}} p^{\text{Gen}}_{g,b,t} \\
& + \sum_{g\in\mathcal{G}^{\text{Thermal}}} \sum_{b\in\mathcal{B}} \sum_{t\in\mathcal{T}_{-1}} C^{\text{Startup}}_{g,t} u_{g,t} + C^{\text{Shutdown}}_{g,t} v_{g,t} \\
& + \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{t\in\mathcal{T}} C^{\text{VOLL}} p^{\text{Unmet}}_{n,b,t} + \sum_{b\in\mathcal{B}} \sum_{t\in\mathcal{T}} C^{\text{UnmetRes}}_{t} p^{\text{UnmetRes}}_{n,b,t} \qquad (1a)
\end{aligned}$$

$$
\begin{aligned}
\text{s.t.}\quad & \sum_{g\in\mathcal{G}_n} p^{\text{Gen}}_{g,b,t} + \sum_{i:(i,n)\in\mathcal{A}} p^{\text{Trans}}_{i,n,b,t} + \sum_{s\in\mathcal{S}_n} \eta^{\text{Discharge}}_s q^{\text{Discharge}}_{s,b,t} + p^{\text{Unmet}}_{n,b,t} = \\
& \quad D_{n,b,t} + \sum_{j:(n,j)\in\mathcal{A}} p^{\text{Trans}}_{n,j,b,t} + \sum_{s\in\mathcal{S}_n} q^{\text{Charge}}_{s,b,t}/\eta^{\text{Charge}}_s + p^{\text{Excess}}_{n,b,t} && \forall n\in\mathcal{N}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1b)}\\
& p^{\text{Gen}}_{g,b,t} - p^{\text{Gen}}_{g,b,t-1} \le R^{\text{Up}}_g(1-u_{g,t}) + \max\{R^{\text{Up}}_g, P^{\min}_g\}u_{g,t} && \forall g\in\mathcal{G}^{\text{Thermal}}, b\in\mathcal{B}, t\in\mathcal{T}_{-1} && \text{(1c)}\\
& p^{\text{Gen}}_{g,b,t-1} - p^{\text{Gen}}_{g,b,t} \le R^{\text{Down}}_g(1-v_{g,t}) + \max\{R^{\text{Down}}_g, P^{\min}_g\}v_{g,t} && \forall g\in\mathcal{G}^{\text{Thermal}}, b\in\mathcal{B}, t\in\mathcal{T}_{-1} && \text{(1d)}\\
& P^{\min}_g w^{\text{On}}_{g,b,t} \le p^{\text{Gen}}_{g,b,t} \le P^{\max}_g w^{\text{On}}_{g,b,t} && \forall g\in\mathcal{G}^{\text{Thermal}}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1e)}\\
& u_{g,b,t} - v_{g,b,t} = w^{\text{On}}_{g,b,t} - w^{\text{On}}_{g,b,t-1} && \forall g\in\mathcal{G}^{\text{Thermal}}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1f)}\\
& u_{g,b,t} + v_{g,b,t} \le 1 && \forall g\in\mathcal{G}^{\text{Thermal}}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1g)}\\
& w^{\text{On}}_{g,b,t} - w^{\text{On}}_{g,b,t-1} \le w^{\text{On}}_{g,b,\tau} && && \text{(1h)}\\
& \qquad \forall g\in\mathcal{G}^{\text{Thermal}}, t\in\mathcal{T}_{-1}, \tau\in[t+1, \min\{t+T^{\text{MinUp}}_g, T\}]\\
& q^{\text{SOC}}_{s,b,t-1}(1-F^{\text{StorageLoss}}_s) + q^{\text{Charge}}_{s,b,t}T^{\text{Step}} = q^{\text{Discharge}}_{s,b,t}T^{\text{Step}} + q^{\text{SOC}}_{s,b,t} && \forall s\in\mathcal{S}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1i)}\\
& q^{\text{Discharge}}_{s,b,t} + q^{\text{Res}}_{s,b,t} \le q^{\text{SOC}}_{s,b,t}/T^{\text{Step}} && \forall s\in\mathcal{S}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1j)}\\
& q^{\text{Discharge}}_{s,b,t} + q^{\text{Res}}_{s,b,t} \le Q^{\max,\text{Discharge}}_s && \forall s\in\mathcal{S}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1k)}\\
& p^{\text{Gen}}_{g,b,t} = F^{\text{CapFactor}}_{g,b,t} P^{\max}_g && \forall g\in\mathcal{G}^{\text{Renew}}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1l)}\\
& p^{\text{Trans}}_{i,j,b,t} \in [0, P^{\text{Trans,max}}_{i,j}] && \forall (i,j)\in\mathcal{A}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1m)}\\
& p^{\text{Excess}}_{n,b,t} \ge 0, p^{\text{Unmet}}_{n,b,t} \ge 0 && \forall n\in\mathcal{N}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1n)}\\
& w^{\text{On}}_{g,b,t}, u_{g,b,t}, v_{g,b,t} \in \{0,1\} && \forall g\in\mathcal{G}^{\text{Thermal}}, b\in\mathcal{B}, t\in\mathcal{T} && \text{(1o)}
\end{aligned}
$$

Constraint description

- Load balance constraints (1b) state that power generated $\left(\sum_{g\in\mathcal{G}} p^{\text{Gen}}_{gbt}\right)$ equals load/demand (D_{nbt}) in every time period and at every node. Curtailment ($p^{\text{Excess}}_{n,b,t}$) and unmet demand ($p^{\text{Unmet}}_{n,b,t}$) variables are included as well.
- Ramping constraints are captured by (1c) and (1d).
- Constraints (1f) ensure that start ups and shut downs are accurately accounted for.
- Constraints (1j) ensure that the power discharged + power allocated to reserves does not exceed the power output equivalent of the current state of charge or the maximum discharge rate, i.e., $q^{\text{Discharge}}_{s,b,t} + q^{\text{Res}}_{s,b,t} \le \min\{q^{\text{SOC}}_{s,b,t}/T^{\text{Step}}, Q^{\max,\text{Discharge}}_s\}$. For example, a 4-hour duration battery with 1 MWh of energy storage can discharge at most 0.25 MW per hour. Hence, this fully charged 4-hour 1 MWh battery cannot allocate 1 MW to reserves, only 0.25 MW.
- Bounds on generator power limits are expressed in (1e).

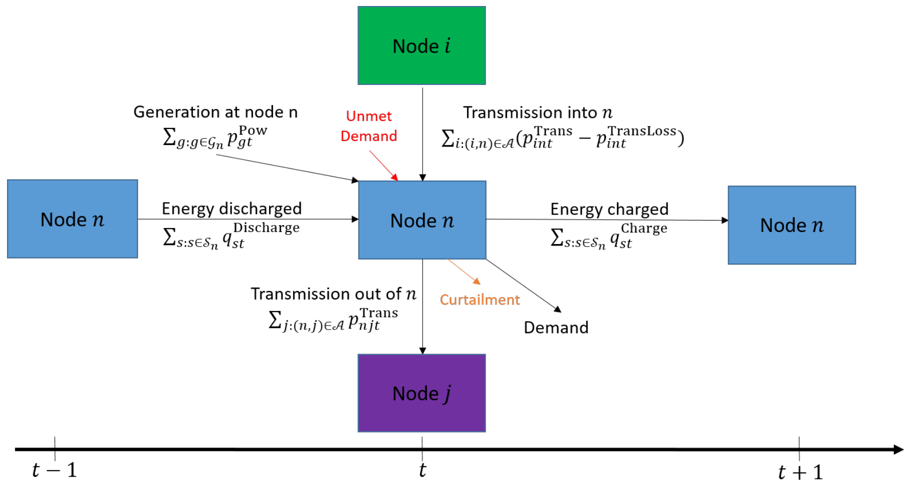

Figure 1: Load balance constraint with transmission losses and energy storage.

Typically, ramping constraints would appear as

$$p_{g,t}^{\text{Gen}} - p_{g,t-1}^{\text{Gen}} \leq R_g^{\text{Up}} \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1\} \tag{2a}$$

$$p_{g,t-1}^{\text{Gen}} - p_{g,t}^{\text{Gen}} \leq R_g^{\text{Down}} \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1\} \tag{2b}$$

However, with minimum power output levels, it may not be possible to ramp up from 0 output to $P_g^{\min}$ in a single period. In Model 1, ramping constraints have been modified, as shown in constraints (1c) and (1d), to allow generators to fully ramp up to or down from $P_g^{\min}$ in a single time step. One can also create a more elaborate constraint to allow for a generator to ramp up to or down from $P_g^{\min}$ over multiple time steps.

A.5 Energy storage

Energy storage equations are straightforward, but do require bookkeeping. Switching between power and energy while handling losses and inefficiencies requires care. Recall that the (gross) power discharged from storage unit s in (b,t) is $q_{s,b,t}^{\text{Discharge}}$. Note that not all of this power makes its way onto the grid due to efficiency losses. Indeed, the (net) amount of energy exiting storage unit s in (b,t) and going onto the grid (and therefore contributing to load) is

$$e_{s,b,t}^{\text{NetOutflow}} = \eta_s^{\text{Discharge}} q_{s,b,t}^{\text{Discharge}} T^{\text{Step}} \ . \tag{3}$$

The (net) amount of energy entering storage unit s in (b,t) is

$$e_{s,b,t}^{\text{NetInflow}} = \eta_s^{\text{Charge}} q_{s,b,t}^{\text{Charge}} T^{\text{Step}} \tag{4}$$

This is the net energy after taking into account energy losses due to charging (in)efficiencies.

A.6 Modifications for reserve constraints

When unit commitment decisions are present, reserve constraints should also be included to account for the fact that we do not have a perfect prediction of load and renewable output. Likewise, some generators or transmission lines may fail, in which case we need backup for the system. There are several forms of reserve constraints. Here we will include two types: spinning reserves and quick-start reserves. Spinning reserves come from generators that are currently on, but operating below capacity. Quick-start reserves come from generators that are currently off, but can quickly turn on. Introduce two new decision variables, $p_{g,t}^{\text{Spin}}$ and $p_{g,t}^{\text{QS}}$, denoting the amount of power allocated to spinning and quick-start reserves. Replace constraints (1e) for committable generators with

$$P_g^{\min} w_{g,b,t}^{\text{On}} \leq p_{g,b,t}^{\text{Gen}} \qquad \forall g \in \mathcal{G}^{\text{Committable}}, b \in \mathcal{B}, t \in \mathcal{T} \tag{5a}$$

$$p_{g,b,t}^{\text{Gen}} + p_{g,b,t}^{\text{Res}} \leq P_g^{\max} w_{g,b,t}^{\text{On}} \qquad \forall g \in \mathcal{G}^{\text{Committable}}, b \in \mathcal{B}, t \in \mathcal{T} \tag{5b}$$

$$p_{g,b,t}^{\text{Res}} \leq F_g^{\text{Res}} P_g^{\max} w_{g,b,t}^{\text{On}} \qquad \forall g \in \mathcal{G}^{\text{Committable}}, b \in \mathcal{B}, t \in \mathcal{T} \tag{5c}$$

$$\sum_{g \in \mathcal{G}^{\text{Committable}}} p_{g,b,t}^{\text{Res}} + \sum_{s \in \mathcal{S}} q_{s,b,t}^{\text{Res}} \geq R_{b,t}^{\text{Res}} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \tag{5d}$$

A.7 Modifications for demand response as deferrable/time-shiftable/flexible loads

We now consider demand response with deadlines because they play a central role in creating load flexibility and enhancing demand response and peak-load shaving programs. Both basic and complex time-shiftable load types are addressed, where the latter includes time-shiftable loads that are uninterruptible, have per-time-slot consumption limits or ramp constraints, or comprise several smaller time-shiftable subloads. See Su and Kirschen [1].

We revisit load balance constraints (1b), but without transmission or storage to ease the presentation. Introducing a new decision variable $d_{n,b,t}^{\text{TotalDR}}$ representing the total amount of flexible load fulfilled at node n in (b,t), we have

$$\sum_{g \in \mathcal{G}_n} p_{g,b,t}^{\text{Gen}} + p_{n,b,t}^{\text{Unmet}} = D_{n,b,t} + d_{n,b,t}^{\text{TotalDR}} + p_{n,b,t}^{\text{Excess}} \qquad \forall n \in \mathcal{N}, b \in \mathcal{B}, t \in \mathcal{T} \tag{6}$$

Let $\mathcal{K}_{n,b,t}$ be the set of demand response bidders participating at node n in time block b and time period t. Let the decision variable $d_{k,b,t}^{\text{DR}}$ denote the amount of bidder k's load that is fulfilled in (b,t). Note that each bidder k is associated with a particular node n and time block b and time interval/window $[t_1^k, t_2^k] = \{t_1^k, \dots, t_2^k\}$.

$$D_k^{\text{MinTotDR}} \leq \sum_{t=t_1^k}^{t_2^k} d_{k,b,t}^{\text{DR}} \leq D_k^{\text{MaxTotDR}} \qquad \forall k \in \mathcal{K}, b \in \mathcal{B} \tag{7a}$$

$$D_{k,b,t}^{\text{MinDR}} \leq d_{k,b,t}^{\text{DR}} \leq D_{k,b,t}^{\text{MaxDR}} \qquad \forall k \in \mathcal{K}, b \in \mathcal{B}, t \in \mathcal{T} \tag{7b}$$

$$-D_{k,b,t}^{\text{RampDown}} \leq d_{k,b,t}^{\text{DR}} - d_{k,b,t-1}^{\text{DR}} \leq D_{k,b,t}^{\text{RampUp}} \qquad \forall k \in \mathcal{K}, b \in \mathcal{B}, t \in \mathcal{T} \tag{7c}$$

More generally, for every bid k, we can define a feasible region $\mathcal{D}_k$, which could include even more complex restrictions on the demand response decision variables $(d^{\mathrm{DR}}_{k,b,t_1^k}, \ldots, d^{\mathrm{DR}}_{k,b,t_2^k})$.

References

[1] C.-L. Su and D. Kirschen. Quantifying the effect of demand response on electricity markets. *IEEE Transactions on Power Systems*, 24(3):1199–1207, 2009.