Channel Modeling and Dataset Generation

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Notations Vectors are denoted by bold lower-case letters. j is the imaginary unit, i.e., $j^2 = -1$. The Euclidean norm of a vector $\mathbf{v}$ is denoted by $\|\mathbf{v}\|$.

We consider a radio system made of a single transmitter and a single receiver. The transmitter is assumed to be fixed, while the receiver is mobile. Let us denote by $\mathbf{p} \in \mathbb{R}^3$ the position of the receiver. The baseband equivalent channel is [1, Section 2.2.2]

$$h(\mathbf{p}) = \sum_i h^{(i)}(\mathbf{p}) \tag{1}$$

where summation is over the paths, and where

$$h^{(i)}(\mathbf{p}) \coloneqq a^{(i)}(\mathbf{p}) \exp\left(-j2\pi \frac{l^{(i)}(\mathbf{p})}{\lambda}\right) \tag{2}$$

where $a^{(i)}(\mathbf{p}) \in \mathbb{R}$ is the overall attenuation, $l^{(i)}(\mathbf{p})$ is the distance traveled by the i^{th} path from the transmitter to the receiver, and λ is the wavelength, i.e., $\lambda = \frac{c}{f_c}$, with c the speed of light and f_c the carrier frequency.

As we consider experimentation of small duration in our simulations (a few milliseconds at most), the overall attenuation $a^{(i)}(\mathbf{p})$ is assumed to be constant over such a small duration, i.e., we neglect its dependency to $\mathbf{p}$. Let us consider a small *linear* displacement $\mathbf{d} \in \mathbb{R}^3$ from an initial position $\mathbf{p}_0$. Then,

$$\begin{aligned} h^{(i)}(\mathbf{p}_0 + \mathbf{d}) &= a^{(i)} \exp\left(-j\left(2\pi\frac{l^{(i)}(\mathbf{p}_0)}{\lambda} + 2\pi\frac{\|\mathbf{d}\|\cos\theta^{(i)}}{\lambda}\right)\right) && (3)\\ &= a^{(i)} \exp\left(-j\left(2\pi\frac{\|\mathbf{d}\|\cos\theta^{(i)}}{\lambda} + \phi_0^{(i)}\right)\right) && (4)\end{aligned}$$

where $\theta^{(i)}$ is the angle between $\mathbf{d}$ and the direction of arrival of the i^{th} path, and $\phi_0^{(i)} \coloneqq 2\pi\frac{l^{(i)}(\mathbf{p}_0)}{\lambda}$ is the initial phase. The initial phase $\phi_0^{(i)}$ is typically chosen randomly and uniformly by channel simulators such as Quadriga.

Let us denote by d the norm of $\mathbf{d}$, i.e., $d \coloneqq \|\mathbf{d}\|$. We can rewrite (4) as a 1D spatial process which corresponds to the channel response along the half-line generated by $\mathbf{d}$ and the initial point $\mathbf{p}_0$,

$$h_{\text{1D}}^{(i)}(d) = a^{(i)} \exp\left(-j\left(2\pi\frac{d\cos\theta^{(i)}}{\lambda} + \phi_0^{(i)}\right)\right). \tag{5}$$

By taking the Fourier transform of (5), one gets

$$H_{1\mathrm{D}}^{(i)}(\xi) = a^{(i)} \exp\left(-j\phi_0^{(i)}\right) \delta\left(\xi + \frac{\cos\theta^{(i)}}{\lambda}\right). \tag{6}$$

Therefore, the spectral representation of the channel response (1) is a sum of Dirac delta functions shifted by $\frac{\cos\theta^{(i)}}{\lambda}$, which take values within the range $(-\frac{1}{\lambda}, \frac{1}{\lambda})$. Thus, the spatial channel process is bandlimited to $\frac{2}{\lambda}$, and sampling it with a period of $\frac{\lambda}{2}$ is sufficient to capture it in all its complexity.

It is easy to get a temporal process from the spatial process. Assuming the user is moving along the half-line generated by $\mathbf{p}_0$ and $\mathbf{d}$ at a constant speed v, we can simply substitute d by vt, t being the time variable, to get

$$h_{1\mathrm{D}}^{(i)}(t) = a^{(i)} \exp\left(-j\left(2\pi\frac{vt\cos\theta^{(i)}}{\lambda} + \phi_0^{(i)}\right)\right). \tag{7}$$

By taking the Fourier transform, one gets

$$H_{1\mathrm{D}}^{(i)}(\nu) = a^{(i)} \exp\left(-j\phi_0^{(i)}\right) \delta\left(\nu + \frac{v\cos\theta^{(i)}}{\lambda}\right) \tag{8}$$

which shows that the temporal channel process is bandlimited to $\frac{2v}{\lambda} = 2\nu_M$, where $\nu_M := \frac{v}{c} f_c$ is the maximum Doppler shift. The spectrum of the temporal process can therefore be obtained by simply scaling the bandwidth of the spatial process by the speed of the user v.

By sampling uniformly the spatial channel process using the spatial period $\frac{\lambda}{2}$, one can reconstruct the equivalent temporal process, for a speed v and a corresponding maximum Doppler shit of $2\nu_M$, using the Sampling Theorem,

$$h_{1\mathrm{D}}^{(i)}(t) = \sum_{n=-\infty}^{+\infty} h_{1\mathrm{D}}^{(i)}[n]\operatorname{sinc}\left(2\nu_M t - n\right) \tag{9}$$

where $\left\{h_{1\mathrm{D}}^{(i)}[n]\right\}_n$ is the sequence of channel samples obtained using the channel simulator. In practice, one can only generate a finite number of samples $N < \infty$. Therefore, the channel coefficients can only be approximated at any instant t by

$$\widetilde{h}_{1\mathrm{D}}^{(i)}(t) = \sum_{n=-N/2}^{N/2-1} h_{1\mathrm{D}}^{(i)}[n]\operatorname{sinc}\left(2\nu_M t - n\right) \tag{10}$$

$$\approx h_{1\mathrm{D}}^{(i)}(t) \tag{11}$$

where N is assumed to be even for convenience. Assuming one requires M samples for its simulations and the bandwidth of the radio system is $W >> 2\nu_M$, the required samples can be generated using (10) by choosing $t_0 = 0$, $t_1 = \frac{1}{W}, \ldots, t_{M-1} = \frac{M-1}{W}$.

Setting N Using a finite number of samples is the first approximation that can lead to inexact channel samples. Finding a right value for N, which controls a trade-off between computational complexity and precision, is still an open problem. It might also be better to have relatively low values N (1 - 2 wavelength), but to oversample the channel, i.e., use a sampling rate higher than $2\nu_M$.

The path coefficients (7) can be expressed using the change in delays due to the mobility $\tau^{(i)}(t) := \frac{vt\cos\theta^{(i)}}{c}$, i.e.,

$$h_{\mathrm{1D}}^{(i)}(t) = a^{(i)} \exp\left(-j\left(2\pi\tau^{(i)}(t) f_c + \phi_0^{(i)}\right)\right) \tag{12}$$

where the initial phase shift can also be expressed from the initial delay, i.e., $\phi_0^{(i)} = 2\pi f_c \tau_0^{(i)}$, with $\tau_0^{(i)} := \frac{l^{(i)}(\mathbf{p}_0)}{c}$.

We now focus on the computation of tap coefficients for the equivalent discrete-time baseband model. Let us denote by K the desired number of taps. For each time step $m = 0, \ldots, M-1$, the k^{th} tap coefficient can be computed by [1, Section 2.2.3]

$$\hbar[m,k] = \sum_i h_{\mathrm{1D}}^{(i)}\left(\frac{m}{W}\right) \operatorname{sinc}\left(k - \left(\tau^{(i)}\left(\frac{m}{W}\right) + \tau_0^{(i)}\right) W\right). \tag{13}$$

To compute the taps coefficients for the discrete-time baseband model, the delays $\tau^{(i)}(t) + \tau_0^{(i)}$ for each path i and for all time steps $\frac{m}{W}$ are needed, in addition to the channel coefficients. When spatially sampling the channel using a channel simulator, such as Quadriga, one gets both samples of the channel coefficients $h_{\mathrm{1D}}^{(i)}[n]$ as well as a corresponding delays for each path i. However, it is not clear to me if one can reconstruct the delays from the samples as done for the channel coefficients. Indeed, it is not even clear if the process generating the delays as a function of space/time is bandlimited, as for the channel coefficient process. However, because of the small duration of the simulations, one can assume that the delays are constant over the duration of a simulation, i.e., ignoring $\tau^{(i)}(t)$. In practice, the sample generated by the channel simulator which is, in space or time, the closest to the simulation location, can be selected. Moreover, the channel coefficients at the desired instants are approximately reconstructed using (10),

$$\widetilde{\hbar}[m,k] = \sum_i \widetilde{h}_{\mathrm{1D}}^{(i)}\left(\frac{m}{W}\right) \operatorname{sinc}\left(k - \tau_0^{(i)} W\right) \tag{14}$$

$$\approx \hbar[m,k] \tag{15}$$

Assuming constant delays The assumption that delays are constant over the duration of the simulation is the second assumption that can lead to inaccuracies in the computation of the tap coefficients. However, I am not aware of any alternative, except running the channel simulator at the rate of the radio system.

Effect of antenna directivity The effect of antenna directivity was no considered in this model. It is not clear if antenna directivity could be neglected over short displacements.

Bibliography

[1] D. Tse and P. Viswanath, *Fundamentals of wireless communication.* Cambridge university press, 2005.