The foundations of the rigorous study of *analysis* were laid in the nineteenth century, notably by the mathematicians Cauchy and Weierstrass. Central to the study of this subject are the formal definitions of *limits* and continuity.

Let D be a subset of \mathbf{R} and let $f:D\to\mathbf{R}$ be a real-valued function on D. The function f is said to be *continuous* on D if, for all $\epsilon>0$ and for all $x\in D$, there exists some $\delta>0$ (which may depend on x) such that if $y\in D$ satisfies

$$|y - x| < \delta$$

then

$$|f(y) - f(x)| < \epsilon.$$

One may readily verify that if f and g are continuous functions on D then the functions f+g, f-g and f.g are continuous. If in addition g is everywhere non-zero then f/g is continuous.