replaced by a small finite number of parallel resistor–capacitor circuits, themselves wired in series, the cell model collapses to the one in Fig. 2.6, with additional resistor–capacitor pairs.

When we discuss implementation of physics-based models, which have a Warburg-type impedance relationship built in intrinsically, we will see how to automatically create ordinary-difference equations that very accurately model the diffusion.

2.7 *Hysteresis voltages*

With the model we have developed so far, if the cell current is set to zero, the voltage drop across R_0 will immediately fall to zero, and the voltage drop across the capacitor C_1 will decay to zero over time as the capacitor discharges through R_1. That is, the cell terminal voltage will converge to open-circuit voltage.

However, what we see in practice is different from this. The cell voltage decays to a value that is somewhat different from OCV, and the difference depends on the recent history of cell usage. For example, we find that if we discharge a cell to 50 % SOC and allow the cell to rest, the equilibrium voltage is lower than OCV. If we charge a cell to 50 % SOC and allow the cell to rest, the equilibrium voltage is higher than OCV. These observations indicate that there is hysteresis in the cell terminal voltage.

Fig. 2.11 shows data collected from a laboratory experiment designed to capture the nature of this hysteresis. The cell is first fully charged before the test begins. Then, the cell is very slowly discharged, at a C/30 rate, down to 0 % SOC. At this slow rate, the $i(t) \times R_0$ and diffusion voltages are very small, so the recorded voltage is very close to the equilibrium rest voltage of the cell. This voltage trace is the lowest line on the plot.

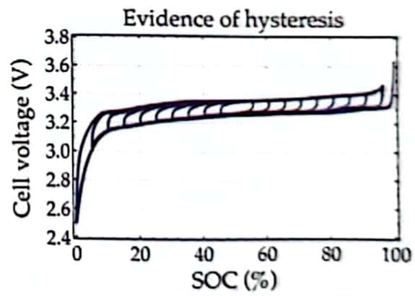

Figure 2.11: Cell test data giving evidence of hysteresis.

Then, the cell is very slowly charged, at a C/30 rate, to 95 % SOC; then the cell is discharged to 5 %, and so forth. Each point on the plot represents (at least approximately) an equilibrium point. Because of the space between the lower and upper curves, we discover that for every SOC there is a range of possible stable rest voltage values.

Vol. II in this book series addresses methods of estimating cell state of charge using equivalent-circuit cell models. To do this well, the cell model must be as accurate as possible. From the figure, we see that omitting hysteresis in our cell models can lead to large SOC estimation errors. For example, if we measure a fully rested terminal voltage of 3.3 V, that could correspond to any SOC between about 20 % and 90 %. We require a good model of hysteresis to know how different we expect the fully rested terminal voltage to be from open-circuit voltage.[10]

[10] Note that this particular cell is a lithium-ion chemistry with an iron-phosphate positive electrode. The very flat OCV characteristics of this type of cell amplify the importance of a good hysteresis model. Other lithium-ion chemistries also exhibit hysteresis, but the higher slope to their OCV relationships dilutes the errors introduced by a poor hysteresis model.

NOTE THE DISTINCTION between hysteresis and diffusion voltages: diffusion voltages change with time but hysteresis voltages change only when SOC changes. They are not directly a function of time. If a cell is allowed to rest, diffusion voltages will decay to zero, but hysteresis voltages will not change at all. Any model of hysteresis must capture this behavior.

The challenge when modeling hysteresis is that it is not very well understood. A simple model jumps immediately from the lower to the upper branch of the plot in Fig. 2.11 when current changes from a discharge to a charge. A somewhat more advanced model changes hysteresis voltage linearly with a change in SOC. We introduce a combination of these two approaches here; however, the resulting model still struggles to explain hysteresis accurately.

2.7.1 *SOC-varying hysteresis*

Let $h(z,t)$ be the dynamic hysteresis voltage as a function of SOC and time, and let $\dot{z} = \mathrm{d}z/\mathrm{d}t$. Then, we model the change in hysteresis voltage as a function of change in SOC as

$$\frac{\mathrm{d}h(z,t)}{\mathrm{d}z} = \gamma\,\mathrm{sgn}(\dot{z})\big(M(z,\dot{z}) - h(z,t)\big),$$

where $M(z,\dot{z})$ is a function that gives the maximum polarization due to hysteresis as a function of SOC and the rate-of-change of SOC. Specifically, we require that $M(z,\dot{z})$ be positive for charge ($\dot{z} > 0$) and negative for discharge ($\dot{z} < 0$). The $M(z,\dot{z}) - h(z,t)$ term in the differential equation states that the rate-of-change of hysteresis voltage is proportional to the distance of the present hysteresis value away from the major hysteresis loop, leading to a kind of exponential decay of voltage to the major loop. The term in front of this has a positive constant γ, which tunes the rate of decay, and $\mathrm{sgn}(\dot{z})$, which forces the equation to be stable both for charge and discharge.

To fit the differential equation for $h(z,t)$ into our model, we must manipulate it to be a differential equation with respect to time, not with respect to SOC. We accomplish this by multiplying both sides of the equation by $\mathrm{d}z/\mathrm{d}t$.

$$\frac{\mathrm{d}h(z,t)}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}t} = \gamma\,\mathrm{sgn}(\dot{z})\big(M(z,\dot{z}) - h(z,t)\big)\frac{\mathrm{d}z}{\mathrm{d}t}.$$

We use the chain rule to write the left-hand side of the equation as $\mathrm{d}h(z,t)/\mathrm{d}t$, and we substitute $\mathrm{d}z/\mathrm{d}t = -\eta(t)i(t)/Q$ into the right-hand side, noting that $\dot{z}\,\mathrm{sgn}(\dot{z}) = |\dot{z}|$. Thus,

$$\dot{h}(t) = -\left|\frac{\eta(t)i(t)\gamma}{Q}\right| h(t) + \left|\frac{\eta(t)i(t)\gamma}{Q}\right| M(z,\dot{z}).$$

This may be converted into a difference equation for our discrete-time application using the technique from Sec. 2.4 (assuming that $i(t)$ and $M(z,\dot{z})$ are constant over the sample period):

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right) h[k] + \left(1 - \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right) M(z,\dot{z}).$$

Note that this is a nonlinear time-varying system as the factors multiplying the state and input change with $i[k]$.

The simplest representation is when $M(z,\dot{z}) = -M\,\mathrm{sgn}(i[k])$, when

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right) h[k] - \left(1 - \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right) M\,\mathrm{sgn}(i[k]).$$

With this representation $-M \le h[k] \le M$ at all times, and $h[k]$ has units of volts. When attempting to find the model parameters, we will find it valuable to rewrite this in an equivalent but slightly different representation, which has unitless hysteresis state $-1 \le h[k] \le 1$,

$$h[k+1] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right) h[k] - \left(1 - \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)\right) \mathrm{sgn}(i[k])$$

$$\text{Hysteresis voltage} = Mh[k].$$

This makes the output equation linear in M, which will make estimating M from lab-test data easier.

2.7.2 *Instantaneous hysteresis*

In addition to the type of dynamic hysteresis that changes as SOC changes, we also often see benefit in modeling an instantaneous change in hysteresis voltage when the sign of current changes. Define

$$s[k] = \begin{cases} \mathrm{sgn}(i[k]), & |i[k]| > 0; \\ s[k-1], & \text{otherwise.} \end{cases}$$

Then, the instantaneous hysteresis is modeled as

$$\text{Instantaneous hysteresis voltage} = M_0 s[k],$$

and overall hysteresis is

$$\text{Hysteresis voltage} = M_0 s[k] + Mh[k].$$

2.8 *Enhanced self-correcting cell model*

The *enhanced self-correcting* (ESC) cell model combines all the aforementioned elements. The model is called *enhanced* because it includes a description of hysteresis, unlike some earlier models. The model is called *self-correcting* because the model's predicted terminal voltage converges to OCV plus hysteresis when the cell rests, and converges to OCV plus hysteresis minus all the current times resistance terms on a constant-current event. The final circuit diagram for this model is shown in Fig. 2.12.

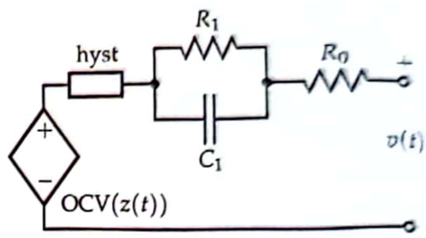

Figure 2.12: The enhanced self-correcting cell model equivalent circuit.

The figure shows an example with a single parallel resistor–capacitor pair, but the model easily accommodates multiple parallel resistor–capacitor pairs. To compact notation, define a resistor–capacitor subcircuit rate factor $F_j = \exp\left(\frac{-\Delta t}{R_j C_j}\right)$, and we have

$$i_R[k+1] = \underbrace{\begin{bmatrix} F_1 & 0 & \cdots \\ 0 & F_2 & \\ \vdots & & \ddots \end{bmatrix}}_{A_{RC}} i_R[k] + \underbrace{\begin{bmatrix} (1-F_1) \\ (1-F_2) \\ \vdots \end{bmatrix}}_{B_{RC}} i[k].$$

Then, if we define $A_H[k] = \exp\left(-\left|\frac{\eta[k]i[k]\gamma\Delta t}{Q}\right|\right)$, we have the dynamic aspects of the model described by the matrix-vector relationship

$$\begin{bmatrix} z[k+1] \\ i_R[k+1] \\ h[k+1] \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & A_{RC} & 0 \\ 0 & 0 & A_H[k] \end{bmatrix} \begin{bmatrix} z[k] \\ i_R[k] \\ h[k] \end{bmatrix} + \begin{bmatrix} -\frac{\eta[k]\Delta t}{Q} & 0 \\ B_{RC} & 0 \\ 0 & (A_H[k]-1) \end{bmatrix} \begin{bmatrix} i[k] \\ \text{sgn}(i[k]) \end{bmatrix}.$$

This is the ESC model *state equation*. The model's *output equation* is

$$v[k] = \text{OCV}(z[k], T[k]) + M_0 s[k] + Mh[k] - \sum_j R_j i_{R_j}[k] - R_0 i[k].$$

The ESC model comprises these two equations with associated parameter values filled in. Note that all model parameters must be nonnegative.

2.9 *Laboratory equipment for cell-data collection*

Parameter values for the ESC model are found by fitting the model equations to data collected from experiments performed on the cells of interest. These experiments are conducted with the aid of specially designed laboratory equipment known as *battery-cell cyclers*

Figure 2.13: Sample cell-test equipment

or *battery-cell test equipment*. An example cell cycler, manufactured by Arbin Instruments and resident in the University of Colorado Colorado Springs' High-Capacity Battery Research and Test Laboratory, is shown in Fig. 2.13. This particular cycler can perform independent experiments on up to 12 different cells simultaneously, controlling each cell's current according to a user-specified profile of demanded current versus time and recording actual cell current, voltage, and temperature. Each of the 10 thick white cables shown in the figure corresponds to a single *test channel*—the power electronics designed to conduct an independent test. Each cable contains four wires internally (two to carry current and two to measure voltage via a *Kelvin connection*) and can be connected to a single 0 V to 5 V cell, commanding current up to ±20 A per test channel. The thinner white cables are connected to *thermistors* to measure cell temperature. The two pairs of thick black cables correspond to two high-current test channels, capable of commanding current up to ±200 A each. Test channels may be connected in parallel if more current is needed than can be supplied or sunk from a single channel.

Battery-cell test equipment is generally custom designed to the user's specifications, so the cycler shown in Fig. 2.13 is discussed as a representative example only. Systems can be configured with more or fewer independent test channels, higher or lower maximum current, specialized measurement circuitry for three-terminal cells or for impedance-spectroscopy measurements, and more.

In addition, most tests must be conducted in controlled temperature settings. Fig. 2.14 shows an *environmental chamber* manufactured by Cincinnati Sub-Zero. This particular unit has 8 ft^3 of interior space and is capable of maintaining constant temperatures between −45 °C and 190 °C and commanding profiles of temperature versus time. Systems can be configured with humidity control, wider temperature ranges, rapid cooling for thermal-shock testing, and more.

Figure 2.14: Sample environmental chamber.

2.10 *Lab tests to determine OCV relationship*

A cell's open-circuit voltage is a static function of its state of charge and temperature. All other aspects of a cell's performance are dynamic in some sense. So, we perform separate experiments to collect data for the OCV versus SOC relationship and for the dynamic relationship. We first discuss experiments to determine the OCV relationship.

The general idea is straightforward. Before the cell test begins, the cell must be fully charged. Then, the cell is very slowly discharged to a minimum operating voltage while continuously measuring cell voltage and accumulated ampere-hours discharged. The cell is then very