Experiment results log.

Planned order of scenarios

Scenarios	Parameters	Week
A1	N	March 23 - March 29
A2	B	March 30 - April 5
A3	Datasets	March 30 - April 5
B1	Chain Strength	April 6 - April 12
B2	Embedding	April 13 - April 19
B3	Shots	April 20 - April 26
B4	Annealing	April 27 - May 3

Actual order of scenarios

Scenarios	Parameters	Week
A1	**N**	**March 23 - March 29**
B1	**Chain Strength**	**March 30 - April 5**
A2	**B**	**March 30 - April 5**
A3	Datasets	April 6 - April 12
B2	Embedding	April 13 - April 19
B3	Shots	April 20 - April 26
B4	Annealing	April 27 - May 3

Sidenotes to research about

- Scenario A1 epsilon values appear to follow a linear trend: `y = (x-8) * 0.0142227624 + 1`
- Find what is the maximum `N` value that is supported by dwave

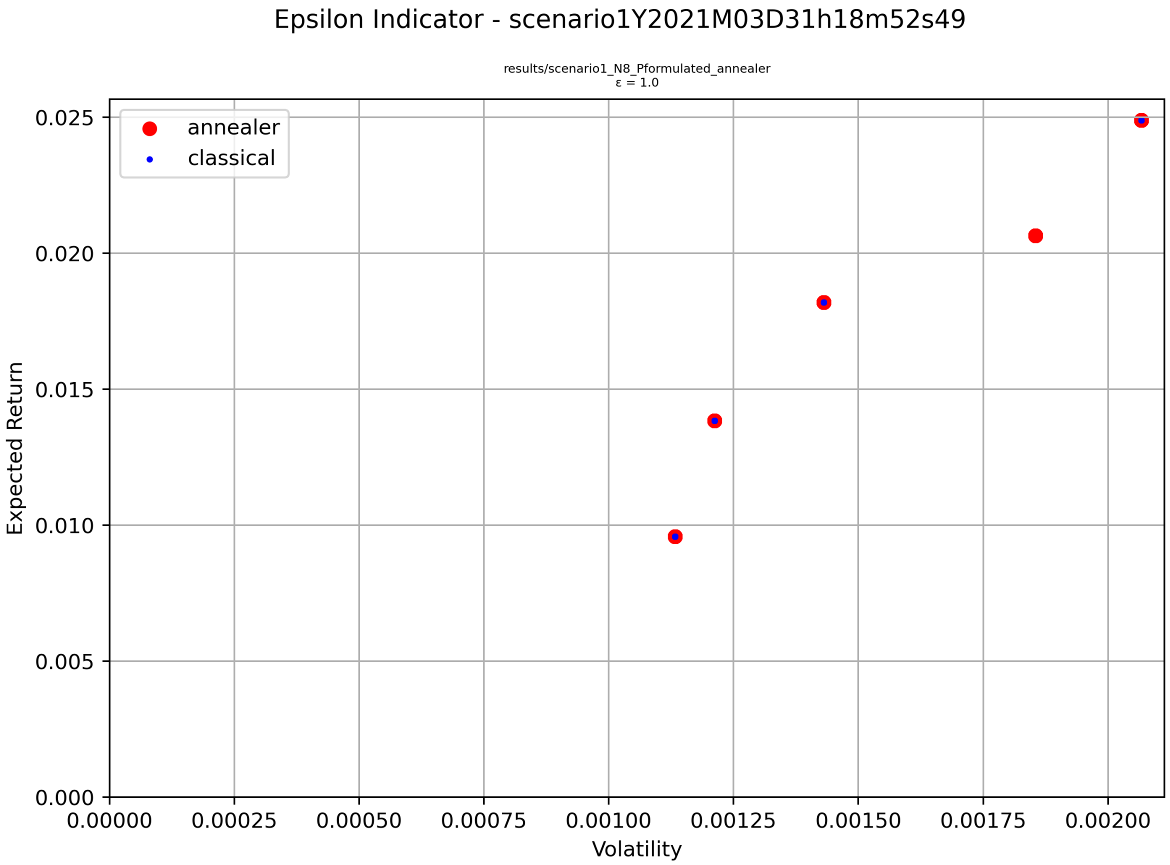
Scenario A1

We started by experimenting several values of `N`, in order to find the maximum possible value of `N` that could be solved in a reasonable time by the classical solver.

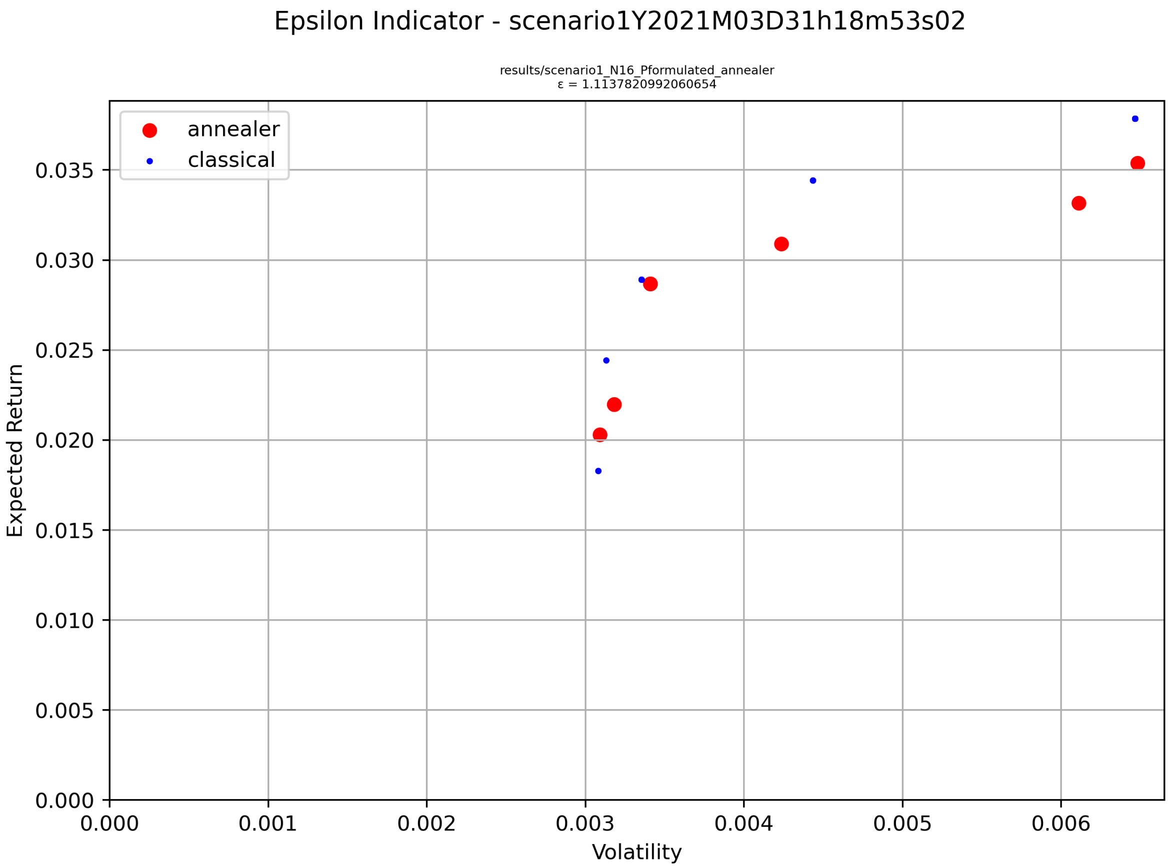
The `N` values are: 8, 16, 32, and 64. P was calculated as `P = -q * min_sigma + max_mu`

For this scenario, we used the "diversified" dataset and 1000 shots per execution. The `q_values` are listed in the following table:

N	q values	Epsilon Indicator
8	0, 11, 20, 54	1.0
16	0, 2, 6, 100, 500	1.114
32	0, 0.4, 0.9, 2, 3, 9, 100	1.340
64	0, 0.2, 0.4, 0.6, 1.1, 1.3, 1.5, 2, 5, 6, 7, 8, 10, 100, 500	1.755



How to interpret: Blue markers are part of the efficient frontier. The epsilon indicator is the minimum factor by which the red set has to be multiplied in the objective so as to weakly dominate the blue set. Hence, the closer to 1 is the epsilon indicator, the better the red set.



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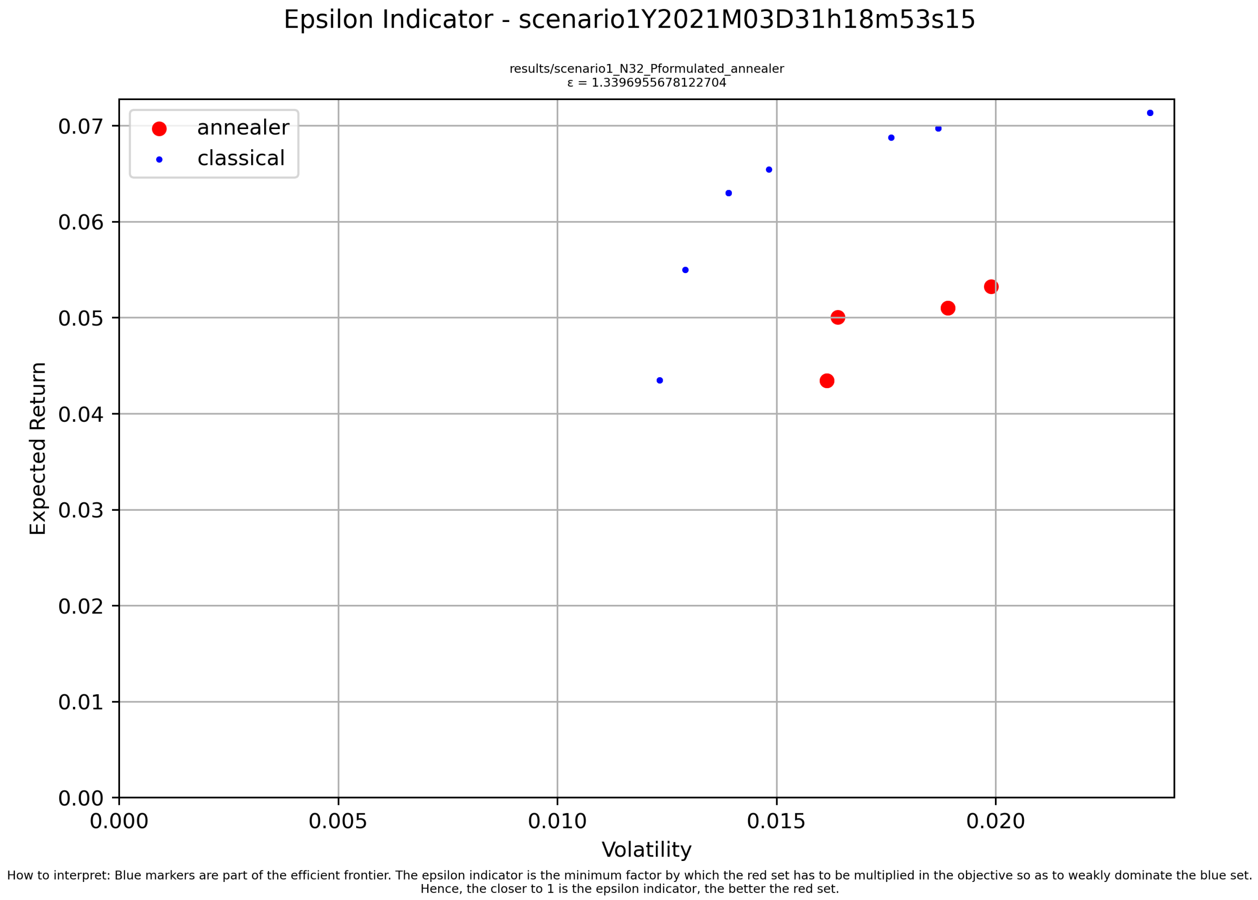
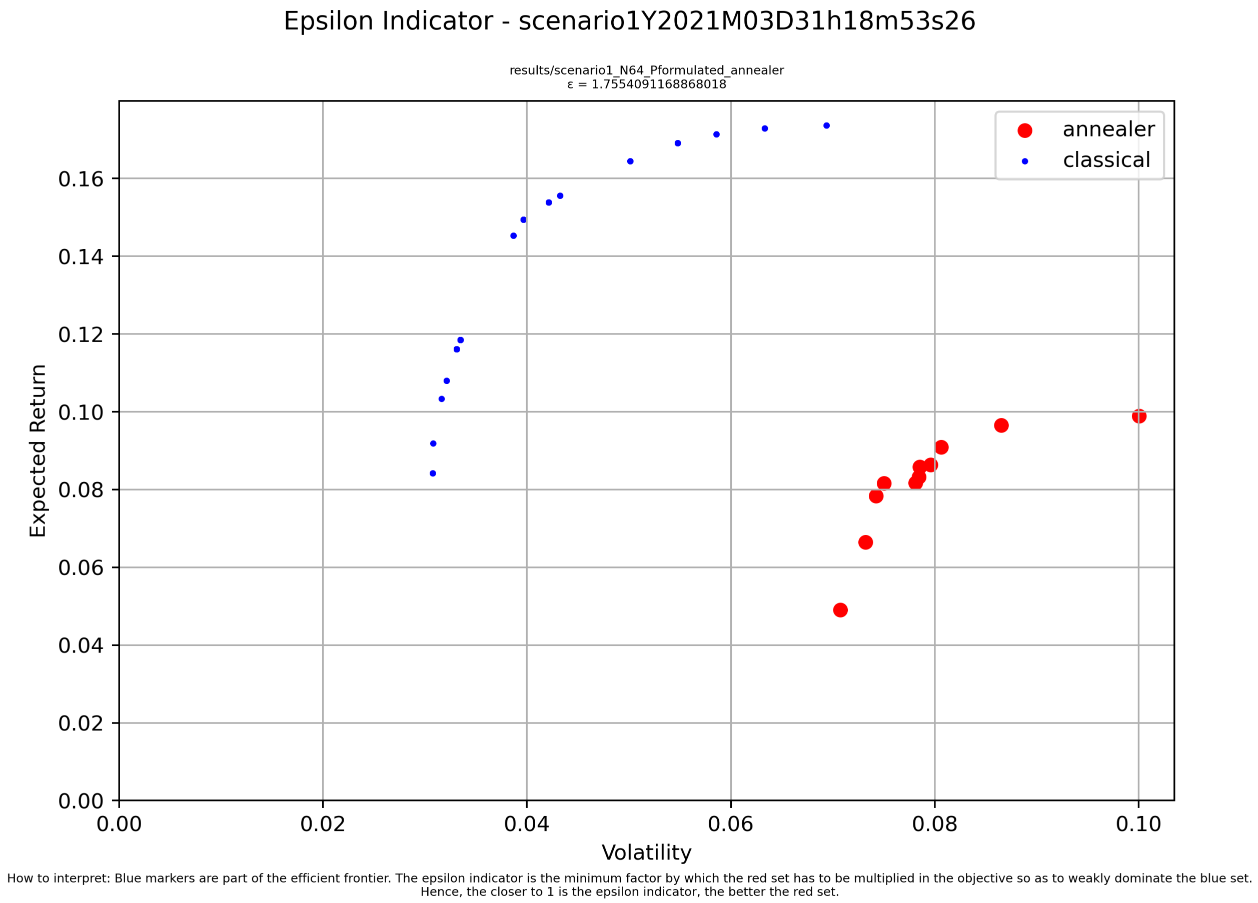




Key Takeaways:

As expected, the epsilon indicator increases with the `N` value. However, during those executions, dwave's problem inspector warned that the chains were too weak, and that, in the case of `N=64`, all samples had broken chains. Based on this warning, we decided to immediately execute scenario B1, changing the original order of scenarios.

Scenario B1

Looking at the fraction of chain breaks in Scenario A1, we know that on average each sample had almost a third (`0.31`) of its chains broken when `N=32`. This fraction increases to over half (`0.54`) when `N=64`! Those values are very high and are another clue that the chain strength needs to be adjusted, especially for those values of `N`.

A good starting value for the chain strength is the maximum absolute value (`maxAbs`) of the QUBO matrix. However, this is not always the most optimal value. We need to test several values based on this initial value. By testing those values, we can find a value near the sweet spot between the probability that the chains are intact and the probability of finding optimal values. Refer to: https://www.dwavesys.com/sites/default/files/2_Wed_Am_PerfTips.pdf

We have three tables, one for the epsilon indicator, one for the fractions of valid solutions, and one for the average fractions of chain breaks.

Starting with the average fractions of chain breaks (Lower is better):

Chain strength	N8	N16	N32	N64
default value	0.00081	0.01153	0.31350	0.54426
0.125 * maxAbs	0.00397	0.02741	0.31014	0.38301
0.250 * maxAbs	0.00034	0.00106	0.00170	0.00683
0.375 * maxAbs	0.00006	0.00032	0.00111	0.00453
0.500 * maxAbs	0.00006	0.00026	0.00149	0.00475
0.625 * maxAbs	0.00006	0.00031	0.00112	0.00453
0.750 * maxAbs	0.00006	0.00029	0.00130	0.00454
0.875 * maxAbs	0.00006	**0.00017**	0.00102	0.00461
1.000 * maxAbs	0.00003	0.00034	**0.00100**	0.00439
1.125 * maxAbs	**0.00000**	0.00030	0.00119	**0.00401**
1.250 * maxAbs	**0.00000**	0.00042	0.00125	0.00419
1.375 * maxAbs	0.00006	0.00028	0.00108	0.00424
1.500 * maxAbs	0.00009	0.00025	0.00201	0.00430

Next, we obtained the following fractions of valid solutions (Higher is better):

Chain strength	N8	N16	N32	N64
default value	0.877	**0.688**	0.121	0.094

Chain strength	N8	N16	N32	N64
0.125 * maxAbs	0.001	0.002	0.076	0.205
0.250 * maxAbs	**0.934**	0.622	**0.395**	**0.243**
0.375 * maxAbs	0.848	0.543	0.325	0.220
0.500 * maxAbs	0.781	0.485	0.299	0.186
0.625 * maxAbs	0.703	0.444	0.261	0.172
0.750 * maxAbs	0.665	0.388	0.252	0.170
0.875 * maxAbs	0.630	0.406	0.242	0.163
1.000 * maxAbs	0.598	0.366	0.235	0.151
1.125 * maxAbs	0.594	0.370	0.219	0.148
1.250 * maxAbs	0.556	0.342	0.223	0.129
1.375 * maxAbs	0.540	0.330	0.212	0.136
1.500 * maxAbs	0.512	0.310	0.198	0.138

Finally, we obtained the following epsilon indicators (Lower is better):

Chain strength	N8	N16	N32	N64
default value	1,000	1,114	1,340	1,755
0,125 * maxAbs	1,368	6,672	1,426	1,640
0,250 * maxAbs	1,000	1,167	1,245	1,504
0,375 * maxAbs	1,000	1,176	1,275	1,429
0,500 * maxAbs	1,000	1,177	1,284	1,524
0,625 * maxAbs	1,000	1,208	1,325	1,388
0,750 * maxAbs	1,000	1,164	1,364	1,423
0,875 * maxAbs	1,000	1,208	1,372	1,358
1,000 * maxAbs	1,000	1,140	1,567	1,520
1,125 * maxAbs	1,000	1,182	1,350	1,421
1,250 * maxAbs	1,000	1,218	1,457	1,518
1,375 * maxAbs	1,000	1,130	1,462	1,488
1,500 * maxAbs	1,000	1,182	1,445	1,583

To validate such results, this scenario has been repeated for `N=16`, `N=32`, and `N=64`.

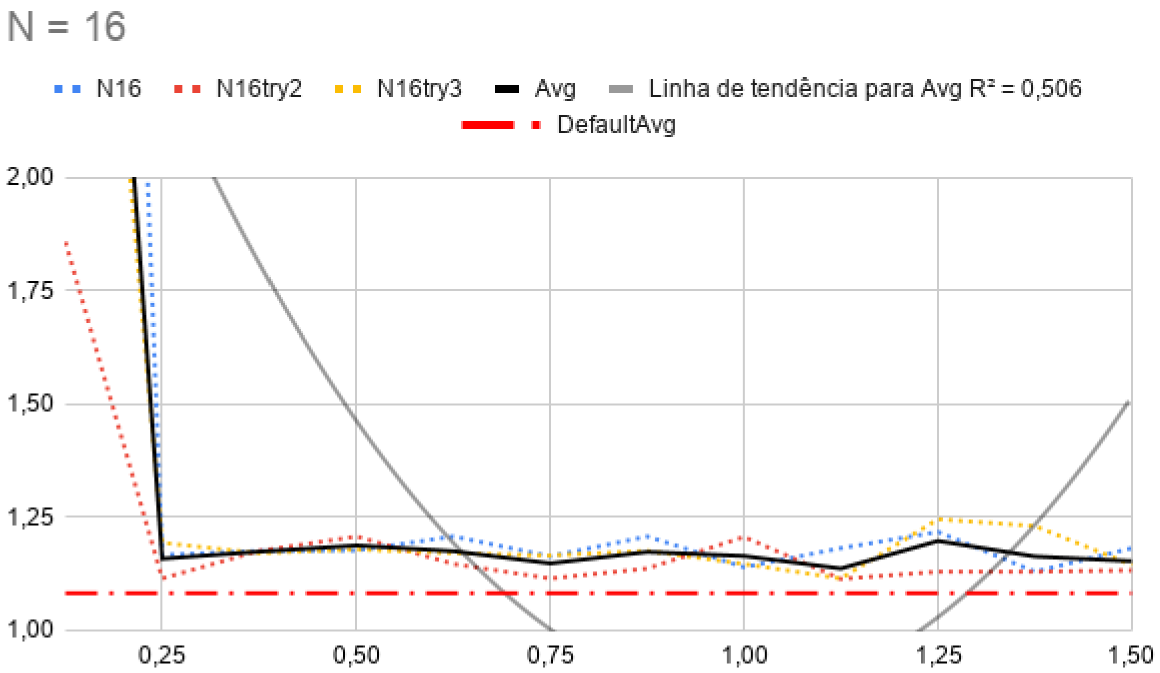
Chain strength	N16	N32	N64

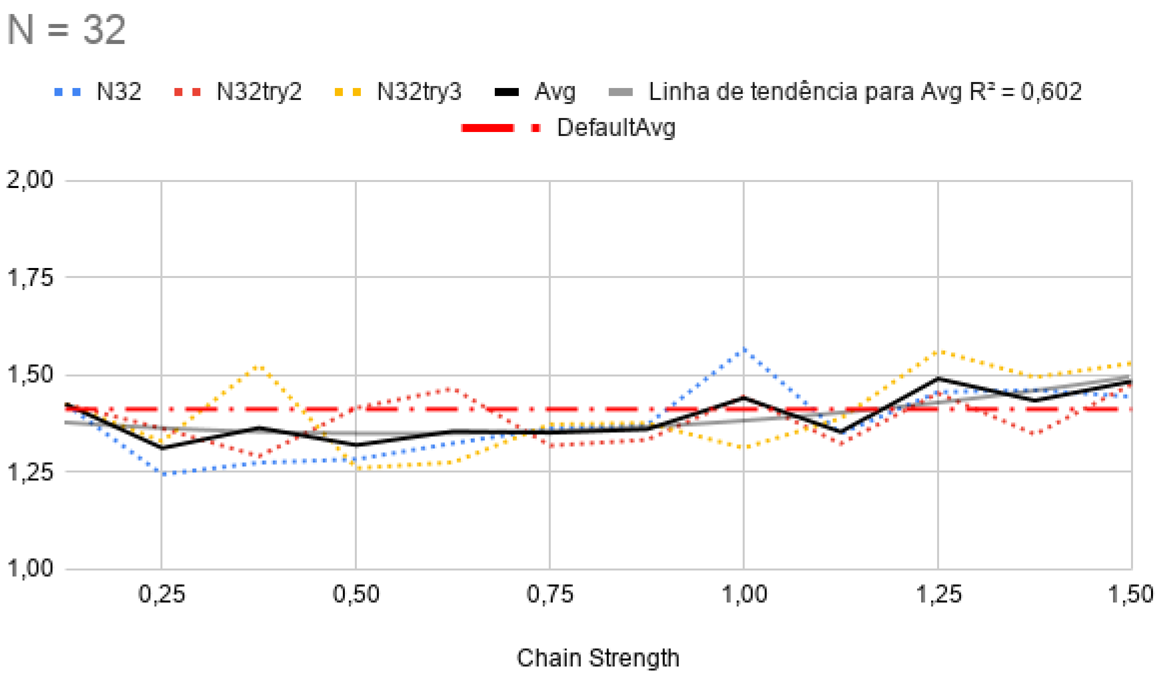
Chain strength	N16	N32	N64
default value	1,072	1,535	1,830
0,125 * maxAbs	1,858	1,426	1,726
0,250 * maxAbs	1,115	1,363	1,459
0,375 * maxAbs	1,176	1,292	1,330
0,500 * maxAbs	1,208	1,416	1,383
0,625 * maxAbs	1,147	1,466	1,485
0,750 * maxAbs	1,115	1,319	1,485
0,875 * maxAbs	1,137	1,334	1,486
1,000 * maxAbs	1,207	1,445	1,543
1,125 * maxAbs	1,114	1,324	1,517
1,250 * maxAbs	1,130	1,454	1,491
1,375 * maxAbs	1,130	1,348	1,472
1,500 * maxAbs	1,133	1,477	1,485

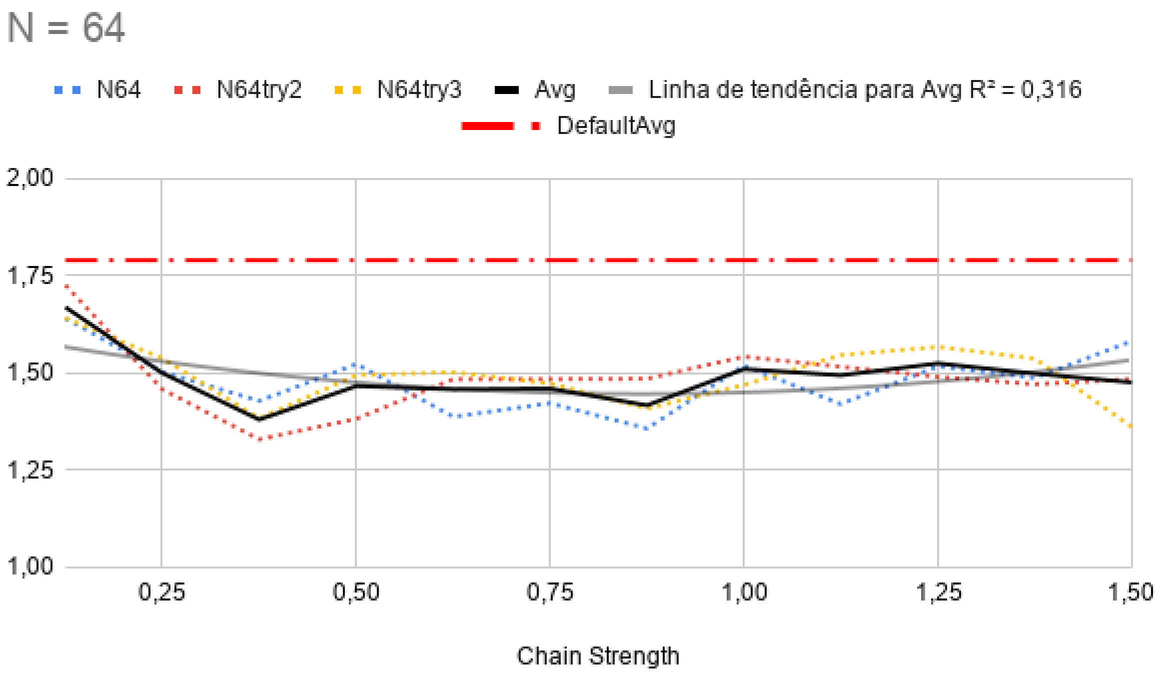
And one more time:

Chain strength	N16	N32	N64
default value	1,062	1,364	1,785
0,125 * maxAbs	3,627	1,426	1,641
0,250 * maxAbs	1,194	1,330	1,539
0,375 * maxAbs	1,171	1,527	1,384
0,500 * maxAbs	1,179	1,261	1,495
0,625 * maxAbs	1,171	1,276	1,503
0,750 * maxAbs	1,166	1,374	1,474
0,875 * maxAbs	1,176	1,377	1,409
1,000 * maxAbs	1,147	1,313	1,470
1,125 * maxAbs	1,115	1,389	1,547
1,250 * maxAbs	1,246	1,563	1,568
1,375 * maxAbs	1,231	1,495	1,538
1,500 * maxAbs	1,145	1,531	1,362
5,000 * maxAbs	1.250	1,249	1,468

The results are summarized in the following charts.







Key Takeaways:

Looking at the results, we notice that the impact of any change to the chain strength is higher for higher values of `N`.

It also becomes clear that, especially for higher values of `N`, the default chain strength is far from being the best value. It seems that for higher values of `N`, the farther is the default chain strength value from the best value.

Another thing that also becomes clear is that the fractions of chain breaks and valid solutions are not directly synonymous with the quality of the solutions.

For the case of `N=8`, every try gave a perfect score of `1.000`.

For the case `N=16`, the epsilon values are so similar that they fall under the margin of variation. Thus we cannot place conclusions based on these results. (Note: in this case, the default strength is always the best!)

There is an exception for both cases of `N=8` and `N=16`. When `chain_strength = 0.125 * maxAbs` there is a high fraction of chain breaks and almost no samples are valid solutions. Thus, for this value of chain strength, the results are very bad.

This behavior is also noticeable for `N=32` and `N=64`, that present a relatively high epsilon indicator with this chain strength.

It seems that, after this very weak chain strength, the following values of chain strength rapidly attain the lowest epsilon indicators registered, with a very slow climb afterwards.

In the end, the results suggest that it is okay to choose any value that is part of the slow climb. However, from theory, we know that we should avoid any value over `1.000 * maxAbs`, since it scales down the problem.

Therefore, for `N=16`, a safe range seems to be between the default value and `1.000 * maxAbs`. For `N=32`, this range seems to be between `0.250 * maxAbs` and `1.000 * maxAbs`. Finally, for `N=64`, this range seems to be between `0.375 * maxAbs` and `1.000 * maxAbs`.

Based on those findings, the case `N=8` will not be tested in the remaining scenarios, since the annealer already achieved optimality.

Scenario A2

For this scenario, we will be looking at how different budgets affect the performance of the annealer. Therefore, different fractions of `B` are going to be tested: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

I will be using `chain_strength = 1.000 * maxAbs`, for the reasons explained in the previous scenario. **Reminder: the fraction used in previous scenarios was `B=0.5`!**

Obviously, for each value of B, we first need to solve it classically. Then, from the results, we get the sequences of `q_values` to be used in the annealer.

N	q values	Budget fraction
16	0, 20, 500	0.1 (1)
32	0, 7, 20, 40	0.1 (3)
64	0, 0.6, 2, 4, 6, 8, 20, 40, 80, 500	0.1 (6)
16	0, 8, 10, 40	0.2 (3)
32	0, 5, 8, 20, 30, 80	0.2 (6)
64	0, 0.3, 0.8, 2, 4, 5, 7, 9, 20, 30, 500	0.2 (12)
16	0, 2, 6, 20, 60	0.3 (4)
32	0, 3, 4, 10, 20, 50	0.3 (9)
64	0, 0.2, 2, 3, 4, 5, 7, 9, 20, 30, 100	0.3 (19)
16	0, 2, 5, 10, 30	0.4 (6)
32	0, 0.2, 0.9, 2, 4, 20, 30, 70, 500	0.4 (12)
64	0, 0.3, 0.6, 1, 2, 3, 4, 6, 8, 20, 30, 90	0.4 (25)
16	0, 2, 6, 100, 500	0.5 (8)
32	0, 0.4, 0.9, 2, 3, 9, 100	0.5 (16)
64	0, 0.2, 0.4, 0.6, 1.1, 1.3, 1.5, 2, 5, 6, 7, 8, 10, 100, 500	0.5 (32)
16	0, 0.1, 0.8, 3, 20, 30	0.6 (9)
32	0, 0.1, 0.5, 1, 2, 3, 7, 8, 20, 30	0.6 (19)
64	0, 0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 2, 3, 7, 9, 20	0.6 (38)
16	0, 0.7, 20	0.7 (11)
32	0, 0.4, 2	0.7 (22)

N	q values	Budget fraction
64	0, 0.1, 0.2, 0.3, 0.7, 1, 2, 3, 4, 6, 20	0.7 (44)
16	0, 4	0.8 (12)
32	0, 0.8, 7, 9	0.8 (25)
64	0, 0.1, 0.2, 0.4, 0.5, 0.6, 1, 2, 3, 6, 20	0.8 (51)
16	0, 50	0.9 (14)
32	0, 0.8, 3	0.9 (28)
64	0, 0.6, 1, 2, 5, 500	0.9 (57)

Question: Is it bad to have different number of samples between cases?

With those results, we obtained the following epsilon indicators:

Budget fraction	N16 (AvgChainBreak)	N32 (AvgChainBreak)	N64 (AvgChainBreak)
0.1	1.000 (0.00406)	1.668 (0.00979)	1.507 (0.01572)
0.2	1.017 (0.00048)	1.274 (0.00151)	2.379 (0.00493)
0.3	1.026 (0.00044)	1.427 (0.00152)	1.406 (0.00473)
0.4	1.172 (0.00024)	1.253 (0.00134)	1.482 (0.00452)
0.5	1.211 (0.00031)	1.297 (0.00127)	1.465 (0.00484)
0.6	1.174 (0.00033)	1.317 (0.00141)	1.533 (0.00470)
0.7	1.042 (0.00038)	1.960 (0.00108)	2.403 (0.00464)
0.8	1.014 (0.00031)	1.419 (0.00144)	inf (0.00462)
0.9	1.120 (0.00034)	inf (0.00129)	inf (0.00447)

Key Takeaways:

The first thing I notice is that there is a high chain break fraction when the budget is `B=0.1`. Afterwards, it attains a consistently low fraction, with small variation.

For `N=32` and `N=64`, as expected from theory, the epsilon indicator is lower when the budget is or is close to `B=0.5`, since this value has the highest number of admissible solutions.

Behavior for `N=16` is hard to grasp. I believe that `N=16` is a too small problem to get any significant benefit from changing parameters.

Nonetheless, budget fraction is still a parameter that is particular to each practitioner.

Scenario A3

For this scenario, we will study the influence from the dataset. Previous scenarios used a `diversified` dataset, with assets as uncorrelated as possible. Therefore, we are going to introduce another dataset, called `strongly_correlated`, from the same source, however, with strongly correlated assets. That is, with assets from the same sub-industry.

The results are executed for sizes `N=32` and `N=64`, with parameters `chain_strength = 1.000 * maxAbs` and `B=0.5`. Since this scenario is small, the results have been repeated two more times, for a total of three tries.

N and Dataset	q values
`N32_diversified`	0, 0.4, 0.9, 2, 3, 9, 100
`N32_strongly_correlated`	0, 1, 6, 10, 70, 90
`N64_diversified`	0, 0.2, 0.4, 0.6, 1.1, 1.3, 1.5, 2, 5, 6, 7, 8, 10, 100, 500
`N64_strongly_correlated`	0, 0.1, 0.2, 0.3, 0.6, 1, 2, 3, 4, 6, 10, 20, 80

Dataset	N32 (AvgChainBreak) (AvgFractionValid)	N64 (AvgChainBreak) (AvgFractionValid)
`diversified` try1	1.433 (0.00075)	1.516 (0.00409)
`diversified` try2	1.505 (0.00092)	1.435 (0.00413)
`diversified` try3	1.500 (0.00074)	1.501 (0.00384)
`strongly_correlated` try1	1.300 (0.00093)	1.701 (0.00370)
`strongly_correlated` try2	1.352 (0.00103)	1.641 (0.00363)
`strongly_correlated` try3	1.482 (0.00076)	1.668 (0.00360)

Key Takeaways:

The dataset choice does make a significant difference in the performance of the annealer. However, it does not seem to be caused by whether it is diversified or not.