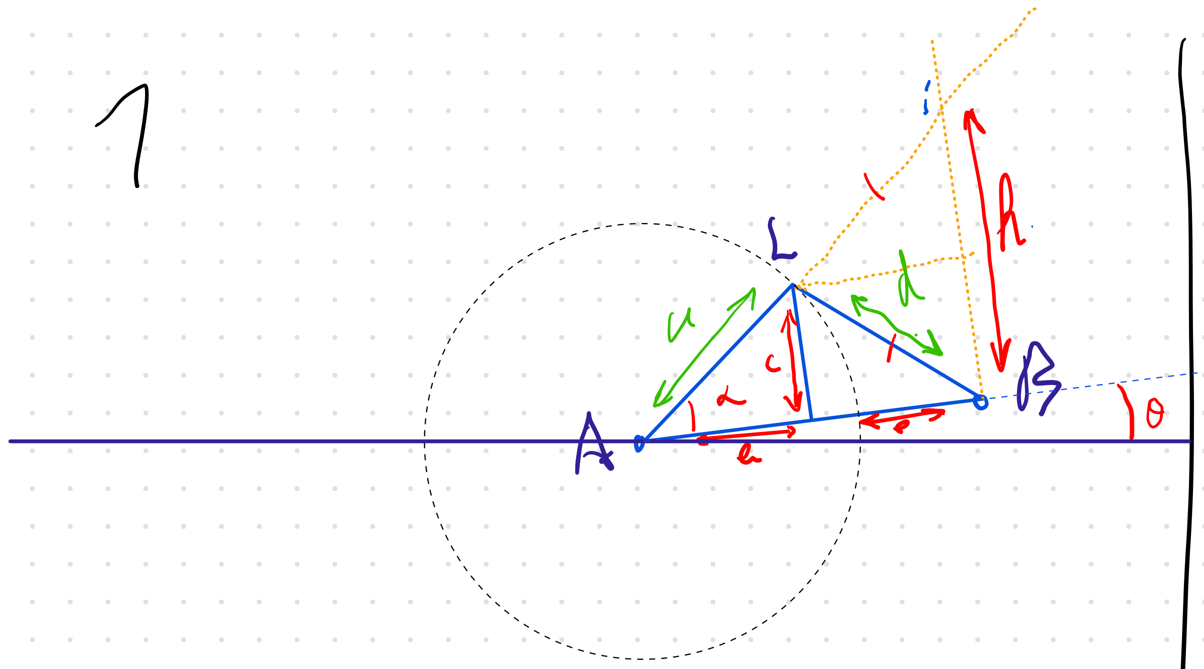
1



$$f : K_{n,3\times 2} \longrightarrow \mathbb{R}^2$$

2

$$f_n\begin{pmatrix} a & d \\ x_A & y_A \\ x_B & y_B \end{pmatrix} = L\begin{pmatrix} x_L \\ y_L \end{pmatrix}$$

$$\frac{h}{h+e} = \frac{a}{a+d}$$

on sait que :

$$h+e = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = z$$

3 alors

$$\frac{h}{z} = \frac{a}{a+d} \iff h = \frac{az}{a+d}$$

on cherche alors c

4 or

$$\frac{c}{R} = \frac{h}{h+e}$$

hors $R = \sqrt{(a+d)^2 - (h+e)^2}$

$$\iff c = \frac{Rh}{h+e}$$

5 alors

$$c = \frac{\left(\frac{az}{a+d}\right)\sqrt{(a+d)^2 - z^2}}{\sqrt{(a+d)^2 - z^2} + z\left(1 - \frac{a}{a+d}\right)}$$

on forme donc $g : K_3 \mapsto \mathbb{R}$ avec $K_3 = \{(a,d,z) \in \mathbb{R}_3 : (Eq_1)\}$

ou

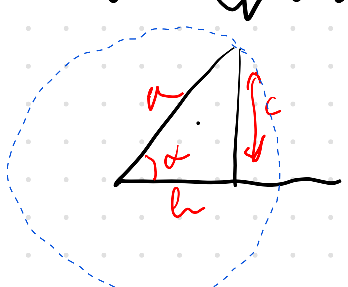
$$Eq_1 = \begin{cases} (a+d)^2 - z^2 > 0 \\ a+d \neq 0 \\ \sqrt{(a+d)^2 - z^2} + z\left(1 - \frac{a}{a+d}\right) \neq 0 \end{cases}$$

et

$$g : (a,d,z) \longmapsto \frac{\left(\frac{az}{a+d}\right)\sqrt{(a+d)^2 - z^2}}{\sqrt{(a+d)^2 - z^2} + z\left(1 - \frac{a}{a+d}\right)}$$

6 on suppose on connait c

on cherche α et θ



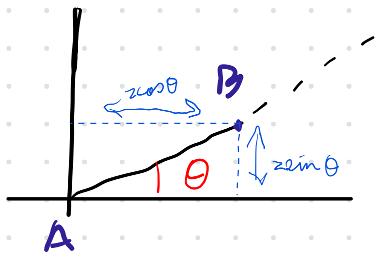
avec le schema on voit que

$c = \sin\alpha$ et $h = \cos\alpha$

on a donc :

$$\alpha = \cos^{-1}\left(\frac{h}{a}\right) = \sin^{-1}\left(\frac{c}{a}\right)$$

maintenant direction θ :



On a : $\vec{AB} = \begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix} = \begin{pmatrix} z\cos\theta \\ z\sin\theta \end{pmatrix}$

alors $\theta = \cos^{-1}\left(\frac{x_B - x_A}{z}\right) = \sin^{-1}\left(\frac{y_B - y_A}{z}\right)$

avec tout cela on peut chercher L

$$L = \begin{pmatrix} a\cos(\theta+\alpha) \\ a\sin(\theta+\alpha) \end{pmatrix} + A$$

$$= a\begin{pmatrix} \cos\left(\cos^{-1}\left(\frac{x_B-x_A}{z}\right) \pm \cos^{-1}\left(\frac{az}{a(a+d)}\right)\right) \\ \sin\left(\cos^{-1}\left(\frac{x_B-x_A}{z}\right) \pm \cos^{-1}\left(\frac{az}{a(a+d)}\right)\right) \end{pmatrix} + A$$

$$= a\begin{pmatrix} \cos\left[\cos^{-1}\left(\frac{x_B-x_A}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}\right) \pm \cos^{-1}\left(\frac{a\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}{d(a+d)}\right)\right] \\ \sin\left[\cos^{-1}\left(\frac{x_B-x_A}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}\right) \pm \cos^{-1}\left(\frac{a\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}{d(a+d)}\right)\right] \end{pmatrix} + A$$

j'ai oublié outler A

On fixe alors f_n comme

$\forall n \in \{0,1\}$ & (Eq1) :

$$f_n : \begin{pmatrix} u & h \\ x_A & y_A \\ x_B & y_B \end{pmatrix} \longmapsto \begin{cases} a \begin{pmatrix} \cos\left[\cos^{-1}\left(\frac{x_B - x_A}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}\right) + \cos^{-1}\left(\frac{a\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}{d(a+d)}\right)\right] \\ \sin\left[\cos^{-1}\left(\frac{x_B - x_A}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}\right) + \cos^{-1}\left(\frac{a\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}{d(a+d)}\right)\right] \end{pmatrix} + A & \text{si } n = 1 \\ a \begin{pmatrix} \cos\left[\cos^{-1}\left(\frac{x_B - x_A}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}\right) - \cos^{-1}\left(\frac{a\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}{d(a+d)}\right)\right] \\ \sin\left[\cos^{-1}\left(\frac{x_B - x_A}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}\right) - \cos^{-1}\left(\frac{a\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}}{d(a+d)}\right)\right] \end{pmatrix} + A & \text{si } n = 0 \end{cases}$$