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$$\nabla \chi^2 (a_0, a_1) = \left[\frac{\partial \chi^2}{\partial a_0}, \frac{\partial \chi^2}{\partial a_1} \right]$$

$$= \left[2 \sum_{i=1}^{m} (y_i - (a_0 + a_1 x_i))(-1), \; 2 \sum_{i=1}^{m} (y_i - (a_0 + a_1 x_i)) (-x_i) \right] = [0, 0]$$

$$\Rightarrow -2 \sum_{i=1}^{m} y_i - a_0 - a_1 x_1 = 0 \; ; \; 2 \sum_{i=1}^{m} y_i x_i - a_0 x_i - a_i x_i^2 = 0$$

$$\sum_{i=1}^{m} y_1 - a_0 m - \sum_{i=1}^{m} a_1 x_1 = 0$$

$$a_0 = \frac{y}{m} \sum_{i=}^{m} y_i - \frac{a_1}{m} \sum_{i=1}^{m} x_i = \bar{y} - a_i \bar{x}$$

$$\sum_{i=1}^{m} y_i x_i - a_0 \sum_{i=1}^{m} x_i - a_i \sum_{i=1}^{m} x_i^2 = 0$$

$$a_1 \sum_{i=1}^{m} x_i^2 - \sum_{i=1}^{m} y_i x_i + \left[\frac{-1}{m} \sum_{i=1}^{m} y_i + a_1 \left(\frac{1}{m} \sum_{i=1}^{m} x_i \right) \right] \left(\sum_{i=1}^{m} x_i \right)$$

$$a_1 \left[\left(\sum_{i=1}^{m} x_i^2 \right) - \frac{1}{m} \left(\sum_{i=1}^{m} x_i \right)^2 \right] = \sum_{i=1}^{m} y_i x_i - \frac{1}{m} \left(\sum_{i=1}^{m} y_i \right) \left(\sum_{i=1}^{m} x_i \right)$$

$$a_i = \frac{\sum_{i=1}^{m} y_i x_i - \frac{1}{m} \left(\sum_{i=1}^{m} y_i \right) \left(\sum_{i=1}^{m} x_i \right)}{\sum_{i=1}^{m} x_i^2 - \frac{1}{m} \left(\sum_{i=1}^{m} x_i \right)^2}$$

$$D\chi^2(a_0, a_1, a_2) = \left[\frac{\partial \chi^2}{\partial a_0}, \frac{\partial \chi^2}{\partial a}, \frac{\partial \chi^2}{\partial a_2}\right] = [0,0,0]$$

$$= \left[-2\sum_{i=1}^{m}(y_i - a_0 - a_1 x_i - a_1 x_i^2) - 2\sum_{i=1}^{m}(y_i - a_0 - a_i x_i - a_2 x_i^2)\, x_i\right.$$

$$\left. - 2\sum_{i=1}^{m}(y_i - a_0 - a_i x_i - a_2 x_i^2)(x_i^2)\right] = [0,0,0]$$

$$-2\left(\sum_{i=1}^{m} y_i - a_0 - a_1 x_i - a_2 x_i^2\right) = 0$$

$$-2\left(\sum_{i=1}^{m} y_i x_i - a_i x_i^2 - a_2 x_i^3\right) = 0$$

$$-2\left(\sum_{i=1}^{m} y_2 x_2^2 - a_0 x_i^2 - a_1 x_i^3 - a_2 x_i^4\right) = 0$$

$$\begin{cases} \sum_{i=1}^{m} y_i = \sum_{i=1}^{m} a_0 + a_1 x_i + a_2 x_2^2 \\ \sum_{i=1}^{m} y_i x_i = \sum_{i=1}^{m} a_0 x_i + a_i x_i^2 + a_2 x_i^3 \\ \sum_{i=1}^{m} y_i x_i^2 = \sum_{i=1}^{m} a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4 \end{cases}$$

(a)

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{m} \left(\frac{y_i - M(x_i, \vec{\theta})}{\sigma_i} \right)^2 ; \quad \sigma_i = 1 \ \forall i$$

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = \sum_{i=1}^{m} 2\left(y_i - M(x_i, \vec{\theta})\right)\left(\frac{-\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right)$$

$$= -2 \sum_{i=1}^{m} \left(y_i - M(x_i, \vec{\theta})\right) \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i}$$

(b) Como

$$\theta_{j+1} = \theta_j - \gamma \nabla \chi^2 (\vec{\theta}_j)$$

y como

$y_i \cdot M(x_i, \vec{\theta}_j)$ en un punto constante

$$\theta_{j+1} = \vec{\theta}_j - \gamma \left(-2 \sum_{i=1}^{m} \left(y_i - M(x_i, \vec{\theta}_j)\right) \left[\frac{\partial M(x_i, \vec{\theta}_j)}{\partial \theta_0}, \frac{\partial M(x_i, \vec{\theta}_j)}{\partial \theta_1}, \frac{\partial M(x_i, \vec{\theta}_j)}{\partial \theta_2} \right] \right)$$

se tiene

$$\vec{\theta}_{j+1} = \vec{\theta}_j - \gamma \left(-2 \sum_{i=1}^{m} \left(y_i - M(x_i, \vec{\theta}_j)\right) \nabla M(x_i, \vec{\theta}_j) \right)$$