

2.) encuentre el error del trapecio Simple

Se sabe que $f(x) = p_1(x) + E(x)$ y

$$E(x) = \frac{f''(\xi)}{2} (x^2 - xb - xa + ab)$$

entonces

$$\int_a^b E(x) = \frac{f''(\xi)}{2} \cdot \left(\frac{x^3}{3} - \frac{x^2}{2}b - \frac{x^2}{2}a + abx \right)$$

$$= \frac{f''(\xi)}{2} \left(\frac{b^3}{3} - \frac{b^3}{2} - \frac{b^2a}{2} + \frac{b^2a}{1} \right) - \left(\frac{a^3}{3} - \frac{a^3}{2} - \frac{a^2b}{2} + a^2b \right)$$

$$= \frac{f''(\xi)}{2} \left(\frac{-b^3}{6} + \frac{b^2a}{2} \right) - \left(\frac{-a^3}{6} + \frac{a^2b}{2} \right)$$

$$= \frac{f''(\xi)}{2} \left(\frac{a^3 - 3a^2b + 3b^2a - b^3}{6} \right)$$

$$= f''(\xi) \cdot \frac{(b-a)^3}{12}$$

4) error del polinomio interpolado

$$\begin{aligned} \varepsilon(x) &= \frac{f'''(\xi)}{3!} (x-a)(x-b)\left(x - \frac{a+b}{2}\right) \\ &= \frac{f'''(\xi)}{3!} \left(x^3 - \frac{3}{2}(a+b)x^2 + \frac{a^2+4ab+b^2}{2}x - \frac{ab(a+b)}{2} \right) \end{aligned}$$

entonces

$$\int_a^b \varepsilon(x) dx = \frac{f'''(\xi)}{3!} \left[\frac{b^4-a^4}{4} - \frac{3}{2}(a+b) \frac{b^3-a^3}{3} + \frac{a^2+4ab+b^2}{4} \cdot \frac{b^2-a^2}{2} - \frac{ab(a+b)(b-a)}{2} \right]$$

$$\frac{f'''(\xi)}{3!} (b^2-a^2) \left[\frac{b^2+a^2}{4} - \frac{b^2+ab+a^2}{2} + \frac{a^2+4ab+b^2}{4} - \frac{ab}{2} \right]$$

$$\frac{f'''(\xi)}{3!} (b^2-a^2) \left[\frac{b^2}{4} - \frac{b^2}{2} + \frac{b^2}{4} - \frac{ab}{2} + \frac{4ab}{4} - \frac{ab}{2} + \frac{a^2}{4} - \frac{a^2}{2} + \frac{a^2}{4} \right]$$

$$\frac{f'''(\xi)}{3!} (b^2-a^2) (0) = 0$$

$$\begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 2/3 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 10/3 \\ 5 \\ 2/3 \end{pmatrix}$$

$$(3 + 5x + x^2) = (3, 5, 1)$$

$$3 + 5x + x^2$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{10}{3} + 5x + \frac{2}{3}x^2$$

$$= \frac{10}{3} P_0(x) + 5 P_1(x) + \frac{2}{3} P_2(x)$$