

3. Hacer pasos intermedios para encontrar la regla de Simpson simple.

$$\int_a^b f(x)dx \cong \int_a^b p_2(x)dx = \frac{h}{3}(f(a) + 4f(x_m) + f(b))$$

$$f(x) \approx p_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)$$

$$x_m = \frac{a+b}{2}$$

$$h = x_m - a$$

$$h = \frac{a+b}{2} - a = \frac{b-a}{2}$$

$$2h = b - a$$

$$b = 2h + a$$

$$b = h + x_m$$

$$a = x_m - h$$

Entonces:

$$= \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \int_a^b (x-b)(x-x_m)dx + \frac{f(x_m)}{(x_m-a)(x_m-b)} \int_a^b (x-a)(x-b)dx$$

$$+ \frac{f(b)}{(b-a)(b-x_m)} \int_a^b (x-a)(x-x_m)dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left[(x_m-a) \frac{(a-b)^2}{2} + \frac{(a-b)^3}{6} \right] + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left[\frac{(a-b)^3}{6} \right]$$

$$+ \frac{f(b)}{(b-a)(b-x_m)} \left[(b-x_m) \frac{(b-a)^2}{2} - \frac{(b-a)^3}{6} \right]$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left[(h) \frac{(-2h)^2}{2} + \frac{(-2h)^3}{6} \right] + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left[\frac{(-2h)^3}{6} \right]$$

$$+ \frac{f(b)}{(b-a)(b-x_m)} \left[(h) \frac{(2h)^2}{2} - \frac{(2h)^3}{6} \right]$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left[\frac{4h^3}{2} - \frac{8h^3}{6} \right] + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left[-\frac{8h^3}{6} \right]$$

$$+ \frac{f(b)}{(b-a)(b-x_m)} \left[\frac{4h^3}{2} - \frac{8h^3}{6} \right]$$

$$\begin{aligned}
&= \frac{f(a)}{(a-b)(a-x_m)} \left[\frac{2h^3}{3} \right] + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left[-\frac{4h^3}{3} \right] + \frac{f(b)}{(b-a)(b-x_m)} \left[\frac{2h^3}{3} \right] \\
&= \frac{2h^3 f(a)}{3(a-b)(a-x_m)} - \frac{4h^3 f(x_m)}{3(x_m-a)(x_m-b)} + \frac{2h^3 f(b)}{3(b-a)(b-x_m)} \\
&= \frac{2h^3 f(a)}{3(-2h)(-h)} - \frac{4h^3 f(x_m)}{3(h)(-h)} + \frac{2h^3 f(b)}{3(2h)(h)} \\
&= \frac{2h^3 f(a)}{6h^2} + \frac{4h^3 f(x_m)}{3h^2} + \frac{2h^3 f(b)}{6h^2} \\
&= \frac{h^3 f(a)}{3h^2} + \frac{4h^3 f(x_m)}{3h^2} + \frac{h^3 f(b)}{3h^2} \\
&= \frac{h}{3} (f(a) + 4f(x_m) + f(b))
\end{aligned}$$