# SEDRA/SMITH

## Microelectronic Circuits

SEVENTH EDITION

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June 2, 2023

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# Chapter 3

# Semiconductors

## Introduction

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2 ways charge can be stored in *pn* junction.

1. charge in depletion region (more visible when reverse bias)
2. minority charge in $n$ and $p$ material (more visible when forward bias)
   - concentration profile by injecting to n-dope
   - " " to p-dope

### 3.5.1 Depletion or Junction Capacitance

**Assumption:** *pn* junction reversed bias with $V_R$, charge on either side of junction:

$$Q_j = A\sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}(V_0 + V_R)} = \alpha\sqrt{(V_0 + V_R)} \tag{3.1}$$

We denote $\alpha$ as $A\sqrt{2\epsilon_s q \frac{N_A N_D}{N_A+N_D}}$ and observe that $Q_j \not\propto V_R$ (also not linearly related)

- Hard to define capacitance that accounts for changing $Q_j$ when $V_R$ changes

  **Assumption:** junction operates as a point $Q$ and define

$$C_j = \left.\frac{dQ_J}{dV_r}\right|_{V_R=V_Q} \tag{3.2}$$

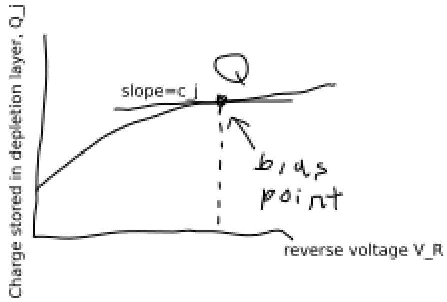



Figure 3.1: The charge stored on either side of the depletion layer as a function of the reverse voltage $V_R$

**Note:-**

The definition of capacitance

$$q = CV \implies C = q/V = \frac{\Delta q}{\Delta V}$$

- Equation 3.2 useful in electronic cct design
- This equation used in this book frequently
- Called the **"incremental-capacitance approach"**

Combining equations 3.2 with 3.1 we obtain:

$$C_j = \frac{\alpha}{2\sqrt{V_0 + V_R}} \tag{3.3}$$

We observe that $C_j$ at reverse bias ($V_R = 0$) is $C_{j0} = \frac{\alpha}{2\sqrt{V_0}}$, so we can write $C_j$ as

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \tag{3.4}$$

Substituting for $\alpha$, we obtain:

$$C_j = A\sqrt{\left(\frac{\epsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)} \tag{3.5}$$

Before leaving concept of junction capacitance, we introduce

**Definition 3.5.1: Terms**

**Abrupt junction** *pn* junction, doping concentration changes abruptly at junction boundary (this is deliberately done)

**Graded junction:** *pn* junction, carrier concentration changes gradually from one side to another. Then $C_j$ becomes:

$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

where $m$ is the **Grading coefficient**

**Note:-**

- $m$ ranges from $1/3$ to $1/2$
- $m$ depends on manner in which concentration changes from $p$ to $n$ side

**Question 1: Exercise 3.14**

For the $pn$ $\ldots$cm$^2$.

***Solution:*** Solution

### 3.5.2 Diffusion Capacitance

**Definition 3.5.2: Limit of Sequence in $\mathbb{R}$**

Let $\{s_n\}$ be a sequence in $\mathbb{R}$. We say

$$\lim_{n\to\infty} s_n = s$$

where $s \in \mathbb{R}$ if $\forall$ real numbers $\epsilon > 0$ $\exists$ natural number $N$ such that for $n > N$

$$s - \epsilon < s_n < s + \epsilon \text{ i.e. } |s - s_n| < \epsilon$$

**Question 2**

Is the set $x-$axis$\backslash\{\text{Origin}\}$ a closed set

***Solution:*** We have to take its complement and check whether that set is a open set i.e. if it is a union of open balls

**Note:-**

We will do topology in Normed Linear Space (Mainly $\mathbb{R}^n$ and occasionally $\mathbb{C}^n$)using the language of Metric Space

**Claim 3.5.1** Topology

Topology is cool

**Example 3.5.1** (Open Set and Close Set)

Open Set:
- $\phi$
- $\bigcup\limits_{x\in X} B_r(x)$ (Any $r > 0$ will do)
- $B_r(x)$ is open

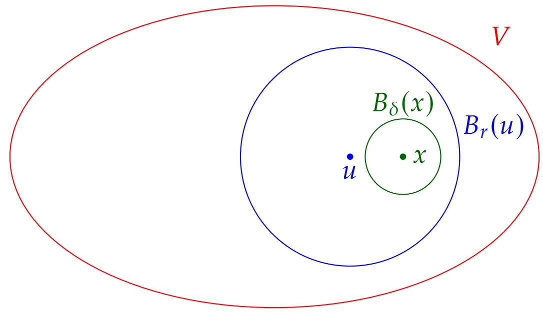
Closed Set:
- $X,\ \phi$
- $\overline{B_r(x)}$

$x-$axis $\cup$ $y-$axis

**Theorem 3.5.1**

If $x \in$ open set $V$ then $\exists$ $\delta > 0$ such that $B_\delta(x) \subset V$

***Proof:*** By openness of $V$, $x \in B_r(u) \subset V$




Given $x \in B_r(u) \subset V$, we want $\delta > 0$ such that $x \in B_\delta(x) \subset B_r(u) \subset V$. Let $d = d(u,x)$. Choose $\delta$ such that $d + \delta < r$ (e.g. $\delta < \frac{r-d}{2}$)

If $y \in B_\delta(x)$ we will be done by showing that $d(u,y) < r$ but

$$d(u,y) \leqslant d(u,x) + d(x,y) < d + \delta < r$$

😄

**Corollary 3.5.1**

By the result of the proof, we can then show...

**Lemma 3.5.1**

Suppose $\vec{v}_1, \ldots, \vec{v}_n \in \mathbb{R}^n$ is subspace of $\mathbb{R}^n$.

**Proposition 3.5.1**

$1 + 1 = 2$.

## 3.6 Random

**Definition 3.6.1: Normed Linear Space and Norm $\|\cdot\|$**

Let $V$ be a vector space over $\mathbb{R}$ (or $\mathbb{C}$). A norm on $V$ is function $\|\cdot\|$ $V \longrightarrow \mathbb{R}_{\geqslant 0}$ satisfying

1. $\|x\| = 0 \iff x = 0 \ \forall \ x \in V$
2. $\|\lambda x\| = |\lambda| \|x\| \ \forall \ \lambda \in \mathbb{R}$(or $\mathbb{C}$), $x \in V$
3. $\|x + y\| \leqslant \|x\| + \|y\| \ \forall \ x, y \in V$ (Triangle Inequality/Subadditivity)

And $V$ is called a normed linear space.

• Same definition works with $V$ a vector space over $\mathbb{C}$ (again $\|\cdot\| \to \mathbb{R}_{\geqslant 0}$) where 2 becomes $\|\lambda x\| = |\lambda| \|x\|$ $\forall \ \lambda \in \mathbb{C}$, $x \in V$, where for $\lambda = a + ib$, $|\lambda| = \sqrt{a^2 + b^2}$

**Example 3.6.1** ($p-$Norm)

$V = \mathbb{R}^m$, $p \in \mathbb{R}_{\geqslant 0}$. Define for $x = (x_1, x_2, \cdots, x_m) \in \mathbb{R}^m$

$$\|x\|_p = \left(|x_1|^p + |x_2|^p + \cdots + |x_m|^p\right)^{\frac{1}{p}}$$

(In school $p = 2$)

**Special Case $p = 1$**: $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_m|$ is clearly a norm by usual triangle inequality.

**Special Case $p \to \infty$ ($\mathbb{R}^m$ with $\|\cdot\|_\infty$)**: $\|x\|_\infty = \max\{|x_1|, |x_2|, \cdots, |x_m|\}$

For $m = 1$ these $p-$norms are nothing but $|x|$. Now exercise

**Question 3**

Prove that triangle inequality is true if $p \geqslant 1$ for $p-$norms. (What goes wrong for $p < 1$ ?)

***Solution:*** **For Property 3 for norm-2**

**When field is $\mathbb{R}$ :**

We have to show

$$\begin{aligned}
&\sum_i (x_i + y_i)^2 \leqslant \left(\sqrt{\sum_i x_i^2} + \sqrt{\sum_i y_i^2}\right)^2 \\
\implies &\sum_i (x_i^2 + 2x_i y_i + y_i^2) \leqslant \sum_i x_i^2 + 2\sqrt{\left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]} + \sum_i y_i^2 \\
\implies &\left[\sum_i x_i y_i\right]^2 \leqslant \left[\sum_i x_i^2\right]\left[\sum_i y_i^2\right]
\end{aligned}$$

So in other words prove $\langle x, y\rangle^2 \leqslant \langle x, x\rangle\langle y, y\rangle$ where

$$\langle x, y\rangle = \sum_i x_i y_i$$

**Note:-**

- $\|x\|^2 = \langle x, x\rangle$
- $\langle x, y\rangle = \langle y, x\rangle$
- $\langle \cdot, \cdot\rangle$ is $\mathbb{R}-$linear in each slot i.e.

$$\langle rx + x', y\rangle = r\langle x, y\rangle + \langle x', y\rangle \text{ and similarly for second slot}$$

Here in $\langle x, y\rangle$ $x$ is in first slot and $y$ is in second slot.

Now the statement is just the Cauchy-Schwartz Inequality. For proof

$$\langle x, y\rangle^2 \leqslant \langle x, x\rangle\langle y, y\rangle$$

expand everything of $\langle x - \lambda y, x - \lambda y\rangle$ which is going to give a quadratic equation in variable $\lambda$

$$\begin{aligned}
\langle x - \lambda y, x - \lambda y\rangle &= \langle x, x - \lambda y\rangle - \lambda\langle y, x - \lambda y\rangle \\
&= \langle x, x\rangle - \lambda\langle x, y\rangle - \lambda\langle y, x\rangle + \lambda^2\langle y, y\rangle \\
&= \langle x, x\rangle - 2\lambda\langle x, y\rangle + \lambda^2\langle y, y\rangle
\end{aligned}$$

Now unless $x = \lambda y$ we have $\langle x - \lambda y, x - \lambda y\rangle > 0$ Hence the quadratic equation has no root therefore the discriminant is greater than zero.

**When field is $\mathbb{C}$ :**

Modify the definition by

$$\langle x, y\rangle = \sum_i \overline{x_i} y_i$$

Then we still have $\langle x, x\rangle \geqslant 0$

## 3.7 Algorithms

**Algorithm 1:** what

**Input:** This is some input
**Output:** This is some output

```
/* This is a comment */
some code here;
x ← 0;
y ← 0;
if x > 5 then
    x is greater than 5 ;                // This is also a comment
else
    x is less than or equal to 5;
end
foreach y in 0..5 do
    y ← y + 1;
end
for y in 0..5 do
    y ← y − 1;
end
while x > 5 do
    x ← x − 1;
end
return Return something here;
```