Show

and

Tell

"Prison Break"

Set Theory

Sets and Elements:

1. Location

Let L be the set of all locations in the prison.

L = { Cell 1, Cell 2, Cell 3, Common Room, Enter, Exit, watch Tower}.

2. Prisoners:

Let $P = \{ P_1, P_2, P_3 \}$, where:

- P_1 represents Prisoner A (In cell 1)
- P_2 represents prisoner B (In cell 2)
- P_3 represents prisoner C (In cell 3)

3. Guards:

Let $G = \{G_1, G_2, G_3, G_4\}$ where:

- G_1: Monitors between Cell 2 → Enter repeatedly.
- G_2: monitors between Cell 3 → Exit repeatedly.
- G_3: Moves between Guard 1's Location → Guard 2's Location → Common Room.
- G_4: Alternates between Enter → Exit repeatedly. From the watch Tower.

4. Edges (connections)

Let $E \subseteq L \times L$ be the set of edges representing valid paths between locations.

E = {(Cell1, Common Room), (Cell 2, Common Room), (Cell 3, Common Room), (Common Room, Enter), (comon Room, Exit), (Common Room, Watch Tower)}.

5. Time

Let T represent the set of all possible times.

T = {Lunch Time, Playtime}.

$T = \{t_o, t_c\}$

open close.

"Logic And Proposition"

Rules:

Rule 1 : Movement constraint

- A prisoner P_i can move from location x to location y at time t if:

1. There is a directi edge between x and y: $E(x,y)$.
2. y is not moniteced by any guard at that time.

$$\neg \exists k\, G_k(y, t).$$

Formal Representation.

$$\forall t \in T, \forall P_i \in P, \forall x, y \in L, (P_i(x,t) \wedge E(x,y) \wedge \neg \exists k\, G_k(y,t) \Rightarrow P_i(y, t+1).$$

Rule 2: Cell gate Access

- A prisoner can leave their cell $cell_i$ only if the cell gate is open at time t:

$\forall t \in T, \forall P_i \in P, (P_i(cell_i, t) \wedge O_{cell_i}(t)) \rightarrow P_i(\text{Common Room}, t+1)$.

where

$cell_i \in Cell$

$Cell = \{cell_1, cell_2, cell_3\}$

O_{cell_i} means that the cell is open at a particular time.

Rule 3: Escape conditions:

- A prisoner P_i escapes successfully if they reach the Exit at time t without being detected.

$\forall P_i \in P, \exists t \in T, (Escape_i(Exit, t) \wedge \neg \exists k G_k(Exit, t))$.

- Where Escape i is a logical predicate that expresses wether prisoner P_i is present at the designated Exit location at a specific time. t.

Conditions

1 Prisoner start in their respective cells
P_1 (Cell 1, 0), P_2 (Cell 2, 0), P_3
($Cell_3$, 0).

2. Cell gates open at time t_0
during Lunch or Play time:
$O_{cell_i}(t_0)$ for $i = 1, 2, 3$.

3. Guards moniter their respective
locations:

- G_1 : G_1 (cell 2, t) $\vee$ G_1 (Enter, t).
- G_2 : G_2 ($Cell_3$, t) $\vee$ G_2 (Exit, t).
- G_3 : G_3 (x, t) where $x \in$
{ Guard 1's location, Guards 2's
Location, Common Room }.
- G_4 : G_4 (x, t) where $x \in$ { Enter, Exit }

4. Prisoners must use valid Edges:
E (x, y) for all connected
locations $x, y \in L$.

Movement to the Common Room:

- Prisoner P_1 starts at $Cell_1$, and the gate $O_{Cell_1}(t_0)$ opens at time t_0. Since: $\neg G_3(\text{Common Room}, t_0)$. P_1 can move to the common Room:

$$P_1(Cell_1, t_0) \land O_{Cell_1}(t_0) \land \neg G_3(\text{Common Room}, t_0) \rightarrow P_1(\text{Common Room}, t_0 + 1).$$

Similarly P_2 and P_3 can move:
i.e

$$P_i(Cell_i, t_0) \land O_{Cell_i}(t_0) \land \neg G_3(\text{Common Room}, t_0) \rightarrow P_i(\text{Common Room}, t+1).$$

Moving to the Exit:

- From the common Room, prisoners can move to the Exit if:

1: The Exit is not being monitered by G_2 or G_4

2. E (common Room, Exit).

For P_1:

$P_1(\text{Common Room}, t_1) \wedge \neg(G_2(\text{Exit}, t_1) \vee G_4(\text{Exit}, t_1)) \Rightarrow P_1(\text{Exit}, t_1 + 1)$.

Similarly for P_2 and P_3:

$P_i(\text{Common Room}, t_i) \wedge \neg(G_2(\text{Exit}, t_i) \vee G_4(\text{Exit}, t_i)) \Rightarrow P_i(\text{Exit}, t_1 + 1)$.

Proving the Escape.

- All prisoners escape succesfully if:

$$\exists t, \left(I(t) \wedge \bigwedge_{i=1}^{3} O_{cell\,i}(t) \wedge \neg \exists k\, G_k(Exit, t) \wedge \bigwedge_{i=1}^{3} P_i(Exit, t) \right).$$