

Classical Graph Theory

Nodes (or Vertices):

- **Definition:** A node (or vertex) is a fundamental part of a graph, representing an entity or object.
- **Properties:**
 - **Degree:** The degree of a node is the number of edges connected to it.
 - If a node has no edges, it is **isolated**.
 - A node with exactly one edge is called a **leaf** in a tree graph.
 - **Labelling:** Vertices are often labelled (e.g., $v_1, v_2, \dots, v_1, v_2, \dots$) to distinguish them.
 - **Types of Nodes:**
 - **Adjacent Nodes:** Two nodes are adjacent if they are connected by an edge.
 - **Connected Nodes:** A graph is connected if there's a path between every pair of nodes.

Edges:

- **Definition:** An edge is a connection between two vertices.
- **Types of Edges:**
 - **Undirected Edge:** Connection without direction (e.g., a friendship relation).
 - **Directed Edge (Arc):** Connection with a direction (e.g., traffic flow).
 - **Weighted Edge:** An edge with an associated value, such as distance or cost.
 - **Loops:** An edge that connects a node to itself.

Movement on Edges

An object can move along edges to which it is connected in a graph, subject to the following conditions:

1. Connectedness:

- An object can move only if there exists an edge connecting its current node to the target node.
- For example, if an object is at node A and there's an edge $A \rightarrow B$, the object can move to B.

2. Directionality:

- **Undirected Graphs:** Movement is bidirectional, allowing traversal in either direction along an edge.
- **Directed Graphs:** Movement is restricted to the direction of the arrow. If an edge

$A \rightarrow B$ exists, movement from A to B is allowed, but not B to A

1. Graph Theory Basics:

- **Edges as Connections:** An edge explicitly represents a direct connection or relationship between two nodes. If there's no edge between a node and another, there's no direct path between them.
- **Adjacency Rule:** A node can interact only with nodes to which it is directly connected by an edge.

For example:

- If you are at node A in a graph, and there's no edge connecting A to node B, you can't move directly to B. You would need a path of intermediate nodes and edges to get there.

2. Practical Implications:

In most practical applications, lack of a connecting edge means movement is impossible:

- **Transportation Networks:** If no road (edge) connects two cities (nodes), travel isn't possible unless new infrastructure is built.
- **Electrical Circuits:** Current can't flow between components unless there's a conductive path (edge).
- **Computer Networks:** Data packets can't travel directly between devices without a physical or logical link.

Conclusion:

In classical graph theory, an object cannot move along an edge to which it is not connected, as the edge explicitly defines the relationship or pathway. However, in specialised or extended graph models, alternative mechanisms like virtual connections, teleportation, or dynamic edge creation could enable such movement.