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A Hybrid Optimization Technique Using Exchange Market and Genetic Algorithms

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ABSTRACT This paper proposes a hybrid optimization technique combining genetic and exchange market algorithms. These algorithms are two evolutionary algorithms that facilitate finding optimal solutions for different optimization problems. The genetic algorithm's high execution time decreases its efficiency. Because of the genetic algorithm's strength in surveying solution space, it can be combined with a proper exploitation-based algorithm to improve the optimization efficiency. The exchange market algorithm is an optimization algorithm that can effectively find the global optimum of the objective functions in an efficient manner. According to the trade's inherent situation, the stock market works under unbalanced and balanced modes. In order to gain maximum profit, shareholders take specific decisions based on the existing conditions. The exchange market algorithm has two searching and two absorbent operators for acquiring the best-simulated form of the stock market. Simulations on twelve benchmarks with the different dimensions and variables prove the effectiveness of this algorithm compared to eight optimization algorithms.

INDEX TERMS Evolutionary algorithm, exchange market algorithm (EMA), genetic algorithm (GA), hybrid algorithm, objective function, optimization algorithm.

NOMENCLATURE

The symbols used throughout this paper are defined below.

g_1	Usual market risk value
$g_{1,min}$	Minimum risk of the market
$g_{1,max}$	Maximum risk of the market
g_2	Third group's variable market risk
$iter_{max}$	Last iteration number
k	Number of program iteration
n_i	First group's n^{th} person
n_j	Second group's n^{th} person
n_k	Third group's n^{th} personn_{pop}
	Last member's number
$n_{t1}t^{th}$	member's all initial shares
$pop_j^{group(2)}$	Second group's j^{th} person
$pop_k^{group(3)}$	Third group's k^{th} person
$pop_{1,i}^{group(1)}$	and $pop_{2,i}^{group(1)}$ First group's i^{th} person
r, r_1, and r_2	Random numbers between [0, 1]
r_s	A random number between [−0.5, 0.5]
s_k	Third group's share variations for the k^{th} member
s_{ty}	t^{th} member's shares
t_{pop}	t^{th} member's number
μ	Each member's constant coefficient
$\Delta n_{t1}$and Δn_{t2}	Random amounts added to some shares
δ	Information of the exchange market
η_1	Second group members' risk level
η_2	Third group members' risk level

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I. INTRODUCTION

Optimization techniques have gained much attention in different fields to efficiently balance the ability of exploitation and exploration for finding the global optimum solution [1]. The two common optimization approaches are mathematical and heuristic approaches. The heuristic approach has shown more accuracy and speed over the mathematical approach [2]. The heuristic algorithms can find the most optimized answer to the complex and constraint-based problems, but mathematical solvers often fail to do so [3]. Heuristic algorithms are inspired by natural processes. Some examples of these algorithms are evolutionary programming [4], improved ant colony search algorithm [5], [6], genetic algorithm (GA) [7], [8], imperialistic harmony search algorithm [9], competitive algorithm [10], [11], multi-objective particle swarm optimization [12], [13], and neural network algorithm, artificial bee colony (ABC) [14]. Evolutionary algorithms are more effective in solving optimization problems in which their objective functions are nonlinear, constrained, and non-smooth. Evolutionary algorithms significantly reduce the computational time and are used in various engineering fields and real-world problems such as electrical engineering, aerospace engineering, industrial engineering, civil engineering, mechanical engineering, biomedical engineering, electronic engineering, reliability engineering, and telecommunications engineering.

Exploration denotes the algorithm's ability to search all sections of the problem. On the other hand, exploitation describes the convergence capability to the optimal solution. According to [15], achieving an acceptable exploration and exploitation behavior in evolutionary computing is a challenging task. In other words, with strengthening one trait, the other may weaken. Hence, the existing heuristic optimization algorithms can only solve a finite set of problems [16]. Combining optimization algorithms is a way to balance exploration and exploitation ability. However, to achieve the optimal results, these meta-heuristic algorithms commonly consume remarkable computing times and various parameter settings [17].

The exchange market algorithm (EMA) is a population-based and meta-heuristic algorithm used for optimizing functions [18]. By inspiration from the stock market, in EMA, shareholders make decisions about the selected stocks based on their procedures and compete to be known as the most successful shareholder. In addition, shareholders with a lower ranking in the market, consider smart risks to gain more profits. EMA facilitates zone searching which in turn facilitates interchanging the bought and sold shares. This highlights the EMA capability in the effective optimization of various functions. In particular, EMA has two searchers and two absorbent operators for individuals, which leads to the creation and organization of random numbers in an efficient manner. Optimization speed, searching zone selectivity, the widespread optimizing range, identical solutions convergence in each program iteration, and high performance in the global optimum finding are some of the salient features of EMA [19]. Moreover, EMA has been implemented on several real-world optimization problems that verify its effectiveness [20]–[23].

The GA is an adaptive search technique that simulates an evolutionary process inspired by the principles of Darwin's theory and uses the idea of selecting the mutation, fittest, and crossing [24], [25]. The adaptive nature and generalizing feature of GA help execute these problems by a noncomplex formation. The GA has been successfully applied in different areas such as neural-fuzzy network, fuzzy control, economic load dispatch, greenhouse climate control, and path planning [26], [27]. The GA-based approach executes a global search and its complexity is lower than other algorithms. Moreover, GA can generate numerous numbers independent of the dimension of basic data. However, the basic GA is not perfectly operative and efficient in finding the solution to the problems with highly required accuracy [28]. After GA approximately locates the optimized solution, it continuously moves back and forth to find the optimal solution. This significantly increases the excitation time which in turn decreases the algorithm efficiency. The efficiency can be increased by reducing the accuracy and replacing the global optimum answer with an approximate solution in the problems that accuracy is not a priority. GA has been merged with many other algorithms in the literature to increase optimization efficiency [29], [30] (e.g., Neural Networks, Dynamic Programming, Lin-Kernighan, Hill-Climbing Methods, Branch and Bound, Tabu search, Bee Colony Algorithm, etc.).

In addition to the heuristic algorithms, mathematical algorithms have several applications in solving the optimization problems in which the dynamic programming method is one of them [31]. A dynamic programming method has excessive application in real-world optimization problems [32], [33]. Although the mathematical algorithms have high accuracy, their main weak points are as follows: they need notable storage space, low speed, disability in dealing with the large-scale and non-convex problems. Consequently, heuristic algorithms are capable to solve many optimization problems. This presented work proposes a new hybrid algorithm with the powerful searching ability and high speed by combing two well-known optimization algorithms.

A. NOVELTY AND CONTRIBUTIONS

This paper proposes to combine EMA and GA to take the advantages of both algorithms. The combined exchange market-genetic algorithm (EMGA) demonstrates a powerful search strategy in finding the optimal solution with fewer iterations. EMGA is applied to twelve standard benchmark functions. The results are compared with the cuckoo search algorithm (CSA) [34], artificial bee colony (ABC) [35], gravitational search algorithm (GSA) [36], particle swarm optimization (PSO) [36], harmony search algorithm (HSA) [37], biogeography-based optimization (BBO) [38], differential evolution (DE) [39], EMA, and the real coded GA (RCGA) [36]. The simulation results highlight the effectiveness of EMGA compared to the conventional optimization algorithms. These algorithms are among the most accurate and efficient optimization techniques that are widely used for solving complex engineering optimization problems.

B. PAPER ORGANIZATION

The rest of the paper is organized as follows: Section II and III review EMA and GA, respectively. Section IV focuses on the implementation pattern of the EMGA. Section V provides some numerical results to show the effectiveness of EMGA. The paper is concluded in Section VI.

II. AN INTRODUCTION TO EMA

EMA is inspired by the stock market in which stockholders may adopt different decisions according to rules and their own experiences and policies [40]. These decisions may either stabilize the market or create fluctuations. Stockholders are seeking to increase their own benefits by dividing stocks while undertaking less loss. In the prevailing balanced condition in the market, unlike fluctuating mode, people can obtain the highest possible profit by predicting the current situation without considering the risk in their transactions. In a stock market, the risk of swinging degrees may be either very beneficial or very harmful for the stockholders [41]. The diverse nature of prevailing situations in the market results in the market complexity and different behaviors of stockholders [42]. A successful stockholder follows performance of other successful stockholders, uses past events to improve current performance, learns from the mistakes to modify the process, avoids investing in sectors that do not comply with stockholder's policy, performs the maximum purchase in favorable conditions, avoids participation in unfavorable conditions, and gives the highest priority to maintaining capital in all market conditions.

In EMA, each answer to the problem resembles a stockholder while its stocks are considered as the parameters related to the optimization problem. At the end of each exchange, the algorithm ranks stockholders in terms of the total value of their shares in the market. The EMA follows the rules and actual conditions of the stock market.

A. FORMULATION OF EMA

The formulation of EMA is elaborated for two states, namely balanced and fluctuations states, as follows:

1) BALANCED STATE OF THE STOCK MARKET

In this state, various stockholders compete with each other to obtain the maximum benefit without taking any risk. In the balanced mode, members of the stock market according to their fitness function are classified into three categories:

- Superior stockholders (10% to 30% of total members) are placed in the first category. Members of this group do not change their stocks in order to maintain their rank in the market. These stockholders are identified as the best answers to the problem.
- Intermediate stockholders (20% to 50% of total members) are in the second category. These stockholders seek to achieve a global optimum by comparing their stocks with stocks of the first category. Each stockholder selects the value of his/her shares based on the values of stocks of the first category members using

$$pop_j^{group(2)} = r \times pop_{1,i}^{group(1)} + (1-r) \times pop_{2,i}^{group(1)} \quad i = 1, 2, 3, \cdots n_i \text{ and } j = 1, 2, 3, \cdots n_j, \tag{1}$$

where $pop_{1,i}^{group(1)}$ and $pop_{2,i}^{group(1)}$ are the selected stocks from the superior stockholders (i.e., first category). $pop_j^{group(2)}$ is the stock value of the intended member of the second category. n_i and n_j are the number of members in the first and second categories, respectively. r is a random number between zero and one.

- The remaining market members are placed in the third category, which has the lowest value of fitness function. These members change their share values using the stocks of the first category by taking more risk and a broader search domain compared to the second category as

$$S_k = 2 \times r_1 \times \left(pop_{1,i}^{group(1)} - pop_k^{group(3)}\right) + 2 \times r_2 \times \left(pop_{2,i}^{group(1)} - pop_k^{group(3)}\right), \tag{2}$$

$$pop_k^{group(3),new} = pop_k^{group(3)} + 0.8 \times S_k, \tag{3}$$

where $pop_{1,i}^{group(1)}$ and $pop_{2,i}^{group(1)}$ are the selected stocks from the superior stockholders (i.e., first category). $pop_k^{group(3)}$ is the stock value of the intended member of the third category. r_1 and r_2 are random numbers between zero and one.

2) FLUCTUATING STATE OF THE STOCK MARKET

When the stock market conditions fluctuate, the stockholders exchange their stocks by intelligently taking risks to achieve a higher rank in the market. In this case, similar to the balanced state, members of the stock market are divided into 3 categories:

- Superior stockholders (10% to 30% of total members) are placed in the first category. These members seek to maintain their rank among the other stockholders and do not change their stocks.
- Intermediate stockholders (20% to 50% of total members) are in the second category. Members of this category seek to improve their rank by changing their stocks. As their rank in the market improves, they are associated with less risk. After modifying the stocks, the total value of stakeholder shares must remain constant according to

$$\Delta n_{t1} = n_{t1} - \delta + (2 \times r \times \mu \times \eta_1), \tag{4}$$

$$\mu = \frac{t_{pop}}{n_{pop}}, \tag{5}$$

$$n_{t1} = \sum_{y=1}^{n} s_{ty} \quad y = 1, 2, 3, \cdots, n, \tag{6}$$

$$\eta_1 = n_{t1} \times g_1, \tag{7}$$

$$g_1^k = g_{1,\max} - \frac{g_{1,\max} - g_{1,\min}}{iter_{\max}} \times k, \tag{8}$$

where Δn_{t1} represents the total changes in the stocks of a member from the second category. This amount of change is deducted from a number of shares of the intended member in a probabilistic manner and is then added to a number of its shares such that the total stock remains constant. n_{t1} is the sum of the intended member's shares before the changes. δ, μ, η_1 and r represent market characteristics, the rank coefficient of the intended member, risk level for the members of the second category, and a random number between zero and one, respectively. In (5), t_{pop} and n_{pop} indicate the rank of the member and the total number of market members, respectively. g_1^k and k are the risk level of the intended member from the second category and the value of the algorithm's iteration counter. In the second category, a portion of Δn_{t1} is randomly added to one of the stocks of a stakeholder in the second category. This process continues until Δn_{t1} is completely added to all stocks of the corresponding stockholder. In this procedure, the total value of stocks for each shareholder must remain constant. Market information (δ) plays an important role to increase the convergence speed of algorithm to the final answer [18].

- The third category includes the stockholders with low rank. Members of this category seek to achieve higher ratings by changing the value of their stocks and a broader search domain. Stock changes in this category are based on

$$\Delta n_{t3} = (4 \times r_s \times \mu \times \eta_2), \tag{9}$$

$$r_s = (0.5 - rand), \tag{10}$$

$$\eta_2 = n_{t1} \times g_2, \tag{11}$$

where Δn_{t2} is the total changes in the shares of the third category. η_2 and g_2 are the risk level of the intended member and the risk taken in the third category, respectively. r_s is a random number in the $[-0.5, 0.5]$ range. *rand* denotes a uniformly distributed random number. In the fluctuating state, the third category members are not required to maintain their total value of stocks at a constant value. In the above equations, g_2 is between zero and one and describes the amount of risk taken in changing stocks.

III. A BRIEF REVIEW OF GA

GA is an evolutionary algorithm. Evolutionary algorithms survey and store a population of answers in each repeat. In GA, the fitness value measures the sufficiency of the answer by a criterion related to the objective of the optimization problem. The answer with a larger fitness value is more appropriate and results in the optimal solution for the problem.

A. OPERATORS OF GA

This algorithm has three fundamental operators.

1) REPRODUCTION AND SELECTION

The purpose of this operator is to conduct searches in the areas with a higher possibility of finding optimum answers.

2) CROSSOVER OR RECOMBINATION

This operator combines the genes of two selected chromosomes and generates two new solutions for the problem. This procedure can be done in a single-point, two-point, multi-point, or uniform way.

3) MUTATION

Mutation provides the feature of randomness and the ability to avoid local optimum points. This operator operates on a single sequence and changes any variable of a probable answer with a small probability named mutation rate.

B. CONVERGENCE OF GA

The convergence of GA towards an optimal solution is a feature that can ensure it with special circumstances. Accomplished researches show, if the best number of each population is placed in the next population, convergence of the algorithm will be guaranteed. In other words, if the best answers are kept at any stage of production of GA and placed in the next population by the probability of one, GA will converge toward the optimal solution. Limited Markov Chain has proven this point.

C. GA TERMINATION CRITERION

The algorithm can stop after a certain period or when reaching the desirable answer as a convergence criterion. In these cases, GA may find a local optimum point as a final solution. In this paper, the algorithm stops if improvement in the answer is not observed after a few successive repetitions.

D. GA ADVANTAGES AND DRAWBACKS

The advantages of GA are listed as follows:

- Parameters work with encoded values, independent of their real values.
- The search begins with a series of points. Therefore, it is not dependent on the initial conditions with a low risk of convergence to a local optimum.
- GA only uses the information of the objective function, so it is not limited to a particular field, and its applications are unlimited.
- The optimization problem is not required to be differentiable, continuous, linear, and so on.
- In GA, rules of transition from one stage to the next stage are probabilistic.

The GA drawbacks are given as follows:

- If the space of search is relatively small, GA is relatively slower than some other algorithms. In general, GA is time-consuming and has a low convergence speed.
- If the objective function is affable and relatively uniform, other optimization algorithms may be more suitable and efficient.

IV. THE EMGA IMPLEMENTATION

Low efficiency is a major drawback of GA. However, to exploit GA's strength in surveying solution space and achieve a high optimization efficiency, this paper combines GA with EMA as a proper exploitation-based algorithm.

TABLE 1. Benchmark characteristics and functions.

#	Function	Characteristics
f_1	Ackley	Multimodal & Non-Separable
f_2	Griewank	Multimodal & Non-Separable
f_3	Penalised function1	Multimodal & Non-Separable
f_4	Penalised function2	Multimodal & Non-Separable
f_5	Quartic	Unimodal & Separable
f_6	Rastrigin	Multimodal & Separable
f_7	Raosenbrock	Unimodal & Non-Separable
f_8	Schwefel's 1.2	Unimodal & Non-Separable
f_9	Schwefel's 2.21	Unimodal & Non-Separable
f_{10}	Schwefel's 2.22	Unimodal & Non-Separable
f_{11}	Sphere	Unimodal & Separable
f_{12}	Step function	Unimodal & Separable

The steps of the proposed hybrid algorithm (EMGA) are given as follows:

After generating a random initial population, the fitness function of the problem is calculated, and members are ranked and categorized based on their fitness function values. Next, considering the balanced and fluctuating conditions of the market, the stocks of the second and third group members are exchanged, the fitness function is recalculated, and members resorted and recategorized. In the next step, the latest population is stored, GA crossover and mutation operators are applied to the population, and a new population is generated. Then, the fitness function for the new population is recalculated. Finally, populations before and after applying GA operators are combined, the total population members are ranked, and half of the new population's best stockholders are selected for the next iteration. The flowchart of the EMGA is shown in Fig. 1.

V. RESULTS AND DISCUSSION

The proposed EMGA is verified on twelve benchmark functions. On each benchmark, different dimensions (10, 20, 30, and 50) are considered. The total population number is 50. In the balanced state of the market, 25%, 50%, and 25% of the total population are assigned to the first, second, and third categories, respectively. In the fluctuating state of the market, the first, second, and third categories include 20%, 60%, and 20% of the entire population, respectively. The benchmark functions are summarized in Table 1 and Table 2. The adjustable parameters of the algorithm that show the amount of risk related to the second and third groups (g_1 and g_2) in fluctuating mode are provided in Table 3. In Table 2, the parameters describe the minimum and maximum value of the 12 benchmark functions. The highlighted ranges for these functions are extracted from the literature. The parameters shown in Table 3 relate to the values of the risk taken by the proposed hybrid algorithm. These parameters are used to adjust the performance and accuracy of the algorithm. It should be noted the accuracy of the optimization algorithm has a low sensitivity to the change of these parameters. Hence, these parameters can be selected by the trial and error method.

The mutation rate must be very low, as low as 0.05 or even smaller. The initial value for the mutation rate has been

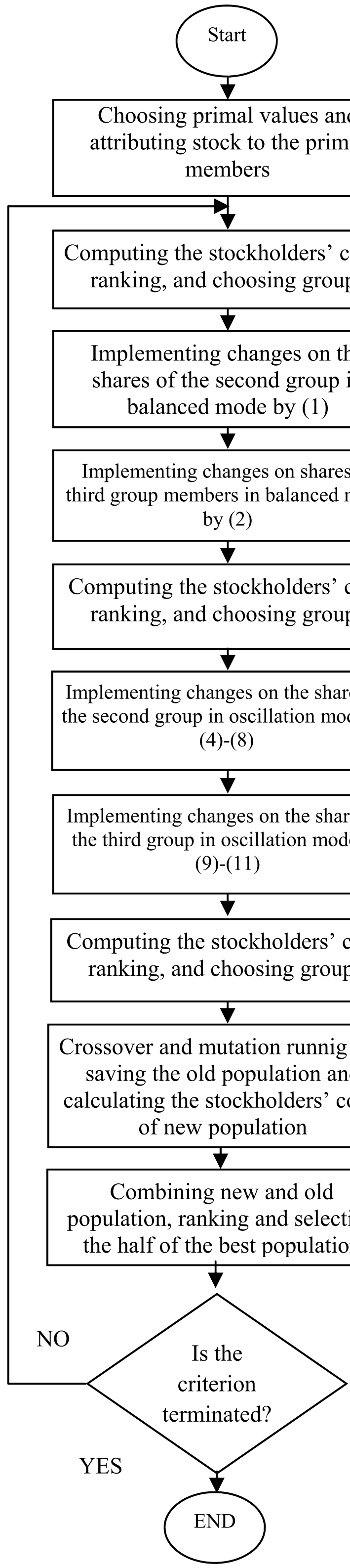

FIGURE 1. Flowchart of the EMGA.

TABLE 2. Benchmark functions explanation.

#	Function	Range
f_1	$f_1(x)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)+20+e$	[-32, 32]
f_2	$f_2(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	[-600, 600]
f_3	$f_3(x)=\frac{\pi}{n}\left\{10\sin^2(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2\left[1+10\sin^2(\pi y_{i+1})\right]+(y_n-1)^2\right\}$ $+\sum_{i=1}^{n}u(x_i,10,100,4)$ $y_i=1+(x_i+1)/4$ $u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m & i>a\\ 0 & -a<x_i<a\\ k(-x_i-a)^m & x_i<-a\end{cases}$	[-50, 50]
f_4	$f_4(x)=0.1\left\{\begin{array}{l}\sin^2(3\pi x_1)+\sum_{i=1}^{n-1}(x_i-1)^2\left[1+\sin^2(3\pi x_{i+1})\right]\\ +(x_n-1)^2\left[1+\sin^2(2\pi x_n)\right]\end{array}\right\}+\sum_{i=1}^{n}u(x_i,5,100,4)$	[-50, 50]
f_5	$f_5(x)=\sum_{i=1}^{n}ix_i^4+random[0,1)$	[-1.28, 1.28]
f_6	$f_6(x)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	[-5.12, 5.12]
f_7	$f_7(x)=\sum_{i=1}^{n-1}\left[100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2\right]$	[-30, 30]
f_8	$f_8(x)=\sum_{i=1}^{n}(\sum_{j=1}^{i}x_j)^2$	[-100, 100]
f_9	$f_9(x)=max_i\left\{\lvert x_i\rvert,1\le i\le n\right\}$	[-100, 100]
f_{10}	$f_{10}(x)=\sum_{i=1}^{n}\lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	[-10, 10]
f_{11}	$f_{11}(x)=\sum_{i=1}^{n}x_i^2$	[-100, 100]
f_{12}	$f_{12}(x)=\sum_{i=1}^{n}\left(\lfloor x_i+0.5\rfloor\right)^2$	[-200, 200]

selected equal to 0.02 by trial and error method within the [0 0.05] interval. This value is the same for all scenarios. Crossover percentage depends on the problem type and can be selected by the trial and error method. The selected value for crossover percentage by trial and error method for all 12 functions is equal to 84%. To find the proper value for this parameter, the trial and error method has been performed in [60% 99%] interval with a step of 1 percent.

In all functions, zero is the optimal value. The EMGA mean errors in finding the optimal value of benchmark functions considering different dimensions are provided in Table 4. Since the optimum value for all the listed functions of the benchmark is equal to zero, the optimal results obtained by algorithms show associated error. The errors less than 10^{-32} are not counted and are approximately considered equal to zero.

The results presented in Table 4 are an average of 50 times the implementation of the proposed algorithm. The results of

TABLE 3. EMGA tunable parameters.

#	g_2	g_3
f_1	[0.35 0.055]	[0.35 0.055]
f_2	[0.35 0.055]	[0.35 0.055]
f_3	[10^{-15} 0]	[0.04 0.00]
f_4	[10^{-15} 10^{-30}]	[0.04 10^{-20}]
f_5	[0.2 0.1]	[0.2 0.1]
f_6	[10^{-1} 10^{-10}]	[0.8 0. 7]
f_7	[10^{-100} 10^{-100}]	[10^{-100} 10^{-100}]
f_8	[0.02 0.01]	[0.02 0.01]
f_9	[0.35 0.055]	[0.35 0.055]
f_{10}	[0.2 0.1]	[0.2 0.1]
f_{11}	[0.35 0.055]	[0.35 0.055]
f_{12}	[0.35 0.055]	[0.35 0.055]

TABLE 4. EMGA's mean errors.

Dimensions	$f_1, f_2, f_3, f_4, f_5, f_6, f_8, f_9, f_{10}, f_{11}, f_{12}$	f_7
10	0	$1.7*10^{-31}$
20	0	$1.54*10^{-27}$
30	0	$3.22*10^{-24}$
50	0	$6.8*10^{-22}$
Optimal	0	0

Table 4 show that the proposed algorithm has obtained an optimal answer for all benchmark functions with different dimensions, except for f_7. In f_7, the answer is very close to zero. To compare the results of the EMGA with other common and efficient algorithms such as PSO, RCGA, DE, HSA, BBO, GSA, ABC, and CSA, optimizations for all benchmark functions are executed and the results are presented in Table 5. The dimension is considered to be 30. The medium ranking of each algorithm is calculated and provided in Table 5. The results of Table 5 show that some algorithms fail to achieve acceptable results for some of the benchmark functions. EMGA renders the most optimal solution for all benchmark functions. Fig. 2 shows the convergence process of algorithms including the EMGA, EMA, GSA, RCGA, and PSO for each of the functions. As seen, EMGA renders a faster and more accurate convergence in comparison with other algorithms. The proposed hybrid algorithm utilizes the GA crossover and mutation operators in addition to EMA searching and absorbing operators. This makes the algorithm to act faster and find the absolute optimal solution instead of the local optimums. The impact of the risk amount on the convergence of EMGA and EMA algorithms for seven different risk values is studied in Fig. 3. The risk values are provided in Table 6 and f_{11} is chosen as the objective function. Table 6 identifies the same parameters that Fig. 3 demonstrates. Fig.3 shows that the proposed algorithm (EMGA) has superiority over all EMA values. As seen in Fig. 3, the convergence of the EMGA

TABLE 5. Mean errors comparison.

#	EMA	RCGA	PSO	GSA	DE	CSA	BB0	ABC	HSA	EMGA
f_1	0	2.15	$2*10^{-02}$	$1.1*10^{-5}$	$3.1*10^{-3}$	$3.47*10^{-02}$	$3.48*10^{-01}$	$8.14*10^{-06}$	$2.94*10^{-03}$	**0**
f_2	0	1.16	$5.5*10^{-2}$	$2.9*10^{-01}$	$1*10^{-3}$	$2.4*10^{-03}$	$4.82*10^{-01}$	$2.48*10^{-04}$	$5*10^{-01}$	**0**
f_3	0	$5.3*10^{-2}$	$2.6*10^{+2}$	$4.2*10^{-13}$	$1.2*10^{-01}$	$3.62*10^{-4}$	$5.29*10^{-03}$	$3.34*10^{-13}$	$1.32*10^{-1}$	**0**
f_4	0	$8.1*10^{-2}$	$7.1*10^{+2}$	$3.2*10^{-32}$	$1.7*10^{-25}$	$1.7*10^{-13}$	$1.42*10^{-01}$	$6.17*10^{-13}$	$2.21*10^{-04}$	**0**
f_5	0	$5.6*10^{-01}$	1.04	$5.33*10^{-01}$	$1.28*10^{-2}$	$1.29*10^{-02}$	$1.9*10^{-02}$	0.2089	$3.75*10^{-02}$	**0**
f_6	0	5.92	72.8	15.32	30.12	16.1867	$8.5*10^{-02}$	$4.66*10^{-02}$	$4.27*10^{-02}$	**0**
f_7	$2*10^{-7}$	$1.1*10^{+3}$	$1.7*10^{+3}$	25.16	$2.4*10^{-01}$	$3.98*10^{-01}$	$9.14*10^{+01}$	1.1836	$7.64*10^{+01}$	$\mathbf{3.22*10^{-24}}$
f_8	0	$5.6*10^{+3}$	$2.9*10^{+3}$	$1.6*10^{+03}$	$1.1*10^{-01}$	$3.14*10^{-10}$	$4.16*10^{+02}$	11173	$3.66*10^{+03}$	**0**
f_9	0	11.78	23.6	$8.5*10^{-06}$	$1.3*10^{-02}$	2.8952	$7.76*10^{-01}$	40.737	3.77	**0**
f_{10}	0	1.07	2.0	$6.09*10^{-05}$	$9*10^{-01}$	$9.8*10^{-12}$	$2.42*10^{-01}$	$5.86*10^{-07}$	$9.75*10^{-03}$	**0**
f_{11}	0	23.45	$5*10^{-02}$	$2.1*10^{-10}$	11.78	$1.71*10^{-16}$	$8.86*10^{-01}$	$4.6*10^{-12}$	$5.14*10^{-04}$	**0**
f_{12}	0	24.52	$2*10^{-02}$	$2.1*10^{-10}$	$1.27*10^{-03}$	0	$2.8*10^{-01}$	0	$2*10^{-02}$	**0**
Ave. Rank	1.083	8.49	8.32	4.99	5.4	4.57	6.74	5.07	6.48	**1**
Sum error	$2*10^{-7}$	6770.74	5669.58	1641.30	43.3	19.5309	510.6653	11215.17	3740.914	$\mathbf{3.22*10^{-24}}$

TABLE 6. Different risk values (*g*).

#	g_1	g_2
a	[0.1, 0.05]	[0.05, 0.02]
b	[0.1, 0.05]	[0.2, 0.1]
c	[0.1, 0.05]	[0.1, 0.05]
d	[0.1, 0]	[0.05, 0]
e	[0.3, 0.2]	[0.05, 0.02]
f	[0.1, 0.05]	[0.3, 0.2]
g	[0.5, 0.3]	[0.5, 0.3]

is faster than the EMA for the same risk values. Moreover, as opposed to EMA that operates more efficiently for low-risk values, the EMGA converges much faster for the high-risk values.

Table 7 highlights the faster response of EMGA compared to other algorithms. Fig. 2 shows that EMGA renders more accurate results with faster convergence time against other algorithms. It should be noted that the ratio of the convergence time to the number of iterations in EMGA is slightly higher than the others. This reason is that EMGA spends more time over a single iteration due to the simultaneous usage of EMA and GA operators. However, EMGA achieves the optimal response in much fewer iterations which increases its response speed accordingly. Fig. 2 and Table 7 highlight the EMGA's advantages from convergence speed and accuracy points of view. Given these features, EMGA can be used as an efficient algorithm to solve engineering optimization problems.

For assessing the performance of the proposed algorithm in finding the solutions to the practical problems, the economic load dispatch in a power system with fifteen generation units is considered. In this optimization problem, the objective function consists of fifteen quadratic mathematical functions according to the generation units. The combination of the considered objective functions is considered as the final objective function, so this problem has fifteen optimization variables in which each of them has its own constraints. The initial inputs of this optimization problem include the objective functions' coefficients, generation boundaries of the power units, initial power of the units, and maximum and minimum ramp rate of the units in an hour which are derived from [43]. The optimization is constrained by the power balance equations among the generation units, demanded load of the network, the power loss of the system. The

TABLE 7. Convergence time of the EMGA, EMA, PSO, RCGA, and GSA algorithms for F1 to F12 benchmarks.

Function	EMGA		EMA		PSO		RCGA		GSA	
	Time	iteration	Time	iteration	Time	iteration	Time	iteration	Time	iteration
F1	0.86	200	1.58	380	23.24	6000	29.91	10000	91.12	10000
F2	0.82	330	1.62	470	25.87	6500	28.09	10000	92.91	10000
F3	4.92	1000	13.21	3200	32.28	6000	39.58	10000	95.22	9000
F4	9.28	1800	28.75	7500	37.92	9000	32.21	9000	104.35	10000
F5	0.71	100	1.23	250	38.48	10000	27.99	10000	22.31	2000
F6	4.28	1000	7.51	2000	33.51	9000	25.47	9000	21.23	2500
F7	4.92	1000	38.2	10000	42.91	10000	40.11	10000	98.30	10000
F8	1.92	600	3.02	1100	3.88	1000	27.86	10000	9.57	1000
F9	3.29	1000	4.52	1500	32.11	7500	28.35	10000	90.51	10000
F10	1.54	400	3.81	1200	4.74	1000	24.51	9000	90.08	10000
F11	0.89	200	2.84	800	30.21	8500	17.21	6000	90.09	10000
F12	0.50	60	1.24	200	3.21	880	2.65	1000	7.21	650
Average	2.82	641	8.96	2383	25.69	6282	26.99	8667	67.74	7096
Time/iteration	0.0044		0.0037		0.0041		0.0031		0.0095	

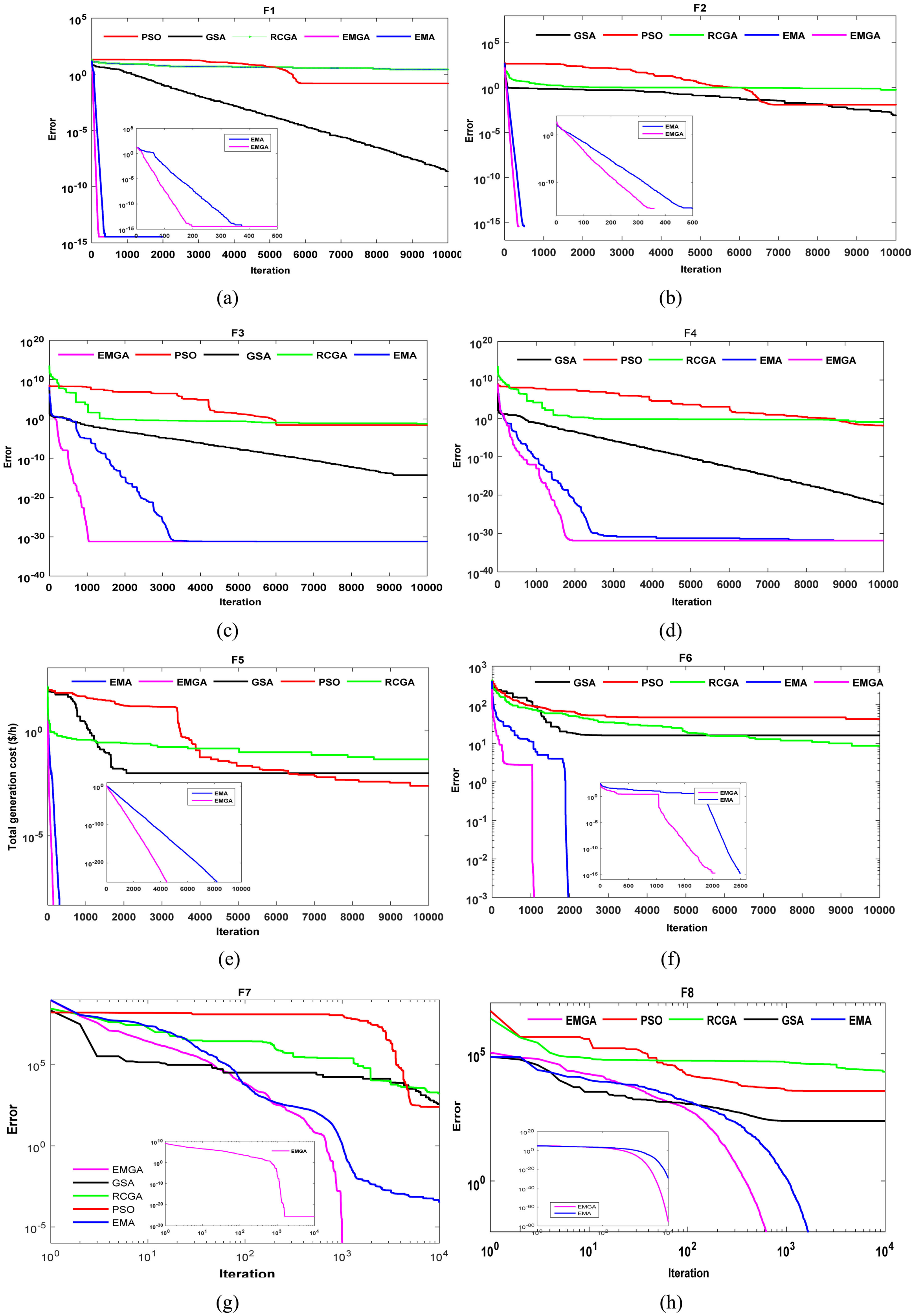

FIGURE 2. Comparison of convergence characteristics of EMGA, EMA, RCGA, PSO, and GSA for different benchmark functions.

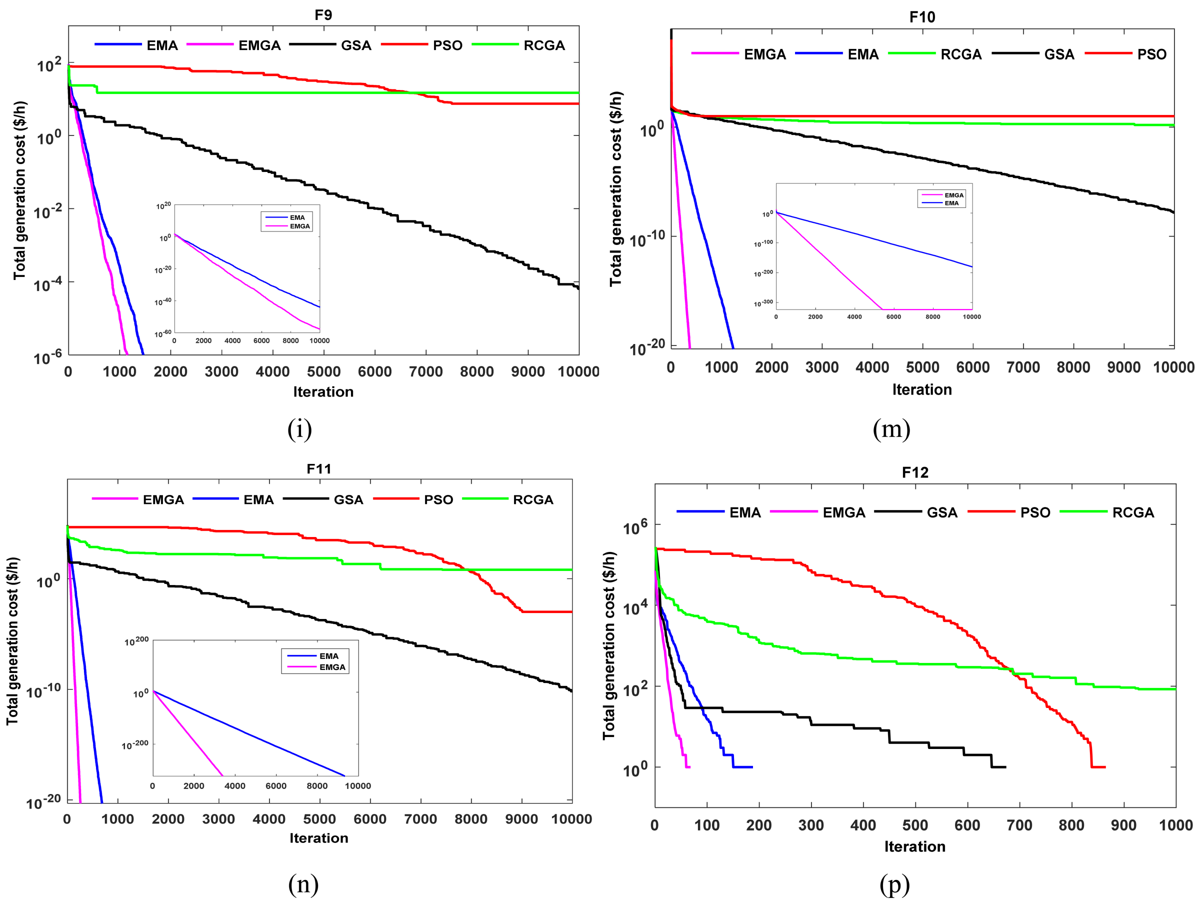

FIGURE 2. ***(Continued.)*** **Comparison of convergence characteristics of EMGA, EMA, RCGA, PSO, and GSA for different benchmark functions.**

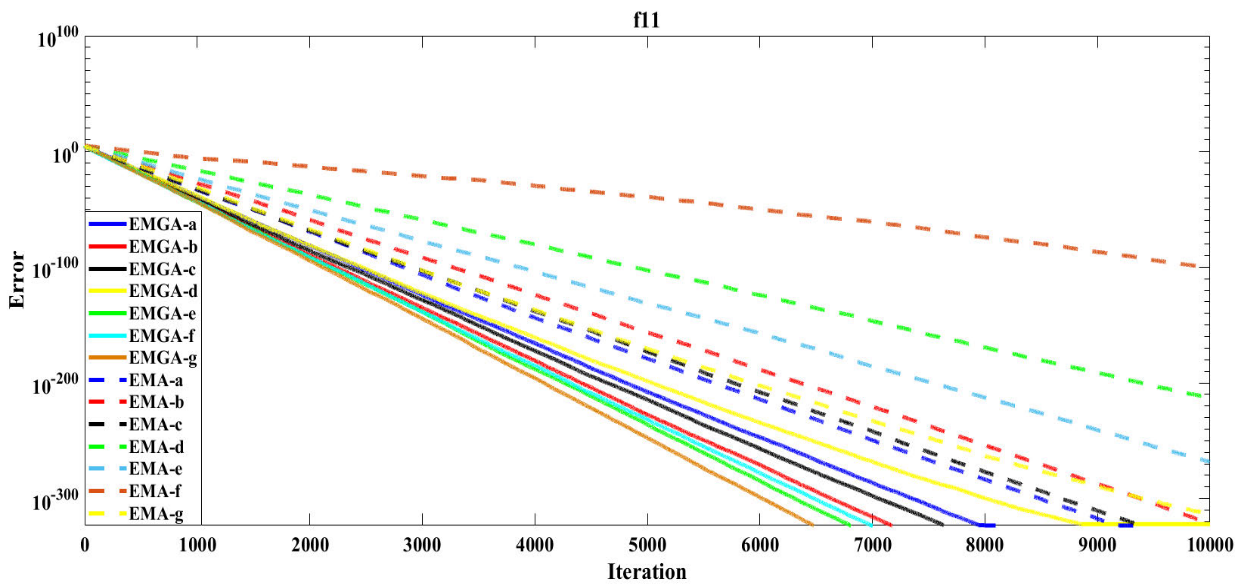

FIGURE 3. **Comparison of convergence characteristics of EMGA and EMA with different risk values.**

required load of the network is considered as 2630 (MW). Reference [43] solved and optimized the presented problem by EMA and compared the obtained results with other optimization algorithms which indicated the advantage of EMA against other considered algorithms in solving the optimization problems from the time and accuracy points of view.

TABLE 8. Results of EMA and EMGA for the economic dispatch of the fifteen unit system.

Unit (MW)	EMA	EMGA
P1	455.0000	455.0000
P2	380.0000	380.0000
P3	130.0000	130.0000
P4	130.0000	130.0000
P5	170.0000	170.0000
P6	460.0000	460.0000
P7	430.0000	430.0000
P8	72.0415	86.0119
P9	58.6212	65.1048
P10	160.0000	139.3867
P11	80.0000	80.0000
P12	80.0000	80.0000
P13	25.0000	25.0000
P14	15.0000	15.0000
P15	15.0000	15.0000
Total cost ($)	32704.4503	32683.3571
Time (Sec)/iteration	0.0033	0.0041
Iteration	5000	2000
Time taken (Sec)	16.5	8.2

Table 8 indicates the optimization results of EMA and EMGA for the considered system with fifteen units.

Table 8 shows that the proposed hybrid algorithm, EMGA, compared to the EMA finds better results for the economic load dispatch problem. Since EMA has better results compared to the other considered optimization algorithms mentioned in [43], EMGA also renders more accurate results against those optimization algorithms. EMGA with the high searching capability of the result space is applicable for practical problems. Even though EMGA uses two more optimizing operators than the EMA, the time of each iteration of the proposed algorithm is close to EMA's. It is remarkable that the EMGA finds the optimal answer in fewer iterations which reduces the total optimization time.

VI. CONCLUSION AND FUTURE WORK

In this paper, a hybrid optimization algorithm (EMGA) is proposed. The EMGA is developed based on the EMA and GA. The EMGA has the advantages of both GA and EMA. EMGA is a powerful tool to find the exact value of the optimal solution with the lowest number of iterations. In the proposed algorithm, two searching operators and one absorbent operator are utilized for the exchange market. The GA crossover operator leads to a faster process of convergence and the GA mutation operator leads to minimizing the risk of falling in local optimum points. The algorithm generates random numbers at each stage and then sorts them to find the best possible form. To verify the effectiveness of the proposed algorithm, 12 objective functions (benchmark functions) with different dimensions were considered. The simulation results show the capability of EMGA to find the most optimal answer in the least number of iterations. The results of the simulations proved that EMGA is a robust, efficient, accurate, and fast algorithm that can obtain the global optimum answer for a variety of discrete and continuous functions with different characteristics. EMGA is computationally efficient and can solve a variety of real-world problems such as linear, nonlinear, mixed-integer linear, and/or mixed-integer nonlinear problems.

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