

Recursive Divide and Conquer algorithm:

(Note for the sake of space we assume that we already know that anything times 1 is 1, anything times 0 is 0 and anything times 2 we shift to the left once )

46 \* 71

46 to eight bit: 00101110

71 to eight bit: 01000111

Divide binary number half and assign as the following:

00101110

A: 0010      B:1110

01000111

C:0100      D:0111

We apply the following formula where n is the number of digits of the original multiplicand:

$$A * C * 10^n + (A * D + B * C) * 10^{(n/2)} + B * D$$

$$0010 * 0100 * 2^8 + (0010 * 0111 + 1110 * 0100) * 2^4 + 1110 * 0111$$

We apply the formula recursively for each multiplicand.

$$0010 * 0100 * 2^8 + (0010 * 0111 + 1110 * 0100) * 2^4 + 1110 * 0111$$

N=4

0010

A: 00      B:10

0100

C: 01      D:00

$$00 * 1 * 10^4 + (00*00 + 10 * 01) * 2^2 + 10 * 00$$

Simplify

$$0 + (10)*2^2 + 0,$$

1000

We return to the original equation but replacing it with our result

$$1000 * 2^8 + (0010 * 0111 + 1110 * 0100) * 2^4 + 1110 * 0111$$

We shift 1000 by eight and apply the formula:

$$A * C * 2^n + (A * D + B * C) * 2^{(n/2)} + B * D$$

$$1000 0000 0000 + (0010 * 0111 + 1110 * 0100) * 2^4 + 1110 * 0111$$

N=4

0010

A: 00            B:10  
 0111  
 C:01            D:11

$$00 * 01 * 2^4 + (00 * 11 + 10 * 01) * 2^2 + 10 * 11$$

We can simplify this as:

$$0 + (0 + 10) * 2^2 + 110,$$

$$1000 + 110$$

$$1110$$

We return to our problem but replacing it with our result.

$$1000 \ 0000 \ 0000 + (1110 + 1110 * 0100) * 2^4 + 1110 * 0111$$

We recursively apply the formula to  $1110 * 0100$ :

$$\mathbf{A * C * 2^n + (A * D + B * C) * 2^{(n/2)} + B * D}$$

N=4  
 1110  
 A: 11            B:10

0100  
 C:01            D:00

$$11 * 01 * 2^4 + (11 * 00 + 10 * 01) * 2^2 + 10 * 00$$

We simplify

$$11 * 2^4 + (00 + 10) * 2^2 + 00,$$

$$110000 + 1000$$

$$111000$$

We return to our problem but replacing the computation with our result.

$$1000 \ 0000 \ 0000 + (1110 + 111000) * 2^4 + 1110 * 0111$$

We recursively apply the formula to  $1110 * 0111$ :

$$\mathbf{A * C * 2^n + (A * D + B * C) * 2^{(n/2)} + B * D}$$

N=4  
 1110  
 A:11            B:10

0111  
 C:01            D:11

$$11 * 01 * 2^4 + (11 * 11 + 10 * 01) * 2^2 + 10 * 11$$

We recursively apply the formula to  $11 * 11$ :

$$A * C * 2^n + (A * D + B * C) * 2^{(n/2)} + B * D$$

$N=2$

11

A: 1                  B: 1

11

C: 1                  D: 1

$$1 * 1 * 2^2 + (1 * 1 + 1 * 1) * 2 + 1 * 1,$$

We simplify

$$1 * 2^2 + (1 + 1) * 2 + 1$$

$$100 + 100 + 1$$

$$1001$$

We return to our problem but replacing the computation with our result.

$$11 * 01 * 2^4 + (1001 + 10 * 01) * 2^2 + 10 * 11$$

We simplify

$$11 * 2^4 + (1001 + 10) * 2^2 + 110,$$

$$110000 + (1011) * 2^2 + 110,$$

$$110000 + 101100 + 110,$$

$$1100010$$

We return to our problem but replacing the computation with our result.

$$100000000000 + (1110 + 111000) * 2^4 + 1100010$$

We now simplify

$$1000\ 0000\ 0000 + (1000110) * 2^4 + 1100010,$$

$$1000\ 0000\ 0000 + 100\ 0110\ 0000 + 110\ 0010$$

We get 1100 1100 0010, which converted to decimal is 3266.