1. Consider the following procedure for dividing X among A, B, and C:
Step 1: A is instructed to cut X into two pieces, X_1 and X_2, such that $\mu A(X_1) = 1/3$ and $\mu A(X_2) = 2/3$.
Step 2: B is then instructed to cut X_2 into pieces X_{21} and X_{22}, so that $\mu B(X_{21}) = \mu B(X_{22})$.
Step 3: The three players then select their pieces from $\{X_1, X_{21}, X_{22}\}$ in the order C, then A, and, finally, B.
Is this procedure correct, i.e., does it accomplish simple fair division for three players? Prove it.
No. Assume that player B believes that X, is greater than X_{21} and X_{22} and player A and C believe that $X_1 = X_{21} = X_{22}$. In this game player A and C pick X, and X_{21} respectively leaving player B to pick X_{22}. He'll complain that player A got the best piece.

2. A, B, and C wish to divide a cake so that A gets 1/2, B gets 1/3, and C gets 1/6. Here is a possible procedure for accomplishing this division:

Step 1: Instruct A and B to play cut-and-choose, so that each receives half of the cake according to his/her own evaluation.

Step 2: Instruct A and B to divide their shares into equivalent thirds.

Step 3: Give C her choice of these six pieces; she exits the game.

Step 4: If C chose one of B's pieces, there is nothing more to be done; otherwise (C must have chosen one of A's pieces, so) A gets to choose one of B's pieces.

Is this procedure correct? Prove it.
Yes. To prove the problem I will prove that all three players are satisfied and that cut and choose is fair towards two players.

1. Cut and Choose
    a. Cut and choose is when one player cuts the cake in what he deems ½ and the second player chooses which piece he wants. Because the player who cut the cake believes both pieces are ½ of a cake it doesn't matter what piece the other player chooses. And the player who chooses will pick the piece he deems is ½ or more.
2. Player C
    a. C is satisfied because he gets to pick any piece he deems to be 1/6th
3. Player A
    a. Case 1: C chooses player's B piece
        i. Player A is still left with ½ of a cake thus he is satisfied.
    b. Case 2: C chooses player's A piece
        i. Because Player A cut the cake he agrees that Player B has ½ of a cake. Because player B divided his ½ cake into three pieces there's at least one piece that is greater than or equal to ⅙ of the original cake. Thus Player A will choose that piece and be satisfied that he has at least ½ of a cake.

4. Player B
    a. Case 1: C chooses player's B piece
        i. Because Player B believes he divided his cake evenly into three pieces it doesn't matter what piece is taken. When a piece is taken he'll have 2/6 or ⅓ of the cake thus he is satisfied.
    b. Case 2: C chooses player A piece
        i. Because Player C choose a piece from Player A, Player A chooses a piece from Player B. Because Player B believes he divided his cake evenly into three pieces it doesn't matter what piece is taken. When a piece is taken he'll have 2/6 or ⅓ of the cake thus he is satisfied.

Thus all players are satisfied thus the procedure is correct.

3. A and B wish to divide a cake in the ratio 6:4, i.e., A is to receive at least 60% of the cake (according to A's valuation), and B is to receive at least 40% of the cake (according to B's valuation). Accordingly, they decide to use this algorithm:

Step 1: B is instructed to cut the cake into six pieces in the ratio 3:2:2:1:1:1.

Step 2: A is instructed to mark those pieces that (A feels) were appropriately cut.

Step 3: If A has identified pieces from which some subset adds up to 6/10, then A gets that subset and B gets the rest; otherwise, from the unmarked pieces, B chooses a subset that adds up to 4/10.

Does this algorithm work? If so, prove it; if not, why not?
Yes. If we treat the ratio 3:2:2:1:1:1 as a stars and bars combinatorics problem with the amount of stars equal to ten and the amount of bars equal to 5 with each bin having at least 1 we cannot get one bin greater than 6. This means that A will always get his 60% of the cake. And since B cut it, he will have his 40%.

4. A, B, and C wish to divide a cake using simple fair division. A was instructed to cut the cake into three equivalent pieces (X_1, X_2, and X_3), then B and C were told to identify pieces that they felt were at least one-third of the cake. Both players identified only X_3. What should they do?

Step 1: Have B pick out X_3 and the second best piece which we'll call X_s.
Step 2: Have B cut a piece off of X_3 until it is equal to X_s. We'll call the trimmings made from X_3 T_1.
Step 3: Have C pick which piece he/she wants.
Step 4: Have B pick a piece
Step 5: Have A pick a piece
Step 6: Have A cut T_1 into three evenly pieces
Step 7: Go back to Step 1 and continue these steps until there are no trimmings.