A Dice Puzzle Analysis

You keep rolling a set of five regular dice until you get a set showing exactly one six or a set showing zero sixes. When that happens, you stop rolling the dice. You win if you stop on a roll with exactly one six. You lose if you stop on a roll with no sixes. What is your probability of winning?

A. Case 1 Three Dice:
    a. Probability of Winning on a single toss:
        i. There are 216 possible outcomes rolling three regular dice. There are 25 possible combinations if the only six rolled is the first die, the same for the second die and the third die giving a possible combination of 75 winning rolls. Assuming the dice are fair and regular the probability of getting a win is 75/216 or roughly 35%
    b. Probability of Losing on a single toss:
        i. There are 216 possible outcomes rolling three dice. There are 125 possible combinations where no six is rolled. Assuming the dice are fair and regular the probability of losing is 125/216 or roughly 58%
    c. General Probability
        i. To get a general probability of winning the game since if we get more than one 6 we have to continue to roll, we assume that there are 100 tosses and according to our probability of winning there should be 35 wins and 58 losses. By adding the wins and loss and exclude the 7 no wins we get 93.
        ii. Our probability of winning is now 35/93 is roughly 37% which means our probability of losing is 1 - .37 = .63 or 63%
B. Case 2 Five Dice:
    a. General Info
        i. There are 7776 possible outcomes rolling 5 dice
    b. Probability of Winning:
        i. There are 625 possible combinations if only the first dice is a 6. We can multiply 625 by 5 to get the possible combinations which is 3125. Which is roughly 40%
    c. Probability of Losing
        i. There are also 3125 of not rolling a 6. Which is roughly 40%
    d. General Probability
        i. To get a general probability of winning and losing we assume that there are 100 tosses which means that there are 40 wins and 40 losses. If we add up the wins and losses and ignoring the third state where we roll again we get 80.
        ii. Our probability of winning is now 40/80 or 50% which means that our probability of losing is 50%
C. Case 3 Eight Dice:
    a. General Info

    i. There are 1679616 possible outcomes rolling 8 dice
b. Probability of Winning:
    i. There are 78125 combinations if only the first dice is 6. To account for the 2nd, 3rd, 4th, 5th, 6th, 7th and 8th dice we can multiply 78125 by 8. We get 625,000 possible combinations of only rolling 1 six. By dividing 625,000 by the total possible outcomes 1679616 we get roughly .37 or 37%.
c. Probability of Losing:
    i. Since there are eight options and each option can be any 5 numbers (not counting the roll of 6) the combinations of not having at least one six is given as 5^8 or 390625. By dividing that by the total possible outcomes 1679616 we get roughly .23 or 23%.
d. General Probability
    i. To get a general probability of winning and losing we assume that there are 100 tosses which means that there are 37 wins and 23 losses. If we add up the wins and losses and ignoring the third state where we roll again we get 60.
    ii. Our probability of winning is now 37/60 or roughly 62% which means that our probability of losing is 38%