Department of Industrial Engineering & Operations Research

IEOR 162: Linear Programming & Network Flows (Spring 2022)

1 Leasing of warehouse space problem

A firm has discovered that its own warehouses will not be sufficient to meet its space requirements over the next five months. Therefore, it plans to lease the additional space on a short term basis. The additional space requirements for the next five months are given by

Month	1	2	3	4	5
Requirement (in 1000 sq. ft.)	25	10	20	5	6

A lease for any amount of space can be taken out at the beginning of any month and can run for 1, 2, 3, 4, or 5 months. It is possible to have more than one leasing agreement in effect at any one time. The costs per thousand sq. feet of leases of various lengths are given by

Length of lease (months)	1	2	3	4	5
Cost (per 1000 sq. ft.)	280	450	600	730	840

Formulate a linear program whose solution will provide a leasing policy that satisfies the requirements at a minimum cost.

Decision variables:

- x_{ij}: Number of 1000 sq. feet leased at the beginning of the month i, for j months, $i \in \{1 \,..\, 5\}$ and $j \in \{1 \,..\, 5 - i + 1\}$

Formulation:

$$
\begin{aligned}
\min \quad & 280(x_{11} + x_{21} + x_{31} + x_{41} + x_{51}) + 450(x_{12} + x_{22} + x_{32} + x_{42}) \\
& + 600(x_{13} + x_{23} + x_{33}) + 730(x_{14} + x_{24}) + 840x_{15} \\
\text{s.t.} \quad & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \geq 25 \\
& x_{12} + x_{13} + x_{14} + x_{15} + x_{21} + x_{22} + x_{23} + x_{24} \geq 10 \\
& x_{13} + x_{14} + x_{15} + x_{22} + x_{23} + x_{24} + x_{31} + x_{32} + x_{33} \geq 20 \\
& x_{14} + x_{15} + x_{23} + x_{24} + x_{32} + x_{33} + x_{41} + x_{42} \geq 5 \\
& x_{15} + x_{24} + x_{33} + x_{42} + x_{51} \geq 6 \\
& x_{ij} \geq 0 \quad \forall i, j
\end{aligned}
$$