Department of Industrial Engineering & Operations Research

IEOR 162: Linear Programming and Network Flows (Spring 2022)

The Simplex Algorithm

Initialization: Let the initial system of equations be:

$$\begin{array}{lllll}
(0) & z - c_1x_1 - c_2x_2 - ... - c_nx_n & & & = 0 \\
(1) & a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + x_{n+1} & & & = b_1 \\
(2) & a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n & + x_{n+2} & & = b_2 \\
\vdots & & & & \\
(m) & a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n & & + x_{n+m} & = b_m
\end{array}$$

Steps:

1. **Selection of a non-basic variable to make basic (entering variable to the basis)**.

   Choose a non-basic variable, x_k, that will increase z at the fastest rate as it is increased; that is, x_k is the non-basic variable that has the most negative coefficient in row (0).

2. **Selection of a basic variable to make non-basic (leaving variable from the basis)**.

   Suppose x_k is the non-basic variable that will become basic. For each row (1) through (m), use the current system of equations to express the basic variable in row (i) as a linear function of the right hand side of row (i) and x_k. The leaving basic variable is the basic variable that reaches zero first as the entering basic variable is increased. This is the **ratio test**.

3. **Updating the tableau**.

   Suppose the leaving basic variable is the current basic variable for row (i). Then use the Gauss-Jordan method of elimination to make the coefficient of x_k a 1 in row (i) and 0 in all other rows; that is, make x_k the basic variable for row (i). The current basic feasible solution is found by setting all non-basic variables equal to zero and the basic variable for each row equal to the right hand side for that row.

Stopping Rule:

Stop when the coefficients of all variables in row (0) are greater than or equal to zero; that is, when increasing any variable cannot increase z. Otherwise, repeat the above steps.