Department of Industrial Engineering & Operations Research

IEOR 162 Linear Programming & Network Flows (Spring 2022)

1 Another example of the general network flow problem

1.1 Problem description

A company manufacturing chairs has four plants located around the country. The cost of manufacture, excluding raw material, per chair and the minimum and maximum monthly production for each plant is shown in the following table.

		Production	
Plant	Cost per chair	Maximum	Minimum
1	\$5	500	0
2	\$7	750	400
3	\$3	1000	500
4	\$4	250	250

Twenty pounds of wood is required to make each chair. The company obtains the wood from two sources. The sources can supply any amount to the company, but contracts specify that the company must buy at least eight tons of wood from each supplier. The cost of obtaining the wood at the sources is:

- Source 1 = \$0.10/pound
- Source 2 = \$0.075/pound

Shipping cost per pound of wood between each source and each plant is shown in cents by the following matrix:

		Plant			
		1	2	3	4
Wood	1	1	2	4	4
Source	2	4	3	2	2

The chairs are sold in four major cities: New York, Chicago, San Francisco and Austin. Transportation costs between the plants and the cities are shown in the following matrix: (All costs are in dollars per chair.)

		Cities			
		NY	A	SF	C
	1	1	1	2	0
Plant	2	3	6	7	3
	3	3	1	5	3
	4	8	2	1	4

The maximum and minimum demand and the selling price for the chairs in each city is shown in the following table:

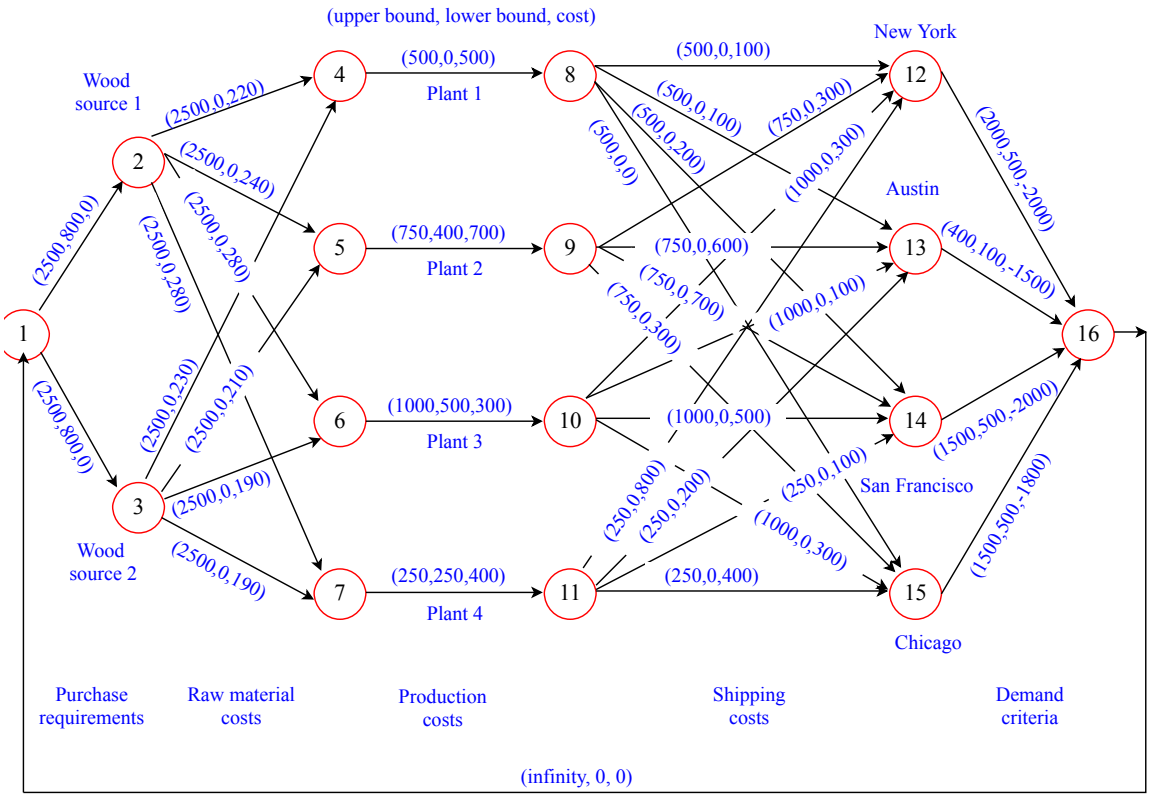
		Demand	
City	Selling price	Maximum	Minimum
New York	\$20	2000	500
Austin	\$15	400	100
San Francisco	\$20	1500	500
Chicago	\$18	1500	500

1.2 Formulation as a minimum-cost network flow problem

Table 3.8. NETWORK LABELS

Arc	*From Node*	*To Node*	*Upper Bound*	*Lower Bound*	*Cost*
1	1	2	2500	800	0
2	1	3	2500	800	0
3	2	4	2500	0	220
4	2	5	2500	0	240
5	2	6	2500	0	280
6	2	7	2500	0	280
7	3	4	2500	0	230
8	3	5	2500	0	210
9	3	6	2500	0	190
10	3	7	2500	0	190
11	4	8	500	0	500
12	5	9	750	400	700
13	6	10	1000	500	300
14	7	11	250	250	400
15	8	12	500	0	100
16	8	13	500	0	100
17	8	14	500	0	200
18	8	15	500	0	0
19	9	12	750	0	300
20	9	13	750	0	600
21	9	14	750	0	700
22	9	15	750	0	300
23	10	12	1000	0	300
24	10	13	1000	0	100
25	10	14	1000	0	500
26	10	15	1000	0	300
27	11	12	250	0	800
28	11	13	250	0	200
29	11	14	250	0	100
30	11	15	250	0	400
31	12	16	2000	500	−2000
32	13	16	400	100	−1500
33	14	16	1500	500	−2000
34	15	16	1500	500	−1800
35	16	1	5400	1600	0

Number of nodes = 16
Number of arcs = 35


(upper bound, lower bound, cost)
Wood source 1
Wood source 2
Plant 1
Plant 2
Plant 3
Plant 4
New York
Austin
San Francisco
Chicago
(500,0,500)
(750,400,700)
(1000,500,300)
(250,250,400)
(500,0,100)
(750,0,600)
(1000,0,500)
(250,0,400)
(infinity, 0, 0)
Purchase requirements
Raw material costs
Production costs
Shipping costs
Demand criteria

1.3 The optimal solution to the chairs problem

Table 3.9. THE OPTIMAL SOLUTION

Node k	π_k	*Arc (i, j)*	f_{ij}	*Arc (i, j)*	f_{ij}
1	1230	1	800	19	750
2	1200	2	1700	20	0
3	1230	3	500	21	0
4	1420	4	300	22	0
5	1440	5	0	23	650
6	1420	6	0	24	100
7	1420	7	0	25	250
8	3230	8	450	26	0
9	2930	9	1000	27	0
10	2930	10	250	28	0
11	3330	11	500	29	250
12	3230	12	750	30	0
13	3030	13	1000	31	1400
14	3430.	14	250	32	100
15	3230	15	0	33	500
16	1230	16	0	34	500
		17	0	35	2500
		18	500		

The original questions can now be answered in terms of the f_{ij} solution variables.

1. Where should each plant buy its raw materials?
   (a) Plant 1 should buy 500 chairs or 10,000 lb from wood source 1 and 0 chairs or 0 lb from wood source 2.
   (b) Plant 2 should buy 300 chairs or 6000 1b from wood source 1 and 450 chairs or 9000 lb from wood source 2.
   (c) Plant 3 should buy 0 chairs or 0 lb from wood source 1 and 1000 chairs or 20,000 lb from wood source 2.
   (d) Plant 4 should buy 0 chairs or 0 lb from wood source 1 and 250 chairs or 5000 lb from wood source 2.
2. How many chairs should be made at each plant?
   (a) Plant 1 should make 500 chairs.
   (b) Plant 2 should make 750 chairs.
   (c) Plant 3 should make 1000 chairs.
   (d) Plant 4 should make 250 chairs.
3. How many chairs should be sold at each city?
   (a) New York should sell 1400 chairs.
   (b) Austin should sell 100 chairs.
   (c) San Francisco should sell 500 chairs.
   (d) Chicago should sell 500 chairs.
4. Where should each plant ship its product?
   (a) Plant 1 ships 500 chairs to Chicago.
   (b) Plant 2 ships 750 chairs to New York.
   (c) Plant 3 ships 650 chairs to New York, 100 to Austin, and 250 to San Francisco.
   (d) Plant 4 ships 250 chairs to San Francisco.