A MAKE-OR-BUY-PROBLEM

A *make-or-buy problem* arises when a firm experiencing rapid growth finds itself with insufficient production capacity to fill its customers' orders. Although the long-run solution may be to expand the production capacity, the short-run solution usually consists of meeting demand with a combination of "in-house" production and "outside" purchases from vendors having excess capacity.

Problem Statement

To illustrate a make-or-buy problem, consider the following scenario:

DuPunt, Inc. manufactures three types of chemicals. For the upcoming month, DuPunt has contracted to supply the following amounts of the three chemicals:

CHEMICAL	CONTRACTED SALES (lb)
1	2000
2	3500
3	1800

DuPunt's production is limited by the availability of processing time in two chemical reactors. Each chemical must be processed first in reactor 1 and then in reactor 2. The following table provides the hours of processing time available next month for each reactor and the processing time required in each reactor per pound of each chemical:

	Reactor Processing Times (hr per lb) Chemical 1	2	3	Reactor Availabilities
Reactor 1	0.05	0.04	0.01	200 hours
Reactor 2	0.02	0.06	0.03	150 hours

Owing to the limited availability of reactor processing time, DuPunt has insufficient capacity to meet its demand with in-house production. Consequently, DuPunt must purchase some chemicals from vendors having excess capacity and resell them to its own customers. The following table provides each chemical's in-house production cost and outside purchase cost:

CHEMICAL	IN-HOUSE PRODUCTION COST ($ per lb)	OUTSIDE PURCHASE COST ($ per lb)
1	2.50	2.80
2	1.75	2.50
3	2.90	3.25

DuPunt's objective is to fill its customers' orders with the cheapest combination of in-house production and outside purchases. In short, DuPunt must decide how much of each chemical to produce in-house and how much of each chemical to purchase outside.

Decision variables:

- x_1: Number of pounds of chemical 1 to produce in-house.
- x_2: Number of pounds of chemical 2 to produce in-house.
- x_3: Number of pounds of chemical 3 to produce in-house.

Auxiliary Variables:

- y_i: Number of pounds of chemical i to purchase outside, $i \in \{1, 2, 3\}$.

Formulation:

$$\begin{array}{lrcll}
\min & 2.5x_1 + 1.75x_2 + 2.9x_3 + 2.8y_1 + 2.5y_2 + 3.25y_3 & & & \\
\text{s.t.} & x_1 + y_1 & = & 2000 & \\
 & x_2 + y_2 & = & 3500 & \\
 & x_3 + y_3 & = & 1800 & \\
 & 0.05x_1 + 0.04x_2 + 0.01x_3 & \leq & 200 & \\
 & 0.02x_1 + 0.06x_2 + 0.03x_3 & \leq & 150 & \\
 & x_i, y_i & \geq & 0 & i \in \{1, 2, 3\}
\end{array}$$

Alternative Formulation:

Decision variables:

- x_1: Number of pounds of chemical 1 to produce in-house.
- x_2: Number of pounds of chemical 2 to produce in-house.
- x_3: Number of pounds of chemical 3 to produce in-house.

Formulation:

$$\begin{array}{lrcll}
\min & 2.5x_1 + 1.75x_2 + 2.9x_3 + 2.8(2000 - x_1) + 2.5(3500 - x_2) + 3.25(1800 - x_3) & & & \\
\text{s.t.} & x_1 & \leq & 2000 & \\
 & x_2 & \leq & 3500 & \\
 & x_3 & \leq & 1800 & \\
 & 0.05x_1 + 0.04x_2 + 0.01x_3 & \leq & 200 & \\
 & 0.02x_1 + 0.06x_2 + 0.03x_3 & \leq & 150 & \\
 & x_i & \geq & 0 & i \in \{1, 2, 3\}
\end{array}$$

note:
$\min 2.5x_1 + 1.75x_2 + 2.9x_3 + 2.8(2000 - x_1) + 2.5(3500 - x_2) + 3.25(1800 - x_3)$ is equal to
$\min -0.3x_1 - 0.75x_2 - 0.35x_3 + [2.8(2000) + 2.5(3500) + 3.25(1800)]$ is equivalent to
$\min -0.3x_1 - 0.75x_2 - 0.35x_3$ is equal to
$-\max 0.3x_1 + 0.75x_2 + 0.35x_3$