Department of Industrial Engineering & Operations Research

IEOR 162 Linear Programming and Network Flows (Spring 2022)

Finding the Minimal Number of Individuals Required to Meet a Fixed Schedule of Tasks

References

1. G.B. Dantzig & D.R. Fulkerson, "Minimizing the Number of Tankers to Meet a Fixed Schedule", Naval Research Logistics Quarterly, Vol. 1 (1954), pp. 217-222.
2. M. Bellmore, G. Bennington, & S. Lubore, "A Multivehicle Tanker Scheduling Problem", Transportation Science, Vol. 5 (1971), pp. 36-47.
3. Saha, J.L., "An Algorithm for Bus Scheduling Problems", Operational Research Quarterly, Vol. 21, No. 4 (December 1970).
4. L.R. Ford & D.R. Fulkerson, Flows in Networks, Princeton University Press, Princeton, N.J. (1962), pp. 64-67.

The table below shows the required start and end times for ten tasks, as well as the minimum time it takes to reassign to task j an individual who has just completed task i.

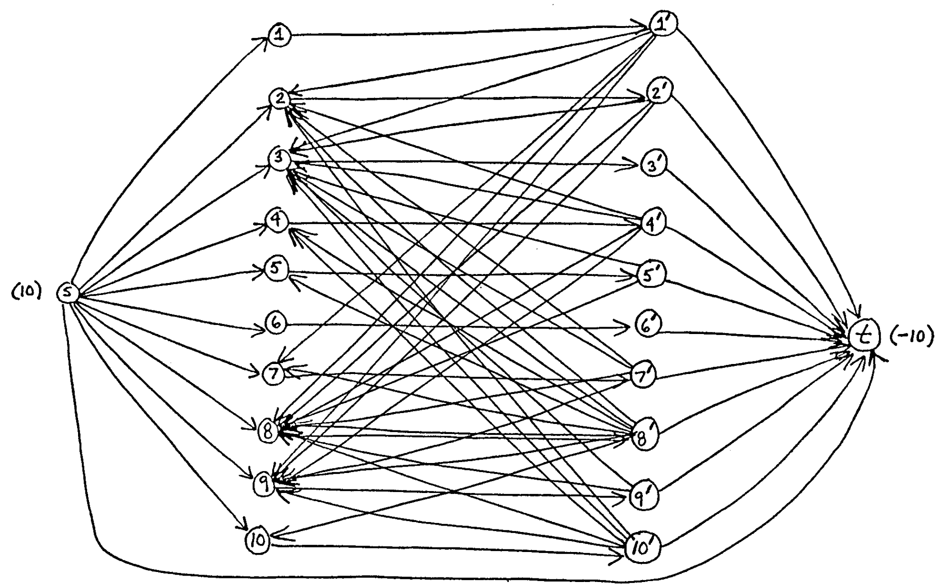
i	**bi**	**ei**	**Rij: Reassignment time from task i to task j (Minutes)**									
Task	**Start**	**End**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1:00 p.m.	1:30 p.m.	----	60	10	230	180	20	15	40	120	30
2	6:00 p.m.	8:00 p.m.	10	----	40	75	40	5	30	60	5	15
3	10:30 p.m.	11:00 p.m.	70	30	----	0	70	30	20	5	120	70
4	4:00 p.m.	5:00 p.m.	0	50	75	----	20	15	10	20	60	10
5	4:00 p.m.	7:00 p.m.	200	240	150	70	----	15	5	240	90	65
6	12:00 noon	1:00 p.m.	20	15	20	75	120	----	30	30	15	45
7	2:00 p.m.	5:00 p.m.	15	30	60	45	30	15	----	10	5	0
8	11:00 p.m.	12:00 Midnite	20	35	15	120	75	30	45	----	20	15
9	8:10 p.m.	9:00 p.m.	25	60	15	10	100	70	80	60	----	120
10	1:45 p.m.	3:00 p.m.	60	60	30	30	120	40	50	60	70	----

Problem: Find the minimum number of individuals required to complete the 10 tasks with no deviation from the schedule and find a schedule for each individual.

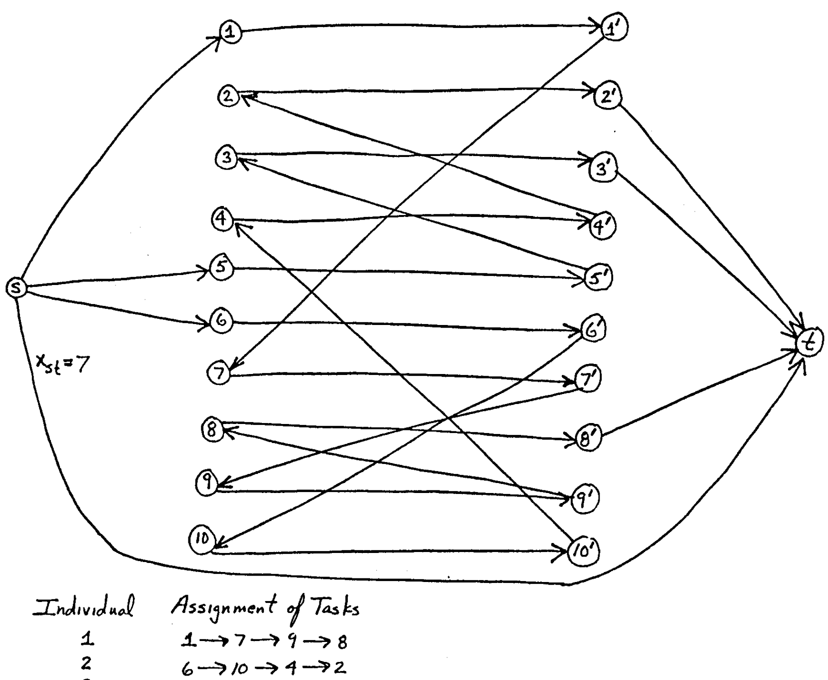
Solution

Node Summary

1. Construct a source s with a supply of 10 individuals; this source has a supply of 10 because this is clearly an upper bound on the minimal number of individuals required.
2. Construct a sink t with a demand for 10 individuals.
3. Construct a node i representing the start of task i and construct a node i' representing the end of task i.



The Network



An Optimal Solution (Only arcs having positive flow are shown, and, except for arc (s, t) which has a flow of $x_{st} = 7$, all other arcs have unit flows.)

Individual	Assignment of Tasks
1	1 → 7 → 9 → 8
2	6 → 10 → 4 → 2
3	5 → 3

Extensions

A The above problem will usually possess multiple optimality in the sense that, if N* denotes the minimal number of individuals required to perform all tasks, there will usually be more than one way to schedule these N* individuals. In such a case, it may be desirable to choose from among these multiple schedules that schedule for the N* individuals that optimizes some criterion. For example, suppose it is desired to find the schedule for the N* individuals that minimizes the sum of the reassignment times. Then after determining N*, the problem should be resolved with the following modifications:

1

2

B Now suppose that the number of individuals available is N. Also, suppose there is a fixed charge of k to use an individual. Finally, suppose that a profit of p_i is earned for performing task i. In this situation, it may be impossible to perform all tasks (e.g., if $N < N^*$) and/or uneconomical to perform all tasks (e.g., if k is very large). The problem now is to determine which tasks should be performed, how many of the N individuals should be used to perform the selected tasks, and what schedule each of these individuals should follow. The problem can be solved by modifying the original network as follows:

1

2

3