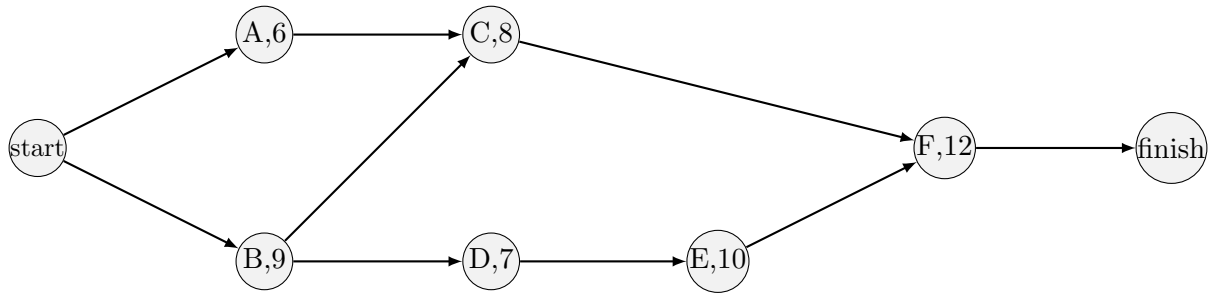
Department of Industrial Engineering & Operations Research

IEOR 162 Linear Programming & Network Flows (Spring 2020)

Linear Programming Formulation of Project Management

Consider the project network below:



Activity	Min. duration	Max. duration	Unit expediting cost ($)
A	3	6	80
B	5	9	50
C	6	8	70
D	5	7	30
E	8	10	50
F	8	12	100

a) Construct an LP to determine the earliest finish time of the project.

t_i = start time of activity i

$$
\begin{aligned}
\min \quad & t_{finish} \\
\text{s.t.} \quad & t_A - t_{start} \geq 0 \\
& t_B - t_{start} \geq 0 \\
& t_C - t_A \geq 6 \\
& t_C - t_B \geq 9 \\
& t_D - t_B \geq 9 \\
& t_E - t_D \geq 7 \\
& t_F - t_C \geq 8 \\
& t_F - t_E \geq 10 \\
& t_{finish} - t_F \geq 12 \\
& t_{start} = 0
\end{aligned}
$$

b) Suppose that the project needs to be completed in 35 days. The activities can be expedited as long as the duration exceeds the minimum duration. The cost of expediting an activity is given in the table above. Write an LP to minimize the expediting cost.

t_i = start time of activity i
x_i = number of days of expediting of activity i

$$
\begin{aligned}
\min \quad & 80x_A + 50x_B + 70x_C + 30x_D + 50x_E + 100x_F \\
\text{s.t.} \quad & t_A - t_{start} \geq 0 \\
& t_B - t_{start} \geq 0 \\
& t_C - t_A \geq 6 - x_A \\
& t_C - t_B \geq 9 - x_B \\
& t_d - t_B \geq 9 - x_B \\
& t_E - t_D \geq 7 - x_D \\
& t_F - t_C \geq 8 - x_C \\
& t_F - t_E \geq 10 - x_E \\
& t_{finish} - t_F \geq 12 - x_F \\
& t_{start} = 0 \\
& t_{finish} \leq 35 \\
& 0 \leq x_A \leq 3 \\
& 0 \leq x_B \leq 4 \\
& 0 \leq x_C \leq 2 \\
& 0 \leq x_D \leq 2 \\
& 0 \leq x_E \leq 2 \\
& 0 \leq x_F \leq 4
\end{aligned}
$$

c) Instead of meeting the deadline exactly, suppose that there is a penalty of \$90 for each day the project extends beyond 35 days. Write an LP to minimize the total cost.

t_i = start time of activity i
x_i = number of days of expediting of activity i
z = number of days beyond 35 that the project takes

$$
\begin{aligned}
\min \quad & 80x_A + 50x_B + 70x_C + 30x_D + 50x_E + 100x_F + 90z \\
\text{s.t.} \quad & t_A - t_{start} \geq 0 \\
& t_B - t_{start} \geq 0 \\
& t_C - t_A \geq 6 - x_A \\
& t_C - t_B \geq 9 - x_B \\
& t_d - t_B \geq 9 - x_B \\
& t_E - t_D \geq 7 - x_D \\
& t_F - t_C \geq 8 - x_C \\
& t_F - t_E \geq 10 - x_E \\
& t_{finish} - t_F \geq 12 - x_F \\
& t_{start} = 0 \\
& 0 \leq x_A \leq 3 \\
& 0 \leq x_B \leq 4 \\
& 0 \leq x_C \leq 2 \\
& 0 \leq x_D \leq 2 \\
& 0 \leq x_E \leq 2 \\
& 0 \leq x_F \leq 4 \\
& z \geq t_{finish} - 35 \\
& z \geq 0
\end{aligned}
$$