

1 Linear programming modeling examples

1.1 Investment problem

You own \$1 on Monday and want to invest it to maximize the amount on Friday. Investments may be made on Monday, Tuesday, Wednesday, Thursday and/or Friday.

The investment scheme is as follows:

Investing \$ x today and \$ $2x$ tomorrow will yield \$ $4x$ the day after tomorrow. For example, investing \$0.10 on Tuesday and \$0.20 on Wednesday will return \$0.40 on Thursday.

Which is the best investment plan that you can come up with? (i.e. How much to invest on each day? What is the total amount you will end on Friday afternoon?)

Decision variables:

- x_m, x_{tu}, x_w : The \$ Amount of initial investment on Monday, Tuesday, and Wednesday.

Formulation:

$$\begin{array}{ll}\max & 1 + x_m + x_{tu} + x_w \\ \text{s.t.} & x_m \leq 1 \\ & 2x_m + x_{tu} \leq 1 - x_m \\ & 2x_{tu} + x_w \leq 1 + x_m - x_{tu} \\ & 2x_w \leq 1 + x_m + x_{tu} - x_w \\ & 0 \leq 1 + x_m + x_{tu} + x_w \\ & x_m, x_{tu}, x_w \geq 0\end{array}$$

Decision variables:

- x_m, x_{tu}, x_w : The \$ Amount of initial investment on Monday, Tuesday, and Wednesday.
- $y_j, j \in \{m, tu, w, th, f\}$: The \$ amount on hand at the end of day j.

Formulation:

$$\begin{array}{ll}\max & y_f \\ \text{s.t.} & x_m + y_m = 1 \\ & x_{tu} + 2x_m + y_{tu} = y_m \\ & x_w + 2x_{tu} + y_w = y_{tu} + 4x_m \\ & 2x_w + y_{th} = y_w + 4x_{tu} \\ & y_f = y_{th} + 4x_w \\ & x_i, y_j \geq 0 \quad i \in \{m, tu, w\}, j \in \{m, tu, w, th, f\}\end{array}$$

Alternative Formulation

Decision variables:

- x_t , $t \in \{1, 2, 3, 4, 5\}$: Amount to invest on day t , where day 1 is Monday, day 2 is Tuesday ...
- y_t , $t \in \{1, 2, 3, 4, 5\}$: Base amount to invest in the scheme that starts on day t , where day 1 is Monday, day 2 is Tuesday ...

Formulation:

$$\begin{aligned} \max \quad & 1 - \sum_{t=1}^5 x_t + \sum_{t=1}^3 4y_t \\ \text{s.t.} \quad & y_1 = x_1 \\ & 2y_{t-1} + y_t = x_t \quad t \in \{2, 3, 4, 5\} \\ & x_1 + x_2 \leq 1 \\ & \sum_{i=1}^t x_i \leq 1 + \sum_{i=1}^{t-2} 4y_i \quad t \in \{3, 4, 5\} \\ & x_t, y_t \geq 0 \quad t \in \{1, 2, 3, 4, 5\} \end{aligned}$$