Department of Industrial Engineering & Operations Research

IEOR 162: Linear Programming & Network Flows (Spring 2022)

Person power planning

Certain types of facilities operate seven days each week and face the problem of allocating person power during the week as staffing requirements change as a function of the day of the week.

Perhaps the most fundamental staffing problem is the assignment of days off to full time employees. In particular, it is regularly the case that each employee is entitled to two consecutive days off per week. If the number of employees required on each of the seven days of the week is given, then the problem is to find the minimum workforce size which will allow these demands to be met and then to determine the days off for the people in this workforce.

To be specific, let us study the problem faced by the Festus City Bus Company. The number of drivers required for each day of the week are as follows:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
14	20	22	5	15	17	16

a) How many drivers should be scheduled to start a five day stint on each day of the week? Formulate this problem as a linear program.

Decision variables:

- x_m: Number of drivers starting stint on Monday.
- x_{tu}: Number of drivers starting stint on Tuesday.
- x_w: Number of drivers starting stint on Wednesday.
- x_{th}: Number of drivers starting stint on Thursday.
- x_f: Number of drivers starting stint on Friday.
- x_{sa}: Number of drivers starting stint on Saturday.
- x_{su}: Number of drivers starting stint on Sunday.

Formulation:

$$
\begin{aligned}
\min \quad & x_m + x_{tu} + x_w + x_{th} + x_f + x_{sa} + x_{su} \\
\text{s.t.} \quad & x_m + x_{th} + x_f + x_{sa} + x_{su} \geq 14 \\
& x_m + x_{tu} + x_f + x_{sa} + x_{su} \geq 20 \\
& x_m + x_{tu} + x_w + x_{sa} + x_{su} \geq 22 \\
& x_m + x_{tu} + x_w + x_{th} + x_{su} \geq 5 \\
& x_m + x_{tu} + x_w + x_{th} + x_f \geq 15 \\
& x_{tu} + x_w + x_{th} + x_f + x_{sa} \geq 17 \\
& x_w + x_{th} + x_f + x_{sa} + x_{su} \geq 16 \\
& x_m, x_{tu}, x_w, x_{th}, x_f, x_{sa}, x_{su} \geq 0
\end{aligned}
$$

b) Daily pay is \$50 per person on weekdays, \$75 on Saturday and \$90 on Sunday. Modify the formulation so that the objective is now to minimize the weekly payroll costs rather than to minimize the workforce size.

Decision variables:

- x_m: Number of drivers starting stint on Monday.
- x_{tu}: Number of drivers starting stint on Tuesday.
- x_w: Number of drivers starting stint on Wednesday.
- x_{th}: Number of drivers starting stint on Thursday.
- x_f: Number of drivers starting stint on Friday.
- x_{sa}: Number of drivers starting stint on Saturday.
- x_{su}: Number of drivers starting stint on Sunday.

Formulation:

$$
\begin{aligned}
\min \quad & 250x_m + 275x_{tu} + 315x_w + 315x_{th} + 315x_f + 315x_{sa} + 290x_{su} \\
\text{s.t.} \quad & x_m + x_{th} + x_f + x_{sa} + x_{su} \geq 14 \\
& x_m + x_{tu} + x_f + x_{sa} + x_{su} \geq 20 \\
& x_m + x_{tu} + x_w + x_{sa} + x_{su} \geq 22 \\
& x_m + x_{tu} + x_w + x_{th} + x_{su} \geq 5 \\
& x_m + x_{tu} + x_w + x_{th} + x_f \geq 15 \\
& x_{tu} + x_w + x_{th} + x_f + x_{sa} \geq 17 \\
& x_w + x_{th} + x_f + x_{sa} + x_{su} \geq 16 \\
& x_m, x_{tu}, x_w, x_{th}, x_f, x_{sa}, x_{su} \geq 0
\end{aligned}
$$