Department of Industrial Engineering & Operations Research

IEOR 162: Linear Programming & Network Flows (Spring 2022)

1 An investment problem

An investor has money-making activities A and B available each year for the next 5 years. Each dollar invested in A at the beginning of a year returns \$1.30 two years later (so \$1.00 invested in year 1 yields \$1.30 available for re-investment at the beginning of year 3). Each dollar invested in B at the beginning of a year returns \$1.50 three years later.

Additionally, money-making activities C and D are available exactly once in the next 5 years. A dollar invested in C at the beginning of year 2 returns \$1.70 at the end of year 5. A dollar invested in D at the beginning of year 5 returns \$1.20 at the end of year 5.

Suppose the investor begins with \$10,000. Formulate a linear program that will produce an investment plan that will maximize his return by the beginning of the 6th year.

2 Solution

Define decision variables:
A_t = the dollar amount invested in A at the beginning of year t, $t = 1, 2, 3, 4$.
B_t = the dollar amount invested in B at the beginning of year t, $t = 1, 2, 3$.
C_2 = the dollar amount invested in C at the beginning of year 2
D_5 = the dollar amount invested in D at the beginning of year 5
R_t = the dollar amount available after investment and returns in year t.

The linear program is:

$$
\begin{aligned}
\text{maximize} \quad & R_6 \\
\text{subject to} \quad & R_1 = 10,000 - A_1 - B_1 \\
& R_2 = R_1 - A_2 - B_2 - C_2 \\
& R_3 = R_2 + 1.3A_1 - A_3 - B_3 \\
& R_4 = R_3 + 1.3A_2 + 1.5B_1 - A_4 \\
& R_5 = R_4 + 1.3A_3 + 1.5B_2 - D_5 \\
& R_6 = R_5 + 1.3A_4 + 1.5B_3 + 1.7C_2 + 1.2D_5 \\
& A_t, B_t, R_t \geq 0, \ t = 1, 2, \ldots, 6, \\
& C, D \geq 0.
\end{aligned}
$$

- The objective function is equal to the total amount of money we have at the beginning of the sixth year, which is equivalent to end of the fifth year.
- Each constraint can be thought of as a "cash balance" constraint – the amount of money we have available after investment and before returns in year t, (R_t), must equal the amount of money we had after investment and before returns of the preceding year, (R_{t-1}), plus any returns we are expecting at the end of year $t-1$, minus any investments we make in year t. Note that this means we have one constraint for each of the 6 years.