

1 Maximizing rent: Example

Problem: We wish to optimally select bids for a 10 day period:

Arrival day	Departure day	Bid (\$)
1	2	2
1	5	7
2	4	2
3	7	4
3	8	11
4	5	1
4	6	6
5	6	3
5	9	7
7	8	4
7	9	5
8	10	3

Solution: Define $D(j)$ to be the optimal renting strategy value from day 1 up till day j . Then

$$D(j) = \max_{i: (i,j) \in A} \{D(i) + \text{weight}(i,j)\},$$

where

$$\text{weight}(i,j) = \begin{cases} \max \{\text{Bid}(i,j)\}, & \text{Bids for days } i \text{ through } j \\ 0, & \text{if no such Bid and } j = i + 1 \end{cases}$$

Then, evaluate $D(j)$ for $j = 1, 2, \dots, 10$ in this order. This process is called **Dynamic Programming**.

The arcs in the graph below represent the available bids. Why is this graph a Directed Acyclic Graph (DAG)? What costs should one assign, so that the shortest (least cost) path in this graph provides the optimal bidding strategy?

