Finding Duals of LP's in Non-standard form

IEOR 162 (Spring 2022)

Method A

1. Convert problem to "standard form" (maximization with ≤ constraints) as indicated by following table:

Non-standard from	Equivalent standard form
Minimize z	Maximize (-z)
$\sum_{j=1}^{n} a_{ij}x_j \geq b_i$	$-\sum_{j=1}^{n} a_{ij}x_j \leq -b_i$
$\sum_{j=1}^{n} a_{ij}x_j = b_i$	$\sum_{j=1}^{n} a_{ij}x_j \leq b_i$ and $-\sum_{j=1}^{n} a_{ij}x_j \leq -b_i$
x_j unconstrained in sign	$(x'_j - x''_j)$, $x'_j \geq 0$, $x''_j \geq 0$

2. Take dual of this equivalent LP in standard form
3. Simplify result, if possible, by "reversing" step 1; that is, replace an expression of a form similar to one in the right column of the table in Step 1 with the corresponding expression in the left column of the table.

Method B

1. Put LP in one of the following forms:
   a) maximize objective with ≤ or = constraints
   b) minimize objective with ≥ or = constraints
2. Construct dual as indicated by following table:

Maximize in objective	Minimize in objective
ith constraint is a ≤ ith constraint is an =	ith variable constrained in sign ith variable unrestricted in sign
jth variable constrained to be ≥ 0 jth variable unrestricted in sign	jth constraint is a ≥ jth constraint is an =

Example of Dual of LP not in standard form:

$$\begin{array}{ll}
\text{min} & 3x1 + 5x2 + 4x3 \\
\text{subject to:} & x1 + 3x2 + x3 \leq 8 \\
 & 2x1 + x2 - 2x3 = 5 \\
 & 3x1 - 2x2 + 4x3 \geq 7 \\
 & x1 \geq 0,\ x2 \geq 0,\ x3 \text{ unrestricted in sign}
\end{array}$$

Method A

Step 1

$$\begin{array}{ll}
\text{max} & -3x1 - 5x2 - 4x'3 + 4x''3 \\
\text{subject to:} & x1 + 3x2 + x'3 - x''3 \leq 8 \\
 & 2x1 + x2 - 2x'3 + 2x''3 \leq 5 \\
 & -2x1 - x2 + 2x'3 - 2x''3 \leq -5 \\
 & -3x1 + 2x2 - 4x'3 + 4x''3 \leq -7 \\
 & x1 \geq 0,\ x2 \geq 0,\ x'3 \geq 0,\ x''3 \geq 0
\end{array}$$

Step 2

$$\begin{array}{ll}
\text{min} & 8y1 + 5y'2 - 5y''2 - 7y3 \\
\text{subject to:} & y1 + 2y'2 - 2y''2 - 3y3 \geq -3 \\
 & 3y1 + y'2 - y''2 + 2y3 \geq -5 \\
 & y1 - 2y'2 + 2y''2 - 4y3 \geq -4 \\
 & -y1 + 2y'2 - 2y''2 + 4y3 \geq 4 \\
 & y1 \geq 0,\ y'2 \geq 0,\ y''2 \geq 0,\ y3 \geq 0
\end{array}$$

Step 3

Replace $y'2 - y''2$, $y'2 \geq 0$, $y''2 \geq 0$ by y2 unconstrained.
Replace last two functional constraints by equivalent equality constraint.
Replace "minimize" with "maximize".
Multiply first two functional constraints by (-1).

The result is

$$\begin{array}{ll}
\text{max} & -8y1 - 5y2 + 7y3 \\
\text{subject to:} & -y1 - 2y2 + 3y3 \leq 3 \\
 & -3y1 - y2 - 2y3 \leq 5 \\
 & -y1 + 2y2 + 4y3 = 4 \\
 & y1 \geq 0,\ y2 \text{ unrestricted},\ y3 \geq 0
\end{array}$$

Method B

Yields directly the dual formulation.