

IEOR 162
Spring 2022

An additional example

$$\max z = 3x_1 + 5x_2$$

$$\begin{aligned} \text{subject to: } & x_1 \leq 4 \\ & 2x_2 \leq 12 \\ & 3x_1 + 2x_2 \leq 18 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

Iteration	Basic Variable	Eq #	z	x1	x2	x3	x4	x5	RHS
0	z	0	1	-3	-5	0	0	0	0
	x3	1	0	1	0	1	0	0	4
	x4	2	0	0	2	0	1	0	12
	x5	3	0	3	2	0	0	1	18

2	z	0	1	0	0	0	3/2	1	36
	x3	1	0	0	0	1	1/3	-1/3	2
	x2	2	0	0	1	0	1/2	0	6
	x1	3	0	1	0	0	-1/3	1/3	2

1. By how much can the coefficient of x_1 in the objective function change while keeping the same optimal solution?

Coefficient	Range in which $(x_1, x_2) = (2, 6)$ remains optimal
$c_1 = 3$	$0 \leq c_1 \leq 7.5$

2. By how much can the coefficient of x_2 in the objective function change while keeping the same optimal solution?

Coefficient	Range in which $(x_1, x_2) = (2, 6)$ remains optimal
$c_2 = 5$	$2 \leq c_2$

3. What is the shadow price of each constraint and the range?

Note that each sensitivity analysis assumes that all other b_i and all c_j and a_{ij} do not change.

Right-hand-side	Range in which x_3, x_2 and x_1 remain the optimal basic variables
$b_1 = 4$	$2 \leq b_1$ (shadow price = 0) → {non-binding constraint}
$b_2 = 12$	$6 \leq b_2 \leq 18$ (shadow price = 3/2)
$b_3 = 18$	$12 \leq b_3 \leq 24$ (shadow price = 1)