New approach to the method of the initial optical design based on the matrix optics

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ABSTRACT

In this work the paraxial optical imaging is generally described by means of three square matrices: one unitary system matrix and two operation matrices with determinants equal the Lagrange-Helmholtz invariant. Elements of system matrix are functions of design parameters while elements of the operation matrix depend on input and output coordinates of characteristic rays. Each matrix has only three independent elements. Internal system parameters are determined from equations created of system matrix elements, which values are dependent on the operation matrix. Matrix approach enables the solution of only three non-linear equations with respect to system parameters. Matrix approach has also another advantage. It enables the determination of number of degrees of freedom. We have a superiority of parameters over the number of equations when the number of components is bigger than 2. The more complex is model the higher degree of freedom it has. There are special ways of reducing the number of degree of freedom: by selection of spaces between component, introduction of additional requirements and criteria of distribution of optical powers. Significant help is in defining all the spaces between components, what means full control of the components position and their dimensions. In such a case the only thing left is the determination of optical powers, while the number of degree of freedom is equal k-1 (k is the number of components). In this work the computer program realizing described algorithms has been developed. This program was tested with specially selected examples. Results of calculation for two interesting applications are also given.

Keywords: predesign of optical system, matrix optics, optical programming

1. INTRODUCTION

Usually the task of initial calculations of optical system is the determination of its thin-component model. There is no information available concerning the development of general approach to the methodology of initial calculations of optical systems. The only exception is the work of authors continued for many years. [1-3]. Earlier authors preferred the step by step method of determination of searched thin-component model parameters [3]. In this paper the method was enriched with conclusions and formulas resulting from the matrix approach, what advantageously influenced application capabilities.

2. THEORETICAL APPROACH

The base for the theoretical approach is description of the complex thin-component system and its operation by means of the following diagram (Fig.1).

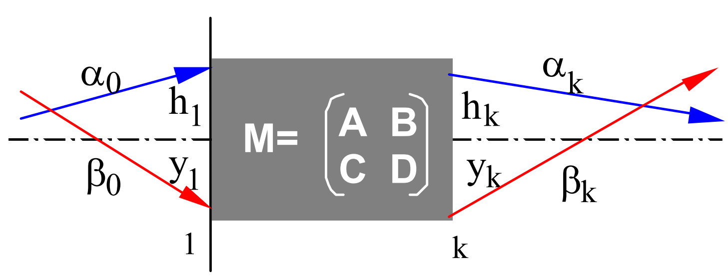

Fig. 1 Diagram of an optical system and its operation

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In this model the optical system is described by unitary square matrix of elements A, B, C and D, and its operation by two operation matrices based on characteristic rays coordinates of an aperture ray and a main field ray fixed at boundaries of system. The advantage of this diagram is that the knowledge of matrix elements is sufficient for the determination of imaging without knowing the internal structure of the system.

2.1 Resultant system matrix

The thin-component system matrix is equal to the product of elementary matrices connected with individual components [4]. Elementary matrices are taken in reversed order due to their non-commutative character. Optical powers of components are denoted by ϕ_i, and spaces by e_i, (where "i" is a component index i=1÷k). When the reference planes are set on the first and last component the resultant system matrix is determined from well known formula

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -e_{k-1} \\ \phi_k & 1-e_{k-1}\cdot\phi_k \end{bmatrix} \cdot \begin{bmatrix} 1 & -e_{k-2} \\ \phi_{k-1} & 1-e_{k-2}\cdot\phi_{k-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 \\ \phi_1 & 1 \end{bmatrix}. \tag{1}$$

In case of three-component system the symbolic multiplication of elementary matrices gives the following result:

$$\begin{aligned} A &= 1-\phi_1\cdot[e_1-e_2\cdot(e_1\cdot\phi_2-1)]-e_2\cdot\phi_2 \\ B &= e_2\cdot(e_1\cdot\phi_2-1)-e_1 \\ C &= \phi_3+\phi_1\cdot[(e_1\cdot\phi_2-1)\cdot(e_2\cdot\phi_3-1)-e_1\cdot\phi_3]-\phi_2\cdot(e_2\cdot\phi_3-1) \\ D &= (e_1\cdot\phi_2-1)\cdot(e_2\cdot\phi_3-1)-e_1\cdot\phi_3 . \end{aligned} \tag{2}$$

Determinant of the resultant system matrix is equal 1, what can be described by the following formula

$$A\cdot D-B\cdot C=1 . \tag{3}$$

System matrix is unitary because the elementary matrices of individual components are unitary. Geometrical interpretation of system matrix elements depends on the type of the system, but independently of this C is always the optical power.

2.2 Operation matrices

In this work two square operation matrices are introduced. These matrices are built of characteristic rays coordinates, which determinants are equal to the Lagrange-Helmholtz invariant. We consider two characteristic rays. The aperture ray is described by coordinates h_1 and α_0 at the entrance of the system, and h_k and α_k at its exit. The main field ray is described by y_1 and β_0 at the entrance and y_k and β_k at the exit of the system. So we have the entrance and exit operation matrix. The structure of operation matrix is given in the following equation

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} y_1 & h_1 \\ \beta_0 & \alpha_0 \end{bmatrix} = \begin{bmatrix} y_k & h_k \\ \beta_k & \alpha_k \end{bmatrix}. \tag{4}$$

Terminal characteristic rays coordinates describe indirectly magnifications, object and image location and also pupils' location. Symbolic solution of equation (4) leads to the following formula on the resultant imaging matrix defined from the characteristic rays coordinates according with the formula

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{J} \begin{bmatrix} \alpha_o y_k-\beta_0 h_k & h_k y_1-y_k h_1 \\ \alpha_0\beta_k-\beta_0\alpha_k & \alpha_k y_1-\beta_k h_1 \end{bmatrix}, \tag{5}$$

where J is the generalized Lagrange-Helmholtz (L-H) invariant, which can be defined either in the entrance of the system as the incident angle version or in the exit of the system as the refracted angle version according to the formula

$$J = y_1\cdot\alpha_0-h_1\cdot\beta_0 = y_k\cdot\alpha_k-h_k\cdot\beta_k . \tag{6}$$

In practice this invariant is determined simply in the object or image plane as the imaging constant. Then it is sufficient to give only 6 coordinates – three in the entrance and three in the exit of the system.

2.3 Number of degrees of freedom of optical system

Matrix approach allows for the solution of only three non-linear equations (1,2) with respect to system parameters. Advantageous for the matrix approach is the capability to define the number of degrees of freedom for the system. Number of parameters in thin-component system (powers and spaces) is equal n=2·k−1, where k is the number of components. If the number of components is higher than 2 we have a superiority of parameter number over the number of equations.

Total number of degrees of freedom is equal n−3=2(k−2). The more complex is the model the higher is the number of degrees of freedom. There are two ways of decreasing the number of degrees of freedom. First consists in defining the spaces between certain components, the second in introduction of additional requirements. Significant help is the defining all the spaces between components, what means full control over the components position and their dimensions. In such case the only thing left is the determination of optical powers, while the number of degrees of freedom is equal k−3. Number of degrees of freedom in relation to number of component is given in Table 1.

Table 1. Number of parameters 2k−1, total number of degrees of freedom 2(k−2) and the number of freedom k−3 (given spaces)

k	2k-1	2(k-2)	k-3
2	3	0	-
3	5	2	0
4	7	4	1
5	9	6	2
6	11	8	3
7	13	10	4
8	15	12	5

3. SPECIAL CASES

3.1 Determination of design parameters in the three-component system

Assume in the three-component system we know the spaces between components e_1, e_2, three coordinates of the characteristic rays in the entrance h_1, α_0, y_1 and in the exit h_3, $\alpha_3=\alpha_4$, y_3 and the coordinates in the image α_4, y_4 (conventionally index i=4 and $h_4=0$), what allows to determine the Lagrange-Helmholtz invariant $J=\alpha_4 \cdot y_4$.

The initial description of his case is show in Table 2, where columns represent the individual components including the object (i=0) and the image (i=4), and the rows represent the system parameters (focal lengths of components $1/\phi$, and spaces e) and characteristic rays coordinates h, α, y and β. Defined values are represented by crossed circles.

Table 2. Defined parameters in the three-component system

i=	0	1	2	3	4
$1/\phi$			⊗		
e		⊗			
h		⊗		⊗	0
α	⊗			⊗ =	⊗
y		⊗		⊗	⊗
β					

Assume the system of equations will be formed by three expressions for A, B and C from system (2). System matrix elements can be determined from formula (5). All his leads to the following system of equations:

$$
\begin{aligned}
&1-\phi_1 \cdot [e_1 - e_2 \cdot (e_1 \cdot \phi_2 - 1)] - e_2 \cdot \phi_2 = A \\
&e_2 \cdot (e_1 \cdot \phi_2 - 1) - e_1 = B \\
&\phi_3 + \phi_1 \cdot [(e_1 \cdot \phi_2 - 1) \cdot (e_2 \cdot \phi_3 - 1) - e_1 \cdot \phi_3] - \phi_2 \cdot (e_2 \cdot \phi_3 - 1) = C\,.
\end{aligned}
\tag{7}
$$

System of non-linear equations (7) has several symbolic solutions depending on which design parameters are determined. The second equation is the simplest what makes the solution much easier. As an example from (7) one can determine the optical powers ϕ_1 and ϕ_3 and space e_2. Then we receive the following formulas:

$$\phi_1 = \frac{B \cdot \phi 2 - A \cdot (1 - e_1 \cdot \phi_2) + 1}{B \cdot (1 - e_1 \cdot \phi_2)}$$

$$e_2 = \frac{B + e_1}{e_1 \cdot \phi_2 - 1} \tag{8}$$

$$\phi_3 = \frac{B \cdot C - A \cdot (1 - e_1 \cdot \phi_2) + 1}{A \cdot B}.$$

After the space e_2 is determined from the equation (8) all other values can be easily defined exploiting the methods used before.

3.2 Determining the height of rays inside the part of a system

Assume in the three-component fragment of a bigger system we know spaces e_{+1}, e_{+2}, four heights of characteristic rays at the boundaries of the fragment h_{+1}, y_{+1} and h_{+3}, y_{+3} and Lagrange-Helmholtz invariant. The initial description of that case can be presented in Table 3.

Table 3. Set parameters of fragment of a system

i_{rel}=	i	i+1	i+2
$1/\phi$			
e	$\otimes$	$\otimes$	
h	$\otimes$		$\otimes$
α			
y	$\otimes$		$\otimes$
β			

To determine the heights of rays h_{i+1}, y_{i+1} inside the fragment of a model it is necessary to create the following system of two equations:

$$h_i \cdot y_{i+1} - h_{i+1} \cdot y_i - J \cdot e_i = 0$$

$$h_{i+1} \cdot y_{i+2} - h_{i+2} \cdot y_{i+1} - J \cdot e_{i+1} = 0. \tag{9}$$

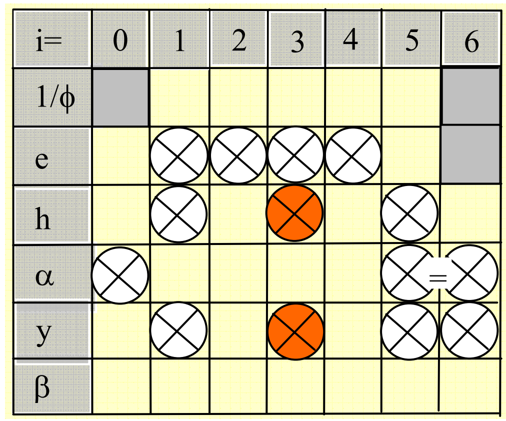
System of equations (9) is based on known dependences described e.g. in [3]. That system has the following symbolic solution:

$$h_{i+1} = \frac{h_{i+2} \cdot e_i + h_i \cdot e_{i+1}}{h_i \cdot y_{i+2} - h_{i+2} \cdot y_i} \cdot J$$

$$y_{i+1} = \frac{y_{i+2} \cdot e_i + y_i \cdot e_{i+1}}{h_i \cdot y_{i+2} - h_{i+2} \cdot y_i} \cdot J. \tag{10}$$

This particular case can be exploited in many ways. One of the examples is the solution of the five-component system with two degrees of freedom. Such system can be illustrated in Table 4. Assume in five-component system we know spaces e_1, e_2, e_3, e_4, three coordinates of characteristic rays in the entrance h_1, α_0, y_1 and in the exit h_5, α_5=α_6, y_5, and coordinates in the image plane α_6, y_6 (formally i=6), which enable us to determine the Lagrange-Helmholtz invariant J=$\alpha_6 \cdot y_6$. Two degrees of freedom are represented by red crossed circles. Using equations (10) twice we can first calculate h_2, y_2, then h_4, y_4 under the condition that the values of freedom parameters were set or selected. Determination of these heights is enough for the system to be fully defined by means of algorithms used before. There are many particular cases but they were disregarded here in order to keep the paper compact.

Table 4. Set parameters of a system with two degrees of freedom



4. SOFTWARE OPERATION AND EXAMPLES

4.1 Operation of software

Input data are the most often coordinates of two characteristic rays at the boundaries of the system only sometimes supplemented with selected design parameters. Introduction of input data and determination of parameters is performed in the two dimensional table. Columns in the table represent the components while the rows – parameters and coordinates of characteristic rays. Each cell of the table is related to other cells by dependences selected from certain groups. These relations bind selected coordinates of characteristic rays and also design parameters with each other. Said relations were described by the authors in previous works [1-3] and now are supplemented with new ones resulting from matrix optics and particular cases. Determination of arbitrary unknown parameter requires the analysis of environment including the input data and previously defined parameters. Dynamic process of determination of parameters consists in multiple local use of similar formulas. In successive cycles (table reviews) the possibility to use various formulas is analyzed and if only such possibility was found it is exploited. Each determination of a parameter increases the number of system data and accelerates the completion of software operation. The possibility to determine the number of degrees of freedom from the matrix approach allows to avoid unpleasant situation, when the software operation is impossible or leads to contradictory results. During the introduction of input data one can constantly check whether it is sufficient to complete the calculations. Program also checks for conflicted input data. During the operation it is possible to modify the input data. This enables us to observe the influence of input data on the final result or on shape of certain characteristics called requirements which are important for the result. Thin-component system being designed can be solved in many ways depending on how its input data were set and how the degrees of freedom were exploited to achieve proper distribution of optical powers and satisfy the requirements. Finally, technical parameters decide about the usefulness of resulting solution.

4.2 System with zero degree of freedom

Let us design the three-component system with zero degree of freedom. The initial description of such system (yellow background) and result of calculations are given in Table 5, while the system diagram is show in Fig.1. In this case formulas (8), resulting from the solution of system of equations (7) with relation to parameters ϕ_1, ϕ_3 and e_2 are exploited.

Table 5. Given parameters (yellow) and parameters determined with the use of the program (white)

i	0	1	2	3	4
1/ϕ		26.2327	-15	24.5745	
e	250.0000	7	8.35227	45.0000	
h	0.0000	10	7.61157	9	0
α	-0.04	0.341204	-0.166234	0.2	0.2
y	-20.0000	-0.6	0.103305	1	4
β	-0.07760	-0.100472	-0.107359	-0.0666667	-0.0666667

Other quantities determined by the program: L-H invariant J=0.8, focal length of the system $1/\phi$=43.9883, transversal magnification m=−0.2, interpupilary magnification v=1.164, location of entrance pupil p_1=7.732 and exit pupil p'_3=-15.

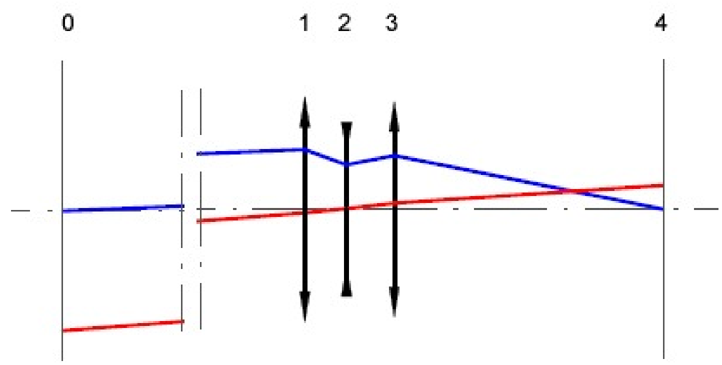


Fig.1 Diagram of three-component system with zero degree of freedom

4.3 System with two degrees of freedom

Let us design the five-component system with two degrees of freedom. Initial description of this system (yellow background), degrees of freedom (brown background) and results (white background) are presented in Table 6, whole the diagram of the system is shown in Fig.2.

Table 6. Given parameters (yellow), degrees of freedom (brown) and parameters determined by the program (white)

i	0	1	2	3	4	5	6
$1/\phi$		152.02	166.655	-56.4921	83.3321	159.324	
e	210	10	10	10	10	99.9999	
h	0.0000	21	20.6186	19	20.7447	20	0
α	-0.1	-0.03814	0.16186	-0.17447	0.07447	0.2	0.2
y	-5.0000	-.4	-0.15464	0.1	0.372344	.6	2.5
β	-0.0219048	-0.024536	-0.0254639	-0.0272341	-0.0227659	-0.019	-0.019

Other quantities determined by the program: L-H invariant J=0.5, focal length of the system $1/\phi$=79.606, transversal magnification m=−0.5, interpupilary magnification v=1.153, location of entrance pupil p_1=18.261 and exit pupil p'_3=-31.58. Values of freedom parameters were selected so that well balanced optical power distribution was achieved.

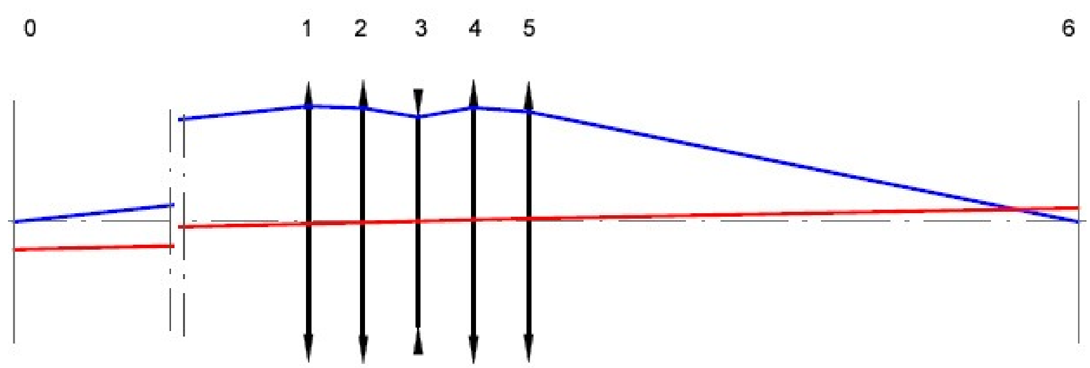


Fig.2 Diagram of the thin-component system with two degrees of freedom

5. CONCLUSIONS

Described here the new matrix approach to initial design of optical systems and its program realization appears to be a very useful tool aiding the work of optical designers.

ACKNOLEDGMENTS

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