Less is More: Optimal Contest Design with a Shortlist

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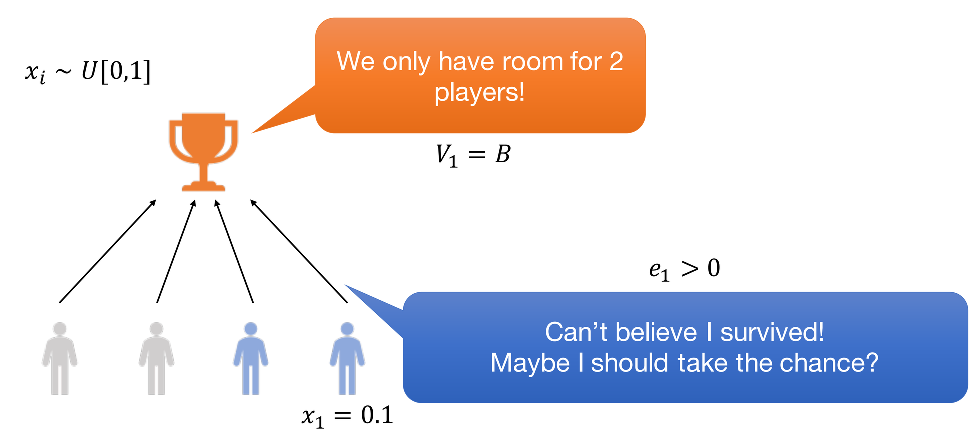
Contest with a Shortlist

Motivation: In practical contest scenarios (e.g. America Got Talent, ICPC Finals), the organizer can't admit all registered contestants, due to resource constraints (e.g. recording duration, venue capacity). A shortlist must be applied beforehand.

How will shortlist affect outcome of a contest?



1. Designer announces contest rules, including shortlist size m and rank-order prize structure $\vec{V}$ (satisfies $V_1 \geq V_2 \geq \ldots \geq V_m \geq 0$), given budget B.
2. n Contestants registers, each associates with a i.i.d private ability $x_i \sim F$. Designer observe their abilities (e.g., by submitted CV) and shortlist contestants with top m abilities, others are eliminated.
3. Each admitted contestant puts effort e_i into contest, which induces an irrevocable cost $g(e_i)/x_i$, where $g(\cdot)$ is non-negative increasing cost function.
4. Designer distribute prizes according to the order of efforts, i.e., the highest prize V_1 is awarded to the contestant with the highest effort e, and so on.

In this work, we answer the following research questions:
RQ1: How will the shortlist affect beliefs and behavior of contestants?
RQ2: How will incorporating a shortlist change the optimal contest design? Will the designer's welfare suffer / benefit from it, and to what extent?

Contestant Equilibrium

Posterior Beliefs

Proposition (Posterior Belief PDF)

For any admitted contestant (w.l.o.g., labeled as Contestant 1):

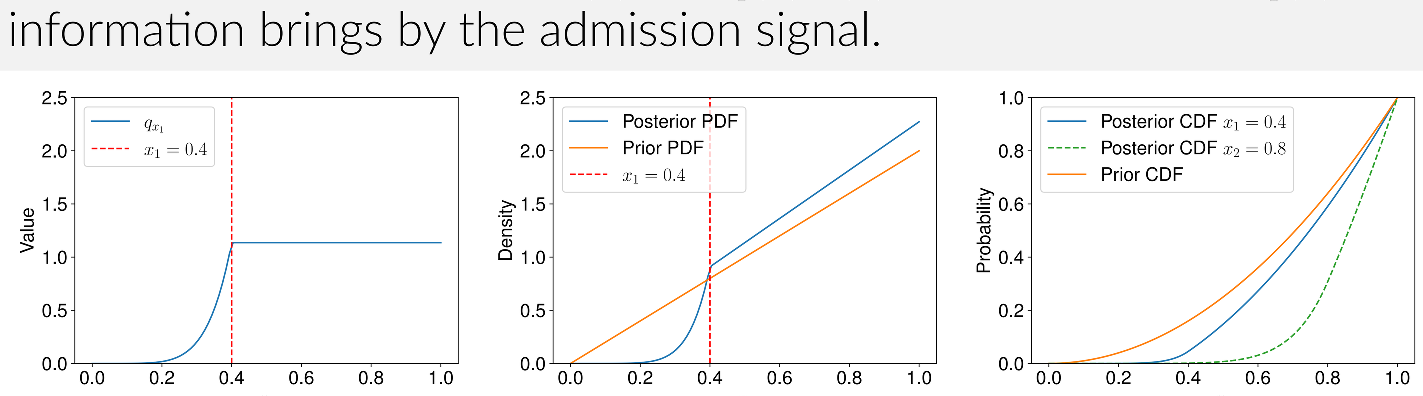
$$\beta_1(\mathbf{x}) = \begin{cases} \dfrac{\binom{n-1}{m-1} F^{n-m}(x^{(1)}) \prod_{i=2}^{m} f(x_i)}{J(F,n,m,x_1)} & \text{if } x^{(1)} \leq x_1, \\ \dfrac{\binom{n-1}{m-1} F^{n-m}(x_1) \prod_{i=2}^{m} f(x_i)}{J(F,n,m,x_1)} & \text{if } x^{(1)} > x_1, \end{cases}$$

where $x^{(1)} := \min_{j \in [m] \setminus \{1\}} x_i$ is the lowest ability level of other admitted contestants. The normalization denominator is defined as $J(F(\cdot), n, m, x) := \binom{n-1}{m-1} F^{n-m}(x)(1-F(x))^{m-1} + \binom{n-1}{m-1}(m-1)\int_0^{F(x)} t^{n-m}(1-t)^{m-2}\,dt$.

After shortlist, posterior beliefs depend on observer's own ability. **Any single opponent is perceived as stronger (FOSD).** Moreover, the stronger the observer is, the stronger she thinks of her opponent.

Example

Figure 1 shows the belief change of contestant x_1 resulting from shortlist. We rewrite the posterior belief as $\beta_1(z) = q_{x_1}(z) f(z)$, then the factor $q_{x_1}(z)$ contains the information brings by the admission signal.



(a) Factor q_{x_1} (b) PDF (c) CDF

Figure 1. Posterior beliefs of player x_1 ($n = 5, m = 2, F(x) = x^2$).

On the contrary, the observer becomes more optimistic about her ability ranking after shortlist. **As a whole, perceived threats of opponents diminish.**

Proposition (Threatens of Opponents Decrease after Shortlist)

A contestant perceives the opponent with the l^{th} highest ability (denoted as $X_{(l)}$, among $[m] \setminus \{1\}$) as weaker after the shortlist, in comparison to herself:

$$\Pr_{\beta_1}(X_{(l)} > x_1) \leq \Pr_{f}(X_{(l)} > x_1).$$

Symmetric Bayesian Nash Equilibrium

Theorem (Unique sBNE of Contestants)

The unique symmetric Bayesian Nash equilibrium exists. And the symmetric strategy $b^* : x_i \mapsto e_i$ under equilibrium can be expressed as:

$$b^*(x) = g^{-1}\left(\int_0^x \frac{\sum_{l=1}^{m-1} \binom{n-1}{l-1}(n-l)(V_l - V_{l+1}) F^{n-l-1}(t)(1-F(t))^{l-1} f(t)}{J(F,n,m,t)} t\,dt \right).$$

Optimal Contest Design

General Guideline

Definition (Simple Contest)

A contest with n participants and a shortlist size m is called a simple contest if all of its non-zero prizes are equal. Moreover, if the number of non-zero prizes is one less than the shortlist size, it's a complete simple contest.

For the two common objectives to be discussed below, **the optimal contest with shortlist is always a complete simple contest** (and is a simple contest for other linear objectives), which simplifies the high-dimension decision problem.

Maximum Individual Effort

Theorem (Optimal Contest for the Ex-ante Maximum Individual Effort)

The optimal contest that maximizes the ex-ante maximum individual effort is a two-contestant winner-take-all contest, i.e. $m = 2$ and $V_1 = B$. The resulting maximum ex-ante individual effort is as follows:

$$\mathbb{E}_{x \sim X_{(1)}}\left[g^{-1}\left(B \int_0^x \frac{(n-1)f(t)t}{F(t) + (n-1)(1-F(t))}\,dt \right) \right],$$

where $X_{(1)}$ denotes the highest realized ability among all contestants.

If the designer seeks to maximize the highest individual effort, the optimal contest is remarkably simple: **a two-contestant winner-take-all contest.** Also, for any ability distribution, **designer will achieve $\Theta(\log n)$ times greater highest effort with a shortlist**, showcasing the effectiveness of the shortlist.

Total Effort

This objective is much harder to tackle, thus we turn to asymptotic analysis.

Proposition (Optimal size is Asymptotically Linear)

The ex-ante total effort in a complete simple contest converges to a function of $k = m/n$ as $n \to \infty$, and the convergence rate is independent of k. Therefore, the optimal shortlist size grows asymptotically linearly with n. Formally, there exists a $k^* \in (0,1)$ such that: $\lim_{n \to \infty} m^*(n)/n = k^*$, where k^* is the solution of the following equation:

$$\int_k^1 F^{-1}(1-q)\left(\frac{1}{q} - (2k - k^2)\frac{1}{q^2}\right)dq = 0.$$

Astonishingly, **the optimal shortlist size for any distribution grows linearly with n**, and we further shows that **designer will achieve $\Theta(n)$ times greater total effort with a shortlist**. Our result also provides a simple way to solve the best slope for any distribution (e.g. 15.07% for $U[0,b]$ and 9.70% for $\text{Exp}(\lambda)$).

Theorem (Tight Upper Bound for the Optimal size)

For arbitrary ability distribution F, when $n \to +\infty$, the optimal shortlist size has the following linear upper bound with respect to n:

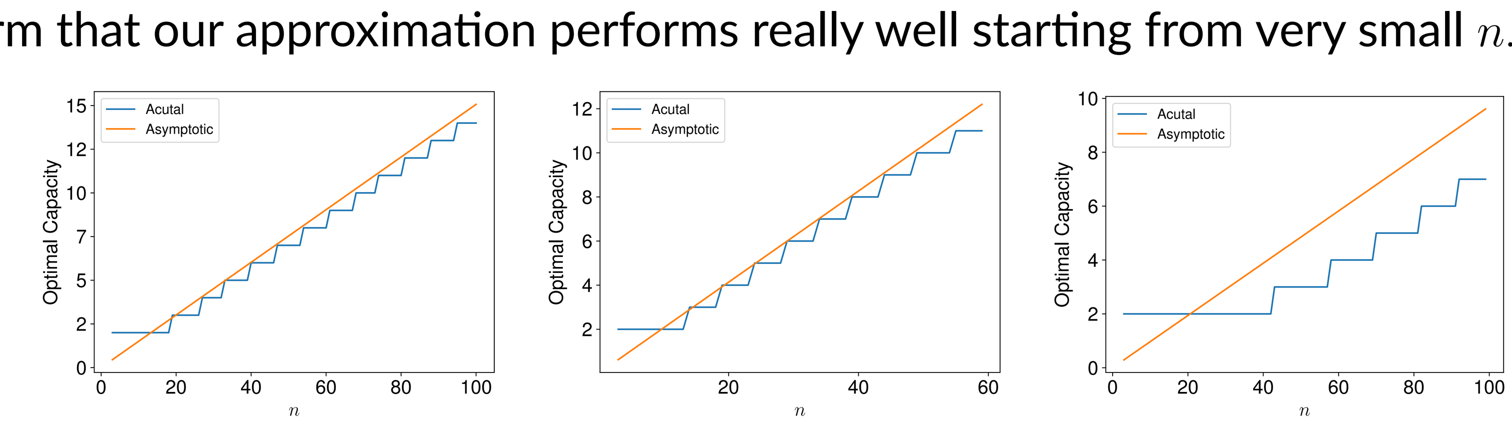
$$\lim_{n \to \infty} \frac{m^*(n)}{n} \leq \bar{k},$$

where $\bar{k} \approx 31.62\%$ is the solution to $\ln k = (2-k)(k-1)$. Moreover, there exists a distribution such that $m^*(n)/n = \bar{k}$, meaning the bound is tight.

💡 As a punchline, we show that the tight bound of the slope is 31.62%, which indicate that: ***Eliminating roughly 2/3 contestants will always incentive total performance without any prior, less is more in a contest!***

Towards Practical Applications

Although our results regarding total effort is asymptotic, **numerical results confirm that our approximation performs really well starting from very small n.**



(a) $F(x) = x$ (b) $F(x) = x^2$ (c) $F(x) = 1 - e^{-x}$

Figure 2. The actual optimal size and m^* predicted by asymptotic relation.

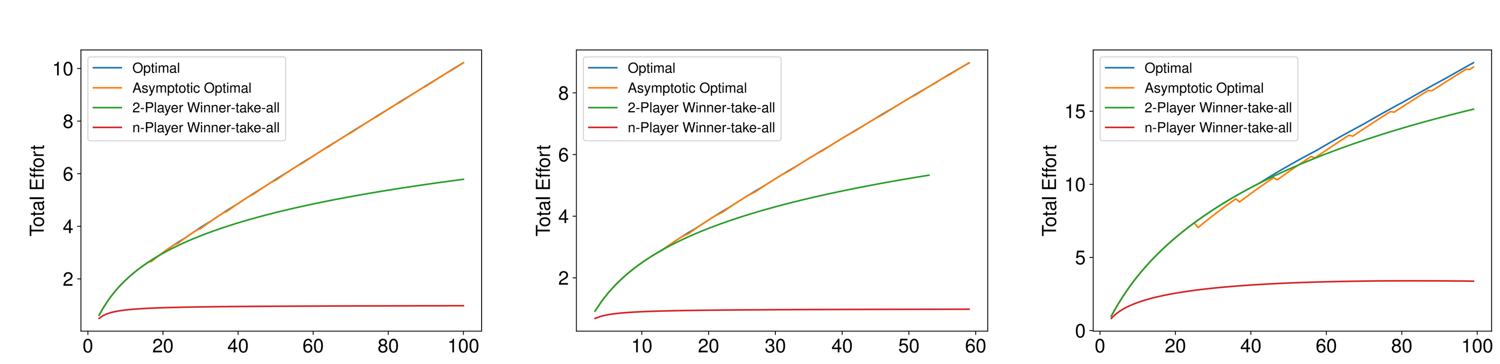


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Figure 3. Total effort performance of different contest designs.

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