

Derivatives

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \sin^{-1} = \frac{1}{\sqrt{1-x^2}}, x \in [-1, 1]$$

$$\frac{d}{dx} \cos^{-1} = \frac{-1}{\sqrt{1-x^2}}, x \in [-1, 1]$$

$$\frac{d}{dx} \tan^{-1} = \frac{1}{1+x^2}, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\frac{d}{dx} \sec^{-1} = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$$

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

$$\frac{d}{dx} \tanh(x) = \text{sech}^2(x)$$

$$\frac{d}{dx} \coth(x) = -\text{csch}^2(x)$$

$$\frac{d}{dx} \text{sech}(x) = -\text{sech}(x) \tanh(x)$$

$$\frac{d}{dx} \text{csch}(x) = -\text{csch}(x) \coth(x)$$

$$\frac{d}{dx} \sinh^{-1} = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx} \cosh^{-1} = \frac{-1}{\sqrt{x^2-1}}, x > 1$$

$$\frac{d}{dx} \tanh^{-1} = \frac{1}{1-x^2}, -1 < x < 1$$

$$\frac{d}{dx} \text{sech}^{-1} = \frac{1}{x\sqrt{1-x^2}}, 0 < x < 1$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} b^x = b^x \ln x$$

Integrals

$$\int \sin(x)dx = -\cos(x)$$

$$\int \cos(x)dx = \sin(x)$$

$$\int \tan(x)dx = -\ln|\sec(x)|$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \sec x dx = \ln|\sec x + \tan x|$$

$$\int \sec^2(x)dx = \tan(x) + C$$

$$\int \sec^3(x)dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x \tan x|)$$

$$\int \csc^2(x)dx = -\cot(x)$$

$$\int \csc(x) \cot(x)dx = -\csc(x)$$

$$\int \csc(x)dx = \ln|\csc(x) - \cot(x)|$$

$$\int \cot(x)dx = \ln|\sin(x)|$$

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int e^x dx = e^x$$

$$\int a^x dx = \frac{1}{\ln a} a^x$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(\frac{x}{a})$$

$$\int \frac{1}{a^2+ x^2} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a})$$

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln|\frac{x-a}{x+a}|$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x)$$

$$\int \sinh(x)dx = \cosh(x)$$

$$\int \cosh(x)dx = \sinh(x)$$

$$\int \tanh(x)dx = \ln|\cosh(x)|$$

$$\int \tanh(x)\text{sech}(x)dx = -\text{sech}(x)$$

$$\int \text{sech}^2(x)dx = \tanh(x)$$

$$\int \text{csch}(x) \coth(x)dx = -\text{csch}(x)$$

$$\int \ln(x)dx = (x \ln(x)) - x$$

$$\int_{-r}^r \sqrt{r^2-x^2} \, dx = \frac{\pi r^2}{2}$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$$

Integrals of inverse trig functions are solved using integration by parts.

U-Substitution

Let $u = f(x)$ (can be more than one variable).

Determine: $du = \frac{f'(x)}{dx} dx$ and solve for dx. Then, if a definite integral, substitute the bounds for $u = f(x)$ at each bounds Solve the integral using u.

Integration by Parts

$$\int u dv = uv - \int v du$$

Functions/ Identities

Most of trig identities work with hyperbolic, exceptions below

$$\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$$

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

$$\sec(\tan^{-1}(x)) = \sqrt{1+x^2}$$

$$\tan(\sec^{-1}(x)) = (\sqrt{x^2-1} \text{ if } x \geq 1)$$

$$= (-\sqrt{x^2-1} \text{ if } x \leq -1)$$

$$\sinh^{-1}(x) = \ln x + \sqrt{x^2+1}$$

$$\sinh^{-1}(x) = \ln x + \sqrt{x^2-1}, x \geq -1$$

$$\tanh^{-1}(x) = \frac{1}{2} \ln x + \frac{1+x}{1-x}, 1 < x < -1$$

$$\text{sech}^{-1}(x) = \ln[\frac{1+\sqrt{1-x^2}}{x}], 0 < x \leq -1$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$$

$$\cos(x \pm y) = \cos(x) \cos(y) \pm \sin(x) \sin(y)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x) \tan(y)}$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cosh(n^2x) - \sinh^2x = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$$

$$\sin(-x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$

$$\tan(-x) = -\tan(x)$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$1 - \tanh^2(x) = \text{sech}^2(x)$$

3D

given two points:
 (x_1, y_1, z_1) and (x_2, y_2, z_2) ,
Distance between them:
 $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Midpoint:
 $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2})$
Sphere with center (h,k,l) and radius r:
 $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$

Vectors

Vector: \vec{u}
Unit Vector: \hat{u}
Magnitude: $||\vec{u}|| = \sqrt{u_1^2 + u_2^2 + u_3^2}$
Unit Vector: $\hat{u} = \frac{\vec{u}}{||\vec{u}||}$

Dot Product

$\vec{u} \cdot \vec{v}$
Produces a Scalar

(Geometrically, the dot product is a vector projection)

$\vec{u} = < u_1, u_2, u_3 >$
 $\vec{v} = < v_1, v_2, v_3 >$
 $\vec{u} \cdot \vec{v} = 0$ means the two vectors are Perpendicular
 θ is the angle between them.
 $\vec{u} \cdot \vec{v} = ||\vec{u}|| ||\vec{v}|| \cos(\theta)$
 $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

NOTE:

$\hat{u} \cdot \hat{v} = \cos(\theta)$
 $||\vec{u}||^2 = \vec{u} \cdot \vec{u}$
 $\vec{u} \cdot \vec{v} = 0$ when \perp
Angle Between \vec{u} and \vec{v} :
 $\theta = \cos^{-1}(\frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| ||\vec{v}||})$
Projection of \vec{u} onto \vec{v} :
 $pr_{\vec{v}}\vec{u} = (\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2})\vec{v}$

Cross Product

$\vec{u} \times \vec{v}$
Produces a Vector
(Geometrically, the cross product is the area of a parallelogram with sides $||\vec{u}||$ and $||\vec{v}||$)
 $\vec{u} = < u_1, u_2, u_3 >$
 $\vec{v} = < v_1, v_2, v_3 >$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$\vec{u} \times \vec{v} = \vec{0}$ means the vectors are parallel
 $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Volume of Parallelepiped

$$(\vec{v} \times \vec{u}) \cdot \vec{w}$$

However, in the case the question gave you three vectors directly

$$Volume = \det \left(\begin{bmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \right)$$

Lines and Planes

Equation of a Plane

(x_0, y_0, z_0) is a point on the plane and
 $< A, B, C >$ is a normal vector
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
 $< A, B, C > \cdot < x - x_0, y - y_0, z - z_0 > = 0$
 $Ax + By + Cz = D$ where
 $D = Ax_0 + By_0 + Cz_0$

Equation of a line

A line requires a Direction Vector
 $\vec{u} = < u_1, u_2, u_3 >$ and a point (x_1, y_1, z_1)

Parametric equation:
 $x = u_1t + x_1 \quad y = u_2t + y_1 \quad z = u_3t + z_1$
Symmetric equations:
 $t = \frac{x-x_1}{u_1} = \frac{y-y_1}{u_2} = \frac{z-z_1}{u_3}$

Distance from a Point to a Plane

The distance from a point (x_0, y_0, z_0) to a plane $Ax+By+Cz=D$ can be expressed by the formula:
 $d = \frac{|Ax_0+By_0+Cz_0-D|}{\sqrt{A^2+B^2+C^2}}$

A line intersects a plane Just put the equations of x, y, z in the equation of the plane given, and solve for t . Then you'll have the points.

Distance between two skew lines

through $P_1P_2, \quad P_3P_4$

$$D = \frac{Vol_{parallelepiped}}{Area_{base}} = \frac{|(P_1\vec{P}_2 \times P_3\vec{P}_4) \cdot P_1\vec{P}_3|}{|P_1\vec{P}_2 \times P_3\vec{P}_4|}$$

Or go for the intuitive approach of forcing perpendicularity.

If two vectors are parallel $|\vec{v}_1 \times \vec{v}_2| = 0$

Distance between a point and a line \vec{v}

$$D = \frac{|P\vec{v}_0 \times \vec{v}|}{|\vec{v}|}$$

Other Information

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
Law of Cosines:
 $a^2 = b^2 + c^2 - 2bc(\cos(\theta))$
Quadratic Formula:
 $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, when $ax^2 + bx + c = 0$