Physics 1 Notes

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Contents

1 Motion **1**
1.1 Formulas 1
1.2 Questions 2

2 Vectors **4**
2.1 Formulas 4
2.2 Motion in 2D & 3D 5
2.2.1 Projectile Motion 5
2.2.2 Circular Motion 6
2.2.3 Questions 6

1 Motion

1.1 Formulas

Displacement:

$$\Delta x = x_2 - x_1$$

Average Velocity:

$$\frac{\Delta x}{\Delta t}$$

Average Speed:

$$\frac{Total\ Distance}{\Delta t}$$

Instantaneous Velocity:

$$v_{ins} = \frac{dx}{dt} \quad (slope)$$

Average Acceleration

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad\qquad a_{ins} = \frac{dv}{dt}$$

Constant Acceleartion
If a particle is moving with constant acceleration, time to SUVAT.

$$v = v_0 + at \qquad \cancel{s}$$

$$s = v_0 t + \frac{1}{2}at^2 \qquad \cancel{v}$$

$$v^2 = v_0^2 + 2as \qquad \cancel{t}$$

$$s = \left[\frac{v_0 + v}{2}\right] t \qquad \cancel{a}$$

$$s = vt - \frac{1}{2}at^2 \qquad \cancel{v}\cancel{0}$$

There are five quantities, there is a law that does not include each of those quantities. Thus, in any problem you will have three quantities at least and you will be able to get the rest.

1.2 Questions

1. An object starts from rest at the origin and moves along the x axis with a constant acceleration of 4 m/s2. Its average velocity as it goes from x = 2 m to x = 8 m is:

   Solution We'll get the time for the whole trip, then time for the 2m trip, and get then the average velocity normally.

   $$s = v_0 t + \frac{1}{2}at^2, 8 = (0)t + \frac{1}{2}(4)(t)^2 \implies Total\ Time = 2s$$

   $$2 = (0)t + \frac{1}{2}(4)(t)^2 \implies Time\ For\ First\ 2m = 1s$$

   $$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{8-2}{2-1} = 6\ m/s$$

2. At time t = 0 a car has a velocity of 16 m/s. It slows down with an acceleration given by -0.50t, in m/s2 for t in seconds. At the end of 4.0 s it has traveled:

   Solution This question is fairly simple, and it can be tricky. You will just integrate the acceleration function normally and equalize the formed velocity function to zero so you can get the time when the car rests. Note that C in x(t) is useless since the question asked for distance **traveled**.

   $$\int -0.5t = \frac{-0.5t^2}{2} + C \implies \frac{-0.5(0)^2}{2} + C = 16 \implies C = 16 \quad (\ v(t)\ )$$

   $$\int v(t) = \frac{-0.25t^3}{3} + 16t + C \implies at\ t = 4, \frac{-0.25(4)^3}{3} + 16(4) \approx 59$$

3. At a stop light, a truck traveling at 15 m/s passes a car as it starts from rest. The truck travels at constant velocity and the car accelerates at 3 m/s2. How much time does the car take tocatch up to the truck?

   Solution This one you will just equate the distance covered by truck to the distance covered by the car, at time they're equal, the shit is done.

   $$S_1 = S_2 \implies v_{truck}t = v_0 t + \frac{1}{2}at^2 \implies 15t = \frac{1}{2}(3)t^2 \implies t = 10s$$

4. What is the average velocity for a ball thrown vertically in the first second?

   $$v_{av} = \frac{v_0 + v_f}{2} = \frac{0 + 9.8}{2} = 4.9m/s$$

5. An object is released from rest. How far does it fall during the second second of its fall?

   Solution This question is very easy, but it took an approach different to the one I usually take solving problems so here is it. The approach is to get the v_{avg} in the second second, then get the distance.

$$v_{t=1} = g \qquad v_{t=2} = 2g \implies v_{avg1->2} = \frac{3g}{2}$$

$$d_{t_1->t_2} = v_{avg}(t) = \left(\frac{3g}{2}\right)(1) = 14.7m$$

6. An object is thrown vertically upward with a certain initial velocity in a world where the acceleration due to gravity is 19.6 m/s2. The height to which it rises is ____ that to which the object would rise if thrown upward with the same initial velocity on the Earth. Neglect friction.

   Solution This question also is fairly easy but maybe tricky if you pushed solving. You will instantly remember $s = v_0 t + \frac{1}{2}at^2$ where you will think that since acceleration is twice of that of earth so the height will be twice, but remember that time will change and it has the power of the second power.

$$a_1 t^2 = k\, a_2 t^2 \qquad t = \frac{v}{a} \implies (19.6)\left(\frac{1}{19.6}\right)^2 = k(9.8)\left(\frac{1}{9.8}\right)^2 \implies k = \frac{1}{2} \quad \#$$

2 Vectors

For this chapter, go visit my notes for calculus, the part of Calculus 2, chapter 1. However, I'll add the TL;DR here.

2.1 Formulas

The Scalar Components of $\vec{a}$, where θ is the angle between $\vec{a}$ and the $xW-axis$.

$$a_x = a\cos\theta \qquad a_y = a\sin\theta \qquad direction:\ \tan\theta = \frac{a_y}{a_x}.$$

Where the magnitude a can be got using $a = \sqrt{a_x^2 + a_y^2}$

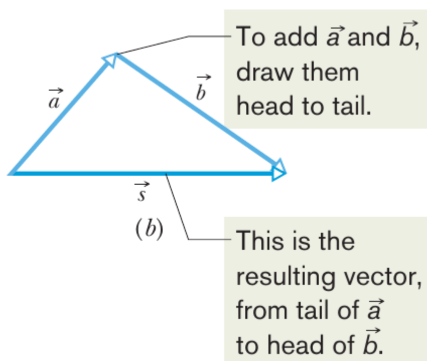

Figure 1: Vectors Addtion

Writing any more shit in this chapter will be beyond pathetic, just go check the calculus 2 notes, they're fresh as hell.

I will include the motion, aka using vectors for real life shit, as a new subsection rather than a whole new section.

2.2 Motion in 2D & 3D

You will notice a major similarity with the normal motion stuff, but since we're in dd, we'll use vectors. Where $\vec{r}$ is the displacement:

$$\vec{v_{avg}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{v_{inst.}} = \frac{d\vec{r}}{dt} \qquad \vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\vec{a}_{avg} = \frac{\vec{v_2} - \vec{v_1}}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a}_{inst} = \frac{d\vec{v}}{dt}$$

2.2.1 Projectile Motion

Welcome bitch, we have now acceleration on the negative direction due to gravity. It's along y axis only. We can write the normal equations of motion in some different way here. θ is the angle measured from the horizontal $x-$axis

For constant speed, we use $d = vt$. For constant acceleration we use these equations and in projectile we plug them for x and y directions

$$v_f = v_0 + at$$
$$v_f^2 = v_0^2 + 2ad$$
$$d = \frac{1}{2}(v_0 + v_f)t \implies d = v_{avg}t$$
$$d = v_0t + \frac{1}{2}at^2$$
$$\implies h = \frac{1}{2}at^2 \qquad R = v_xt$$

On the other hand, the trajectory path of a particle in the projectile equation is

$$y = (\tan\theta)x - \frac{gx^2}{2(v_0\cos\theta)^2}$$

To get the horizontal range R we also manipulate till

$$R = \frac{v_0^2}{g}\sin 2\theta$$

Remember There is constant horizontal motion and vertical motion which can be got using the normal component laws.

Also, note that to get the direction angle, you can use $\tan^{-1}\frac{v_y}{v_x}$, with normal positive numbers, and then use alternate angles to find the real angle based on the signs, that's easier than using negative that will screw you up.

2.2.2 Circular Motion

A particle moving along a circular path with radius r, with constant acceleration a

$$a = \frac{v^2}{r}$$

With $\vec{a}$ directed towards center of the circular path, and $\vec{v}$ is perpendicular to the acceleration vector. The period of revolution can be got using

$$T = \frac{2\pi r}{v}$$

2.2.3 Questions

1. A child whirls a stone in a horizontal circle 1.9 m above the ground by means of a string 1.4 m long. The string breaks, and the stone flies off horizontally, striking the ground 11 m away. What was the centripetal acceleration of the stone while in circular motion?

   Solution For this question, when the string breaks the stone moves in a projectile motion with initial velocity v_i. To answer, just analyze the projectile to get the velocity. The acceleration along x-axis is zero (only acceleration is g)

$$s_x = v_i \cos\theta t + \frac{1}{2}a_x t^2 \implies 11 = v_i \cos 0t - \frac{1}{2}(0)t^2 \implies 11 = v_i t$$
$$s_y = v_i \cos\theta t - \frac{1}{2}gt^2 \implies -1.9 = v_i \cos\frac{\pi}{2}t - 4.9t^2 \implies t = 0.6227\ s$$

   From the previous two equations

$$v_i = 17.665\ m/s$$

$$a = \frac{v^2}{r} = \frac{17.665^2}{1.4} = 222.89\ m/s^2$$

2. A particle started a journey in a plane with a constant acceleration pointing towards the east with an initial speed of 5 m/s. If the total displacement through the journey is along the north direction, calculate the final speed of the particle.

   Solution

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}}{d\vec{r}} \cdot \frac{d\vec{r}}{dt} \qquad \& \ \vec{v} = \frac{d\vec{r}}{dt} \implies \vec{a} = \frac{d\vec{v}}{d\vec{r}} \cdot \vec{v} \qquad (0)$$

$$\int_{r_i}^{r_f} \vec{a}\ d\vec{r} = \vec{a} \cdot (r_f - r_i) \qquad (1)$$

$$\int_{v_i}^{v_f} \vec{v}\, d\vec{v} = \frac{v_f^2 - v_i^2}{2} \tag{2}$$

$$\vec{a}(r_f - r_i) = \frac{v_f^2 - v_i^2}{2} \tag{3}$$

From 0, 1, 2, 3, and since total displacement is in north, and acceleration is in east, then angle between them is $\pi/2$, and $\vec{a}\ .\ \Delta r = 0$

$$\implies v_f^2 = v_i^2 \implies v_f = v_i = 5\ m/s$$

3. A jet plane in straight horizontal flight passes over your head. When it is directly above you, the sound seems to come from a point behind the plane in a direction 30 degrees from the vertical. The speed of the plane is

   Solution If the plane is a distance d directly overhead when you heard the sound, the sound waves must have traveled a distance $\frac{d}{\cos(30)} = \frac{2d}{\sqrt{3}}$. The plane has traveled $d\tan(30) = \frac{d}{\sqrt{3}}$, which is half the distance the sound waves traveled. Therefore the plane's speed is half the speed of sound.

4. An airplane makes a gradual 90○ turn while flying at a constant speed of 200 m/s. The process takes 20.0 seconds to complete. For this turn the magnitude of the average acceleration of the plane is?

   Solution Since $v_i = 200\vec{i}$ and $v_f = 200\vec{j}$, therefore $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{200\vec{j} - 200\vec{i}}{20} =< -10, 10 > \implies |a_{avg}| = \sqrt{10^2 + 10^2}$

5. A boat is traveling upstream at 14 km/h with respect to a river that is flowing at 6 km/h (with respect to the ground). A man runs directly across the boat, from one side to the other, at 6 km/h (with respect to the boat). The speed of the man with respect to the ground is:

   Solution Pythagoras. Net velocity is 8 m/s, velocity of the guy is 6 m/s perpendicular to the first one. Use Pythagoras and you'll get 10 m/s. Easy as fuck.