

Derivatives

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \sin^{-1} = \frac{1}{\sqrt{1-x^2}}, x \in [-1, 1]$$

$$\frac{d}{dx} \cos^{-1} = \frac{-1}{\sqrt{1-x^2}}, x \in [-1, 1]$$

$$\frac{d}{dx} \tan^{-1} = \frac{1}{1+x^2}, \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\frac{d}{dx} \sec^{-1} = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$$

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x)$$

$$\frac{d}{dx} \coth(x) = -\operatorname{csch}^2(x)$$

$$\frac{d}{dx} \operatorname{sech}(x) = -\operatorname{sech}(x) \tanh(x)$$

$$\frac{d}{dx} \operatorname{csch}(x) = -\operatorname{csch}(x) \coth(x)$$

$$\frac{d}{dx} \sinh^{-1} = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx} \cosh^{-1} = \frac{-1}{\sqrt{x^2-1}}, x > 1$$

$$\frac{d}{dx} \tanh^{-1} = \frac{1}{1-x^2}, -1 < x < 1$$

$$\frac{d}{dx} \operatorname{sech}^{-1} = \frac{1}{x\sqrt{1-x^2}}, 0 < x < 1$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} b^x = b^x \ln x$$

Integrals

$$\int \sin(x)dx = -\cos(x)$$

$$\int \cos(x)dx = \sin(x)$$

$$\int \tan(x)dx = -\ln|\cos(x)|$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \sec x dx = \ln|\sec x + \tan x|$$

$$\int \sec^2(x)dx = \tan(x) + C$$

$$\int \sec^3(x)dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x \tan x|)$$

$$\int \csc^2(x)dx = -\cot(x)$$

$$\int \csc(x) \cot(x)dx = -\csc(x)$$

$$\int \csc(x)dx = \ln|\csc(x) - \cot(x)|$$

$$\int \cot(x)dx = \ln|\sin(x)|$$

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int e^x dx = e^x$$

$$\int a^x dx = \frac{1}{\ln a} a^x$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(\frac{x}{a})$$

$$\int \frac{1}{a^2+\frac{1}{x^2}} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a})$$

$$\int \frac{1}{x^2-\frac{1}{a^2}} dx = \frac{1}{2a} \ln|\frac{x-a}{x+a}|$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x)$$

$$\int \sinh(x)dx = \cosh(x)$$

$$\int \cosh(x)dx = \sinh(x)$$

$$\int \tanh(x)dx = \ln|\cosh(x)|$$

$$\int \tanh(x)\operatorname{sech}(x)dx = -\operatorname{sech}(x)$$

$$\int \operatorname{sech}^2(x)dx = \tanh(x)$$

$$\int \operatorname{csch}(x) \coth(x)dx = -\operatorname{csch}(x)$$

$$\int \ln(x)dx = (x \ln(x)) - x$$

$$\int_{-r}^r \sqrt{r^2-x^2} \, dx = \frac{\pi r^2}{2}$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$$

Integrals of inverse trig functions are solved using integration by parts.

Weierstrass

$$t = \tan \frac{x}{2} \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2} \quad dx = \frac{2}{1+t^2} dt$$

$\int \sin^m x \cos^n x$   
*m* is odd factor sines, then sub *u* = cos *x* If *n* is odd factor cosines, then sub *u* = sin *x*.  
If both even, use  $\frac{1-\cos(2x)}{2}$ ,  $\frac{1+\cos(2x)}{2}$ . If both odd, factor one with less power.

$\int \tan^m x \sec^n x$   
*n* even save one sec<sup>2</sup> *x* and sub *u* = tan *x*.  
*m* odd save one sec *x* tan *x* and sub *u* = sec *x*

Integration by Parts

$$\int u dv = uv - \int v du$$

Functions/ Identities

**Most of trig identities work with hyperbolic, exceptions below**

$$\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$$

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

$$\sec(\tan^{-1}(x)) = \sqrt{1+x^2}$$

$$\tan(\sec^{-1}(x)) = (\sqrt{x^2-1} \text{ if } x \geq 1)$$

$$= (-\sqrt{x^2-1} \text{ if } x \leq -1)$$

$$\sinh^{-1}(x) = \ln x + \sqrt{x^2+1}$$

$$\sinh^{-1}(x) = \ln x + \sqrt{x^2-1}, x \geq -1$$

$$\tanh^{-1}(x) = \frac{1}{2} \ln x + \frac{1+x}{1-x}, -1 < x < 1$$

$$\operatorname{sech}^{-1}(x) = \ln[\frac{1+\sqrt{1-x^2}}{x}], 0 < x \leq 1$$

$$\sinh(x) = \frac{e^x-e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x+e^{-x}}{2}$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$$

$$\cos(x \pm y) = \cos(x) \cos(y) \pm \sin(x) \sin(y)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x) \tan(y)}$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cosh(n^2 x) - \sinh^2 x = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$$

$$\sin(-x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$

$$\tan(-x) = -\tan(x)$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

3D

given two points:  
(*x*<sub>1</sub>, *y*<sub>1</sub>, *z*<sub>1</sub>) and (*x*<sub>2</sub>, *y*<sub>2</sub>, *z*<sub>2</sub>),  
Distance between them:  
 $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$   
Midpoint:  
( $\frac{x_1+x_2}{2}$ ,  $\frac{y_1+y_2}{2}$ ,  $\frac{z_1+z_2}{2}$ )  
Sphere with center (h,k,l) and radius r:  
(*x* − *h*)<sup>2</sup> + (*y* − *k*)<sup>2</sup> + (*z* − *l*)<sup>2</sup> = *r*<sup>2</sup>

Vectors

Vector: *u*  
Unit Vector: *u*  
Magnitude:  $||\vec{u}|| = \sqrt{u_1^2 + u_2^2 + u_3^2}$   
Unit Vector:  $\hat{u} = \frac{\vec{u}}{||\vec{u}||}$

Dot Product

*u* · *v*  
Produces a Scalar  
(Geometrically, the dot product is a vector projection)  
 $\vec{u} = \langle u_1, u_2, u_3 \rangle$   
 $\vec{v} = \langle v_1, v_2, v_3 \rangle$   
*u* · *v* = 0 means the two vectors are Perpendicular *θ* is the angle between them.  
*u* · *v* =  $||\vec{u}|| \, ||\vec{v}|| \cos(\theta)$   
*u* · *v* = *u*<sub>1</sub> *v*<sub>1</sub> + *u*<sub>2</sub> *v*<sub>2</sub> + *u*<sub>3</sub> *v*<sub>3</sub>

NOTE:  
*u* · *v* = cos(*θ*)  
 $||\vec{u}||^2 = \vec{u} \cdot \vec{u}$   
*u* · *v* = 0 when ⊥  
Angle Between *u* and *v*:  
 $\theta = \cos^{-1}(\frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| \, ||\vec{v}||})$   
Projection of *u* onto *v*:  
 $pr_{\vec{v}}\vec{u} = (\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2})\vec{v}$

Cross Product

*u* × *v*  
Produces a Vector  
(Geometrically, the cross product is the area of a parallelogram with sides  $||\vec{u}||$  and  $||\vec{v}||$ )  
 $\vec{u} = \langle u_1, u_2, u_3 \rangle$   
 $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

*u* × *v* = 0 means the vectors are parallel  
*a* × (*b* × *c*) = (*a*·*c*)*b* − (*a*·*b*)*c*

Volume of Parallelepiped

$$(\vec{v} \times \vec{u}) \cdot \vec{w}$$

However, in the case the question gave you three vectors directly

$$Volume = \det \left( \begin{bmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \right)$$

Lines and Planes

Equation of a Plane

(*x*<sub>0</sub>, *y*<sub>0</sub>, *z*<sub>0</sub>) is a point on the plane and  
< *A*, *B*, *C* > is a normal vector  
*A*(*x* − *x*<sub>0</sub>) + *B*(*y* − *y*<sub>0</sub>) + *C*(*z* − *z*<sub>0</sub>) = 0  
< *A*, *B*, *C* > · < *x* − *x*<sub>0</sub>, *y* − *y*<sub>0</sub>, *z* − *z*<sub>0</sub> > = 0  
*Ax* + *By* + *Cz* = *D* where  
*D* = *Ax*<sub>0</sub> + *By*<sub>0</sub> + *Cz*<sub>0</sub>

Equation of a line

A line requires a Direction Vector  
 $\vec{u} = \langle u_1, u_2, u_3 \rangle$  and a point (*x*<sub>1</sub>, *y*<sub>1</sub>, *z*<sub>1</sub>)

Parametric equation:  
*x* = *u*<sub>1</sub>*t* + *x*<sub>1</sub>    *y* = *u*<sub>2</sub>*t* + *y*<sub>1</sub>    *z* = *u*<sub>3</sub>*t* + *z*<sub>1</sub>  
Symmetric equations:  
 $t = \frac{x-x_1}{u_1} = \frac{y-y_1}{u_2} = \frac{z-z_1}{u_3}$

Distance from a Point to a Plane

The distance from a point (*x*<sub>0</sub>, *y*<sub>0</sub>, *z*<sub>0</sub>) to a

plane *Ax*+*By*+*Cz*=*D* can be expressed by the formula:  
 $d = \frac{|Ax_0+By_0+Cz_0-D|}{\sqrt{A^2+B^2+C^2}}$

**A line intersects a plane** Just put the equations of *x*, *y*, *z* in the equation of the plane given, and solve for *t*. Then you'll have the points.

Distance between two skew lines through *P*<sub>1</sub>*P*<sub>2</sub>, *P*<sub>3</sub>*P*<sub>4</sub>

$$D = \frac{Vol_{parallelepiped}}{Area_{base}} = \frac{|(P_1\vec{P}_2 \times P_3\vec{P}_4) \cdot P_1\vec{P}_3|}{|P_1\vec{P}_2 \times P_3\vec{P}_4|}$$

Or go for the intuitive approach of forcing perpendicularity.

**If two vectors are parallel**  $|\vec{v}_1 \times \vec{v}_2| = 0$

Distance between a point and a line *v*

$$D = \frac{|P\vec{v}_0 \times \vec{v}|}{|\vec{v}|}$$

Other Information

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$   
Law of Cosines:  
*a*<sup>2</sup> = *b*<sup>2</sup> + *c*<sup>2</sup> − 2*bc*(cos(*θ*))  
Quadratic Formula:  
 $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ , when *ax*<sup>2</sup> + *bx* + *c* = 0