

Derivatives

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \sin^{-1} = \frac{1}{\sqrt{1-x^2}}, x \in [-1, 1]$$

$$\frac{d}{dx} \cos^{-1} = \frac{-1}{\sqrt{1-x^2}}, x \in [-1, 1]$$

$$\frac{d}{dx} \tan^{-1} = \frac{1}{1+x^2}, \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\frac{d}{dx} \sec^{-1} = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$$

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

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$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x)$$

$$\frac{d}{dx} \coth(x) = -\operatorname{csch}^2(x)$$

$$\frac{d}{dx} \operatorname{sech}(x) = -\operatorname{sech}(x) \tanh(x)$$

$$\frac{d}{dx} \operatorname{csch}(x) = -\operatorname{csch}(x) \coth(x)$$

$$\frac{d}{dx} \sinh^{-1} = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx} \cosh^{-1} = \frac{-1}{\sqrt{x^2-1}}, x > 1$$

$$\frac{d}{dx} \tanh^{-1} = \frac{1}{1-x^2}, -1 < x < 1$$

$$\frac{d}{dx} \operatorname{sech}^{-1} = \frac{1}{x\sqrt{1-x^2}}, 0 < x < 1$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} b^x = b^x \ln x$$

Integrals

$$\int \sin(x)dx = -\cos(x)$$

$$\int \cos(x)dx = \sin(x)$$

$$\int \tan(x)dx = -\ln|\cos(x)|$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \sec x dx = \ln|\sec x + \tan x|$$

$$\int \sec^2(x)dx = \tan(x) + C$$

$$\int \sec^3(x)dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x \tan x|)$$

$$\int \csc^2(x)dx = -\cot(x)$$

$$\int \csc(x) \cot(x)dx = -\csc(x)$$

$$\int \csc(x)dx = \ln|\csc(x) - \cot(x)|$$

$$\int \cot(x)dx = \ln|\sin(x)|$$

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int e^x dx = e^x$$

$$\int a^x dx = \frac{1}{\ln a} a^x$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(\frac{x}{a})$$

$$\int \frac{1}{a^2+ x^2} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a})$$

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln|\frac{x-a}{x+a}|$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}|$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x)$$

$$\int \sinh(x)dx = \cosh(x)$$

$$\int \cosh(x)dx = \sinh(x)$$

$$\int \tanh(x)dx = \ln|\cosh(x)|$$

$$\int \tanh(x) \operatorname{sech}(x)dx = -\operatorname{sech}(x)$$

$$\int \operatorname{sech}^2(x)dx = \tanh(x)$$

$$\int \operatorname{csch}(x) \coth(x)dx = -\operatorname{csch}(x)$$

$$\int \ln(x)dx = (x \ln(x)) - x$$

$$\int_{-r}^r \sqrt{r^2-x^2} \, dx = \frac{\pi r^2}{2}$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$$

Integrals of inverse trig functions are solved using integration by parts.

Weierstrass

$$t = \tan \frac{x}{2} \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2} \quad dx = \frac{2}{1+t^2} dt$$

$\int \sin^m x \cos^n x$
m is odd factor sines, then sub *u* = cos *x*
n is odd factor cosines, then sub *u* = sin *x*.
If both even, use $\frac{1-\cos(2x)}{2}$, $\frac{1+\cos(2x)}{2}$. If both odd, factor one with less power.

$\int \tan^m x \sec^n x$
n even save one sec² *x* and sub *u* = tan *x*.
m odd save one sec *x* tan *x* and sub *u* = sec *x*

$$\int \sqrt{a^2-x^2} dx \implies x = a \sin \theta$$

$$\int \sqrt{a^2+x^2} dx \implies x = a \tan \theta$$

$$\int \sqrt{x^2-a^2} dx \implies x = a \sec \theta$$

$$\int_1^\infty \frac{1}{x^p}, p > 1 : \text{converge}, p \leq 1 : \text{diverges}$$
Integration by Parts

$$\int u dv = uv - \int v du$$

Functions/ Identities

Most of trig identities work with hyperbolic, exceptions below
sin(cos⁻¹(*x*)) = $\sqrt{1-x^2}$
cos(sin⁻¹(*x*)) = $\sqrt{1-x^2}$
sec(tan⁻¹(*x*)) = $\sqrt{1+x^2}$
tan(sec⁻¹(*x*)) = $(\sqrt{x^2-1} \text{ if } x \geq 1)$
= $(-\sqrt{x^2-1} \text{ if } x \leq -1)$
sinh⁻¹(*x*) = ln *x* + $\sqrt{x^2+1}$
sinh⁻¹(*x*) = ln *x* + $\sqrt{x^2-1}$, *x* ≥ -1
tanh⁻¹(*x*) = $\frac{1}{2} \ln x + \frac{1+x}{1-x}$, -1 < *x* < 1
sech⁻¹(*x*) = ln[$\frac{1+\sqrt{1-x^2}}{x}$], 0 < *x* ≤ -1
sinh(*x*) = $\frac{e^x-e^{-x}}{2}$
cosh(*x*) = $\frac{e^x+e^{-x}}{2}$
sin²(*x*) + cos²(*x*) = 1
1 + tan²(*x*) = sec²(*x*)
1 + cot²(*x*) = csc²(*x*)
sin(*x* ± *y*) = sin(*x*) cos(*y*) ± cos(*x*) sin(*y*)
cos(*x* ± *y*) = cos(*x*) cos(*y*) ∓ sin(*x*) sin(*y*)
tan(*x* ± *y*) = $\frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x) \tan(y)}$
sin(2*x*) = 2 sin(*x*) cos(*x*)
cos(2*x*) = cos²(*x*) - sin²(*x*)
cosh(*n*²*x*) - sinh² *x* = 1
1 + tan²(*x*) = sec²(*x*)
1 + cot²(*x*) = csc²(*x*)
sin²(*x*) = $\frac{1-\cos(2x)}{2}$
cos²(*x*) = $\frac{1+\cos(2x)}{2}$
tan²(*x*) = $\frac{1-\cos(2x)}{1+\cos(2x)}$
sin(-*x*) = -sin(*x*)
cos(-*x*) = cos(*x*)
tan(-*x*) = -tan(*x*)
cosh²(*x*) - sinh²(*x*) = 1
1 - tanh²(*x*) = sech²(*x*)
sin *A* cos *B* = $\frac{1}{2}(\sin(A-B) + \sin(A+B))$
sin *A* sin *B* = $\frac{1}{2}(\cos(A-B) - \cos(A+B))$
cos *A* cos *B* = $\frac{1}{2}(\cos(A-B) + \cos(A+B))$

$$\sinh^{-1}(x) = \ln x + \sqrt{x^2+1}$$

$$\sinh^{-1}(x) = \ln x + \sqrt{x^2-1}, x \geq -1$$

$$\tanh^{-1}(x) = \frac{1}{2} \ln x + \frac{1+x}{1-x}, -1 < x < 1$$

$$\operatorname{sech}^{-1}(x) = \ln[\frac{1+\sqrt{1-x^2}}{x}], 0 < x \leq -1$$

$$\sinh(x) = \frac{e^x-e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x+e^{-x}}{2}$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$$

$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x) \tan(y)}$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cosh(n^2x) - \sinh^2x = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$$

$$\sin(-x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$

$$\tan(-x) = -\tan(x)$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$$

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

3D

given two points:
(*x*₁, *y*₁, *z*₁) and (*x*₂, *y*₂, *z*₂),
Distance between them:
 $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

Midpoint:
($\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}$)
Sphere with center (h,k,l) and radius r:
(*x* - *h*)² + (*y* - *k*)² + (*z* - *l*)² = *r*²

Vectors

Vector: *u*
Unit Vector: *u*
Magnitude: $||\vec{u}|| = \sqrt{u_1^2+u_2^2+u_3^2}$
Unit Vector: $\hat{u} = \frac{\vec{u}}{||\vec{u}||}$

Dot Product

u · *v*
Produces a Scalar
(Geometrically, the dot product is a vector projection)
 $\vec{u} = < u_1, u_2, u_3 >$
 $\vec{v} = < v_1, v_2, v_3 >$
u · *v* = 0 means the two vectors are Perpendicular
θ is the angle between them.
 $\vec{u} \cdot \vec{v} = ||\vec{u}|| \, ||\vec{v}|| \cos(\theta)$
u · *v* = *u*₁*v*₁ + *u*₂*v*₂ + *u*₃*v*₃
NOTE:
u · *v* = cos(*θ*)
 $||\vec{u}||^2 = \vec{u} \cdot \vec{u}$
u · *v* = 0 when ⊥
Angle Between *u* and *v*:
 $\theta = \cos^{-1}(\frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| \, ||\vec{v}||})$
Projection of *u* onto *v*:
 $pr_{\vec{v}}\vec{u} = (\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2})\vec{v}$

Cross Product

u × *v*
Produces a Vector
(Geometrically, the cross product is the area of a parallelogram with sides $||\vec{u}||$ and $||\vec{v}||$)
 $\vec{u} = < u_1, u_2, u_3 >$
 $\vec{v} = < v_1, v_2, v_3 >$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

u × *v* = *0* means the vectors are parallel
a × (*b* × *c*) = (*a*·*c*)*b* - (*a*·*b*)*c*

Volume of Parallelpiped

$$(\vec{v} \times \vec{u}) \cdot \vec{w}$$

However, in the case the question gave you three vectors directly

$$Volume = \det \left(\begin{bmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \right)$$

Lines and Planes

Equation of a Plane

(*x*₀, *y*₀, *z*₀) is a point on the plane and
< *A*, *B*, *C* > is a normal vector
A(*x* - *x*₀) + *B*(*y* - *y*₀) + *C*(*z* - *z*₀) = 0
< *A*, *B*, *C* > · < *x* - *x*₀, *y* - *y*₀, *z* - *z*₀ > = 0
Ax + *By* + *Cz* = *D* where
D = *Ax*₀ + *By*₀ + *Cz*₀

Equation of a line

A line requires a Direction Vector
 $\vec{u} = < u_1, u_2, u_3 >$ and a point (*x*₁, *y*₁, *z*₁)

Parametric equation:
x = *u*₁*t* + *x*₁ *y* = *u*₂*t* + *y*₁ *z* = *u*₃*t* + *z*₁
Symmetric equations:
 $t = \frac{x-x_1}{u_1} = \frac{y-y_1}{u_2} = \frac{z-z_1}{u_3}$

Distance from a Point to a Plane

The distance from a point (*x*₀, *y*₀, *z*₀) to a plane *Ax*+*By*+*Cz*=*D* can be expressed by the formula:
 $d = \frac{|Ax_0+By_0+Cz_0-D|}{\sqrt{A^2+B^2+C^2}}$

A line intersects a plane Just put the equations of *x*, *y*, *z* in the equation of the plane given, and solve for *t*. Then you'll have the points.

Distance between two skew lines through P₁P₂, P₃P₄

$$D = \frac{Vol_{parallelepiped}}{Area_{base}} = \frac{|(P_1\vec{P_2} \times P_3\vec{P_4}).P_1\vec{P_3}|}{|P_1\vec{P_2} \times P_3\vec{P_4}|}$$

Or go for the intuitive approach of forcing perpendicularity.

If two vectors are parallel $|\vec{v_1} \times \vec{v_2}| = 0$

Distance between a point and a line v

$$D = \frac{|P\vec{v_0} \times \vec{v}|}{|\vec{v}|}$$

Other Information

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
Law of Cosines:
 $a^2 = b^2 + c^2 - 2bc(\cos(\theta))$
Quadratic Formula:
 $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$, when *ax*² + *bx* + *c* = 0