

Derivatives

$\frac{d}{dx}e^x = e^x$
 $\frac{d}{dx}\sin(x) = \cos(x)$
 $\frac{d}{dx}\cos(x) = -\sin(x)$
 $\frac{d}{dx}\tan(x) = \sec^2(x)$
 $\frac{d}{dx}\cot(x) = -\csc^2(x)$
 $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
 $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
 $\frac{d}{dx}\sin^{-1} = \frac{1}{\sqrt{1-x^2}}, x \in [-1, 1]$
 $\frac{d}{dx}\cos^{-1} = \frac{-1}{\sqrt{1-x^2}}, x \in [-1, 1]$
 $\frac{d}{dx}\tan^{-1} = \frac{1}{1+x^2}, \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$
 $\frac{d}{dx}\sec^{-1} = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$
 $\frac{d}{dx}\sinh(x) = \cosh(x)$
 $\frac{d}{dx}\cosh(x) = \sinh(x)$
 $\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x)$
 $\frac{d}{dx}\coth(x) = -\operatorname{csch}^2(x)$
 $\frac{d}{dx}\operatorname{sech}(x) = -\operatorname{sech}(x)\tanh(x)$
 $\frac{d}{dx}\operatorname{csch}(x) = -\operatorname{csch}(x)\coth(x)$
 $\frac{d}{dx}\sinh^{-1} = \frac{1}{\sqrt{x^2+1}}$
 $\frac{d}{dx}\cosh^{-1} = \frac{-1}{\sqrt{x^2-1}}, x > 1$
 $\frac{d}{dx}\tanh^{-1} = \frac{1}{1-x^2}, -1 < x < 1$
 $\frac{d}{dx}\operatorname{sech}^{-1} = \frac{1}{x\sqrt{1-x^2}}, 0 < x < 1$
 $\frac{d}{dx}\ln(x) = \frac{1}{x}$
 $\frac{d}{dx}b^x = b^x \ln b$

Integrals

$\int \sin(x)dx = -\cos(x)$
 $\int \cos(x)dx = \sin(x)$
 $\int \tan(x)dx = -\ln|\cos(x)|$
 $\int \sec x \tan x dx = \sec x$
 $\int \sec x dx = \ln|\sec x + \tan x|$
 $\int \sec^2(x)dx = \tan(x) + C$
 $\int \sec^3(x)dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x \tan x|)$
 $\int \csc^2(x)dx = -\cot(x)$
 $\int \csc(x)\cot(x)dx = -\csc(x)$
 $\int \csc(x)dx = \ln|\csc(x) - \cot(x)|$
 $\int \cot(x)dx = \ln|\sin(x)|$
 $\int \frac{1}{x}dx = \ln|x|$
 $\int e^x dx = e^x$
 $\int a^x dx = \frac{1}{\ln a} a^x$
 $\int e^{ax} dx = \frac{1}{a} e^{ax}$
 $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(\frac{x}{a})$
 $\int \frac{1}{a^2+1+x^2} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a})$
 $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln|\frac{x-a}{x+a}|$
 $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}|$
 $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}(x)$
 $\int \sinh(x)dx = \cosh(x)$
 $\int \cosh(x)dx = \sinh(x)$
 $\int \tanh(x)dx = \ln|\cosh(x)|$
 $\int \tanh(x)\operatorname{sech}(x)dx = -\operatorname{sech}(x)$
 $\int \operatorname{sech}^2(x)dx = \tanh(x)$
 $\int \operatorname{csch}(x)\coth(x)dx = -\operatorname{csch}(x)$
 $\int \ln(x)dx = (x \ln(x)) - x$
 $\int_{-r}^r \sqrt{r^2-x^2} dx = \frac{\pi r^2}{2}$
 $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$

Integrals of inverse trig functions are solved using integration by parts.

Weierstrass

$t = \tan \frac{x}{2} \quad \cos x = \frac{1-t^2}{1+t^2}$
 $\sin x = \frac{2t}{1+t^2} \quad dx = \frac{2}{1+t^2} dt$

$\int \sin^m x \cos^n x$
 m is odd factor sines, then sub $u = \cos x$
 n is odd factor cosines, then sub $u = \sin x$.

If both even, use $\frac{1-\cos(2x)}{2}, \frac{1+\cos(2x)}{2}$.
If both odd, factor one with less power.

$\int \tan^m x \sec^n x$
 n even save one $\sec^2 x$ and sub $u = \tan x$.
 m odd save one $\sec x \tan x$ and sub $u = \sec x$

$\int \sqrt{a^2-x^2} dx \implies x = a \sin \theta$
 $\int \sqrt{a^2+x^2} dx \implies x = a \tan \theta$
 $\int \sqrt{x^2-a^2} dx \implies x = a \sec \theta$
 $\int_1^\infty \frac{1}{x^p}, p > 1$: converge, $p \leq 1$: diverges
e.o=o e.e=e o.o=e e+e=e o+o=o

Integration by Parts
 $\int u dv = uv - \int v du$

Functions/ Identities

$\sin(\cos^{-1}(x)) = \sqrt{1-x^2}$
 $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$
 $\sec(\tan^{-1}(x)) = \sqrt{1+x^2}$
 $\tan(\sec^{-1}(x)) = (\sqrt{x^2-1} \text{ if } x \geq 1)$
 $= (-\sqrt{x^2-1} \text{ if } x \leq -1)$
 $\sin^2(x) + \cos^2(x) = 1$
 $1 + \tan^2(x) = \sec^2(x)$
 $1 + \cot^2(x) = \csc^2(x)$
 $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
 $\cos(x \pm y) = \cos(x)\cos(y) \pm \sin(x)\sin(y)$
 $\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$
 $\sin(2x) = 2\sin(x)\cos(x)$
 $\cos(2x) = \cos^2(x) - \sin^2(x)$
 $1 + \tan^2(x) = \sec^2(x)$
 $1 + \cot^2(x) = \csc^2(x)$
 $\sin^2(x) = \frac{1-\cos(2x)}{2}$
 $\cos^2(x) = \frac{1+\cos(2x)}{2}$
 $\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$
 $\sin(-x) = -\sin(x)$
 $\cos(-x) = \cos(x)$
 $\tan(-x) = -\tan(x)$
 $\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))$
 $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$
 $\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$

$\sinh(x) = \frac{e^x - e^{-x}}{2}$
 $\cosh(x) = \frac{e^x + e^{-x}}{2}$
 $\cosh^2 x - \sinh^2 x = 1$
 $1 - \tanh^2(x) = \operatorname{sech}^2(x)$
 $\sinh^{-1}(x) = \ln x + \sqrt{x^2+1}$
 $\sinh^{-1}(x) = \ln x + \sqrt{x^2-1}, x \geq -1$
 $\tanh^{-1}(x) = \frac{1}{2} \ln x + \frac{1+x}{1-x}, 1 < x < -1$
 $\operatorname{sech}^{-1}(x) = \ln[\frac{1+\sqrt{1-x^2}}{x}], 0 < x \leq -1$
 $\cosh^2(x) - \sinh^2(x) = 1$
 $\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$
 $\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$

3D

given two points:
 (x_1, y_1, z_1) and (x_2, y_2, z_2) ,
Distance between them:
 $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$
Midpoint:
 $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2})$
Sphere with center (h,k,l) and radius r:
 $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$

Vectors

Vector: \vec{u}
Unit Vector: \hat{u}
Magnitude: $||\vec{u}|| = \sqrt{u_1^2 + u_2^2 + u_3^2}$
Unit Vector: $\hat{u} = \frac{\vec{u}}{||\vec{u}||}$

Dot Product
 $\vec{u} \cdot \vec{v}$
Produces a Scalar
(Geometrically, the dot product is a vector projection)
 $\vec{u} = \langle u_1, u_2, u_3 \rangle$
 $\vec{v} = \langle v_1, v_2, v_3 \rangle$
 $\vec{u} \cdot \vec{v} = 0$ means the two vectors are Perpendicular
 θ is the angle between them.
 $\vec{u} \cdot \vec{v} = ||\vec{u}|| ||\vec{v}|| \cos(\theta)$
 $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$
NOTE:
 $\hat{u} \cdot \hat{v} = \cos(\theta)$
 $||\vec{u}||^2 = \vec{u} \cdot \vec{u}$
 $\vec{u} \cdot \vec{v} = 0$ when \perp
Angle Between \vec{u} and \vec{v} :
 $\theta = \cos^{-1}(\frac{\vec{u} \cdot \vec{v}}{||\vec{u}|| ||\vec{v}||})$
Projection of \vec{u} onto \vec{v} :
 $pr_{\vec{v}}\vec{u} = (\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2})\vec{v}$

Cross Product
 $\vec{u} \times \vec{v}$
Produces a Vector
(Geometrically, the cross product is the area of a parallelogram with sides $||\vec{u}||$ and $||\vec{v}||$)
 $\vec{u} = \langle u_1, u_2, u_3 \rangle$
 $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$\vec{u} \times \vec{v} = \vec{0}$ means the vectors are parallel
 $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Volume of Parallelepiped

$(\vec{v} \times \vec{u}) \cdot \vec{w}$

However, in the case the question gave you three vectors directly

$$Volume = \det \left(\begin{bmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \right)$$

Lines and Planes

Equation of a Plane
 (x_0, y_0, z_0) is a point on the plane and $\langle A, B, C \rangle$ is a normal vector
 $A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$
 $\langle A, B, C \rangle \cdot \langle x-x_0, y-y_0, z-z_0 \rangle = 0$
 $Ax + By + Cz = D$ where
 $D = Ax_0 + By_0 + Cz_0$

Equation of a line

A line requires a Direction Vector
 $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and a point (x_1, y_1, z_1)

Parametric equation:
 $x = u_1t + x_1 \quad y = u_2t + y_1 \quad z = u_3t + z_1$
Symmetric equations:
 $t = \frac{x-x_1}{u_1} = \frac{y-y_1}{u_2} = \frac{z-z_1}{u_3}$
Distance from a Point to a Plane
The distance from a point (x_0, y_0, z_0) to a plane $Ax+By+Cz=D$ can be expressed by the formula:
 $d = \frac{|Ax_0+By_0+Cz_0-D|}{\sqrt{A^2+B^2+C^2}}$

A line intersects a plane Just put the equations of x, y, z in the equation of the plane given, and solve for t . Then you'll have the points.

Distance between two skew lines through P_1P_2, P_3P_4

$$D = \frac{Vol_{parallelepiped}}{Area_{base}} = \frac{|(P_1\vec{P}_2 \times P_3\vec{P}_4) \cdot P_1\vec{P}_3|}{|P_1\vec{P}_2 \times P_3\vec{P}_4|}$$

Or go for the intuitive approach of forcing perpendicularity.

If two vectors are parallel $|\vec{v}_1 \times \vec{v}_2| = 0$

Distance between a point and a line \vec{v}
 $D = \frac{|P\vec{v}_0 \times \vec{v}|}{|\vec{v}|}$

Other Information

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
Law of Cosines:
 $a^2 = b^2 + c^2 - 2bc(\cos(\theta))$
Quadratic Formula:
 $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$, when $ax^2 + bx + c = 0$

Logarithmic

$\log(xy) = \log(x) + \log(y)$
 $\log(\frac{x}{y}) = \log(x) - \log(y)$
 $\log({}^y\sqrt{x}) = \frac{\log(x)}{y}$
 $\log_b(1) = 0$
 $\log_a(x) = \frac{1}{\log_x(a)}$

MOSTAFA’S SHIT

Integration by parts

$\int u dv = uv - \int v du$

Case 1

The integration is the product of 2 functions: 1 easy to integrate and the other vanishes by differentiation. use Table method:

Alternate Signs	Keep Taking Derivative of u	Keep Taking Integrals of v, dv
+	x^n	$\frac{1}{n+1}x^{n+1}$
-	$4x^3$	$\frac{1}{4}x^4$
+	$12x^2$	$\frac{1}{3}x^3$
-	$24x$	$\frac{1}{2}x^2$
+	24	$\frac{1}{24}x^{24}$
-	0	$-\frac{1}{32}x^{32}$

Case 2

The presence of ln, log, inverse trig, inverse hyp, or any weird function without its derivative
put u = weird function and dv = the other function(or dx)

Case 3

$\int e^a x * \sin bx || \cos b x dx$
N.B (does not work with sinh and cosh as they can be factorized)
you can put any of the functions as u or dv but remain with the same choice till the end. Integrate till you reach the same integration then solve as an algebraic equation.

Case 4: The reduction formula

In = solution + In-m

Integration by partial fraction

- A rational function $\frac{p(x)}{q(x)}$
- proper (degree of p(x) < q(x)) if improper use long division
- q(x) can be factorized
- to get A, B and C you can cover the denominator of each one alone and substitute the x with its root. Or unify the denominator and substitute any three numbers in x.

Case 1

q(x) is a product of distinct linear functions.
Ex: $\int \frac{p(x)}{(x-a)(x+b)(x-c)} dx = \frac{A}{x-a} + \frac{B}{x+b} + \frac{C}{x-c}$

Case 2

q(x) is a product of linear functions but one of them is repeated.
Ex: $\int \frac{p(x)}{(x+a)(x-b)^2} dx = \frac{A}{x+b} + \frac{B}{(x-b)^2} + \frac{C}{(x-c)}$

Case 3

q(x) contain an irreducible and not repeated quadratic function.
use a polynomial with a degree less then q(x) by 1
Ex: $\int \frac{p(x)}{(x-a)(x^2+e^2)} dx = \frac{A}{x-a} + \frac{Bx+C}{x^2+e^2}$
use $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan(\frac{x}{a}) + C$

Case 4

Rationalization: when you have p(x) is a root take u = p(x) and proceed.

Volume

Disk Method

volume of any revolving graph = $\pi \int_a^b f(x)^2 dx$
where f(x) represents the radius of the disk and a,b are the end points. (When revolving about x the integral is in terms of x and same for the y)

Washer Method

Volume of a graph between two functions = $\pi \int_a^b f(x)^2 - g(x)^2 dx$
where f(x) represents the outer radius, g(x) the inner radius and a,b are the end points.

Shell Method

(When revolving about y the integral is in terms of x and vice versa)
 $V = 2\pi \int_a^b x f(x) dx$ If two graphs then f(x) will be the difference between the inner and the outer functions.

Area

$\int_a^b f(x) - g(x) dx$ Where f(x) is the larger function (further from the axis).

Improper integration

Case 1

$\int_a^\infty f(x) = \lim_{t \rightarrow \infty} \int_a^t f(x)$
 $\int_{-\infty}^b f(x) = \lim_{t \rightarrow -\infty} \int_t^b f(x)$
 $\int_{-\infty}^\infty f(x) =$
 $\lim_{t \rightarrow \infty} \int_a^t f(x) + \lim_{v \rightarrow -\infty} \int_v^b f(x)$

Case 2

f(x) is continuous on [a, b]
 $\int_a^b f(x) = \lim_{t \rightarrow b^-} \int_a^t f(x)$
f(x) is continuous on (a, b]
 $\int_a^b f(x) = \lim_{t \rightarrow a^+} \int_t^b f(x)$
f(x) is discontinuous on a point c in [a,b]
 $\int_a^b f(x) =$

$\lim_{t \rightarrow c^-} \int_a^t f(x) + \lim_{t \rightarrow c^+} \int_t^b f(x)$

Comparison

$g(x) \leq f(x)$
if $\int_a^\infty f(x) dx$ is convergent then $\int_a^\infty g(x) dx$ is convergent
if $\int_a^\infty g(x) dx$ is divergent then $\int_a^\infty f(x) dx$ is divergent.

Sequence

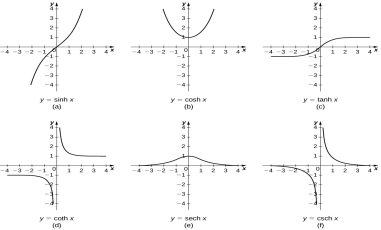
$\lim_{n \rightarrow \infty} an = L$ if exists then the sequence is convergent if not it is divergent.
 $\lim_{n \rightarrow \infty} |an| = 0$ Then $\lim_{n \rightarrow \infty} an = 0$
if $\lim_{n \rightarrow \infty} an = L$ then $\lim_{n \rightarrow \infty} f(an) = f(L)$
The sequence is bounded above if there is a number M such that $an < M$ for every n.
The sequence is bounded below if there is a number m such that $an > m$ for every n.
The sequence is monotonic if it is only decreasing or increasing.
If the sequence is bounded and monotonic then it is convergent.

Some Limit Rules

$\lim_{x \rightarrow \infty} e^x = \infty$
 $\lim_{x \rightarrow -\infty} e^x = 0$
 $\lim_{x \rightarrow \infty} \ln(x) = \infty$
 $\lim_{x \rightarrow 0^+} \ln(x) = -\infty$
 $\lim_{x \rightarrow \infty} \frac{c}{x^r} = 0$ when $r > 0$
 $\lim_{x \rightarrow \infty} \frac{c}{x^r} = 0$ when $r > 0$ and x^r is real on $x < 0$
 $\lim_{x \rightarrow \pm \infty} x^r = \infty$ when r is even
 $\lim_{x \rightarrow \infty} x^r = \infty$ and $\lim_{x \rightarrow -\infty} x^r = -\infty$ when r is odd
Use l'hospital's rule when $(\frac{0}{0}, \frac{\infty}{\infty}, 0\infty, 1^\infty, \infty^0, 0^0, \infty - \infty)$

Use squeeze theorem with trig and e^x

Randoms



Hyperbolic Functions			
Definitions		Hyperbolic Identities	
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$	$\sinh(-x) = -\sinh x$	$\cosh(-x) = \cosh x$
$\cosh^2 x - \sinh^2 x = 1$	$1 - \tanh^2 x = \operatorname{sech}^2 x$	$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$	$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\cosh x = \frac{e^x + e^{-x}}{2}$		
Derivatives of Hyperbolic Functions			
$\frac{d}{dx}(\sinh x) = \cosh x$	$\frac{d}{dx}(\cosh x) = \sinh x$	$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$	$\frac{d}{dx}(\operatorname{coth} x) = -\operatorname{csch}^2 x$
$\frac{d}{dx}(\sinh x) = \cosh x$	$\frac{d}{dx}(\cosh x) = \sinh x$	$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$
$\frac{d}{dx}(\sinh x) = \cosh x$	$\frac{d}{dx}(\cosh x) = \sinh x$	$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$	$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$
 $\cosh x = \frac{1}{2}(e^x + e^{-x})$
 $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
 $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}, x \neq 0$
 $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$
 $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}, x \neq 0$

