CAMBRIDGE UNIVERSITY ENGINEERING TRIPOS PART IB

IB INTEGRATED COURSEWORK: EXTENDED EXERCISE REPORT

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Title: How can multiple vibration absorbers be used to optimise damping of the harmonic response to an input signal?

Main topic area(s): *(delete as appropriate)* Vibration

Marker: .... Date: ....

Please highlight one mark in each row:

Technical content mark:	8	7	6	5	4	3	2	1	0
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Marker's Comments:

How can multiple vibration absorbers be used to optimise damping of the harmonic response to an input signal?

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Proposal Outline:

In lab A1 – Vibration Absorber, we studied the effect of adding a vibration absorber to a single degree of motion system. Specifically, we considered how we could reduce the maximum amplitude of the frequency domain response of the system by tuning the vibration absorber to the resonant frequency of the system.

During the lab we found that adding one tuned vibration absorber reduced the maximum amplitude of the system significantly, but there was still a significant amplitude of vibration remaining in the damped system.

In our extended exercise we set out to investigate the potential benefits of deploying multiple vibration absorbers for the purpose of reducing the maximum amplitude of the frequency domain response of the system. Our experimental aims where as follows:

- To find the critical mass of absorbers required to reduce the maximum amplitude of the frequency domain response to 10% of undamped value.
- To investigate the relationship between the total number of absorbers deployed and the maximum amplitude of the frequency domain response of the system.
- Find the optimal parameter values for total damper mass and number of dampers looking to minimise the maximum amplitude of the frequency domain response.

I expect to see that as we increase the number of vibration absorbers deployed there will be a decrease in maximum amplitude in the frequency domain response. I expect to see diminishing returns as we increase the number of vibration absorbers and the maximum amplitude in the frequency domain response will reach a horizontal asymptote as the decrease in max amplitude for an increase in the number of vibration absorbers becomes negligible. Furthermore, as we increase the number of vibration absorbers, I expect to see the range of frequencies where the response is damped to increase as well, since each vibration absorber will be tuned to a different frequency.

Technical Report

Background information

To further reduce the amplitude of vibration we need to consider the frequency at which maxima occur and tune the next vibration absorber to this frequency. With each additional vibration absorber, the frequency at which maxima occur changes, so it is important to consider this effect in our experiment. Using this principle, we could theoretically continue to add vibration absorbers until the maximum amplitude is at an acceptable amplitude. This was the methodology that we employed for our multiple absorber implementation.

When carrying out our experiment we decided to introduce some limits to our parameters. Firstly, we set an upper limit for the total mass of absorbers deployed to be half the total mass of the system (approximately 1.9kg) This was because a vibration absorber system where the total mass of absorbers is larger than the mass of the system is not a feasible engineering solution. We also limited the number of absorbers considered to 100, to reduce the programme execution time.

Lastly, to have each vibration absorber tuned to the most problematic frequency, we used a recursive method (attached in appendix 1) to assign the spring stiffness of each new absorber. Initially, we considered using a decay factor to reduce the spring stiffness of a new absorber by a constant factor (approximately 0.85) from the previous spring stiffness. This was a good approximation for small numbers of absorbers but quickly became inaccurate.

Simulating multiple absorbers

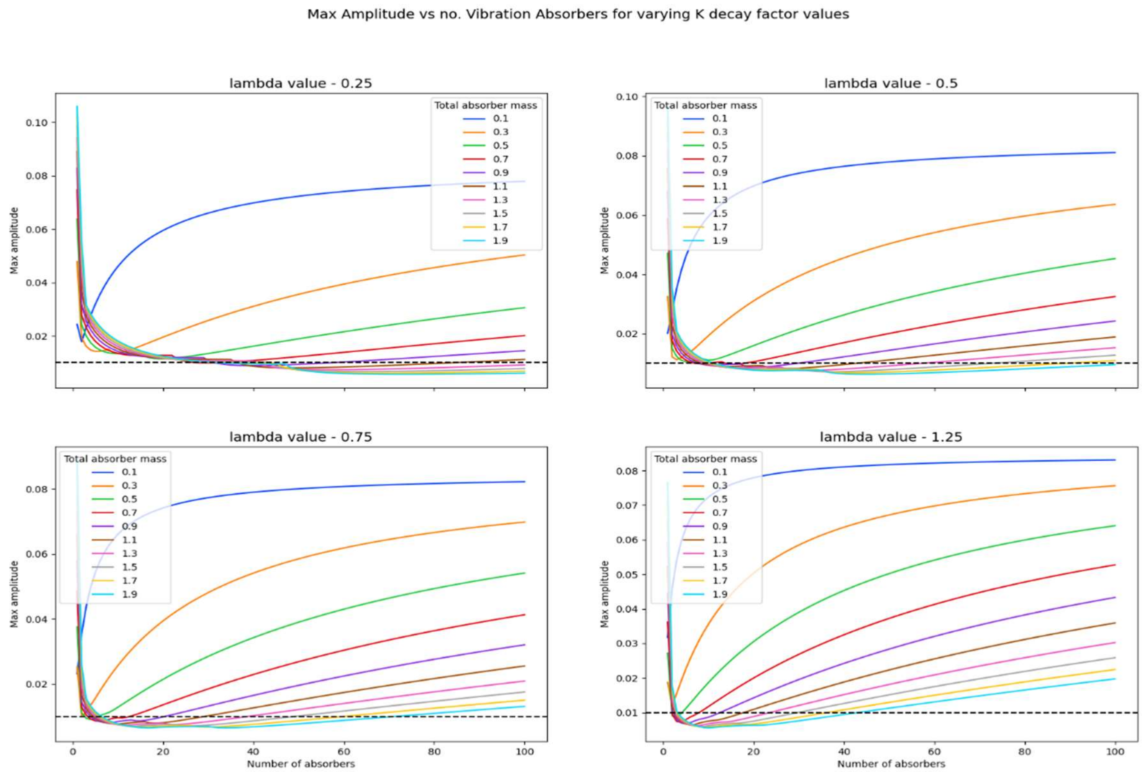
To collect results, we used numerous embedded for loops to run every possible parameter combination. In these simulations we considered:

- Total absorber mass between 0.1kg – 1.9kg in steps of 0.2kg.
- Number of absorbers between 1 – 100 in steps of 1
- Damping rate between $0.25Nsm^{-1}$ – $1.25Nsm^{-1}$ in steps of 0.25

Initially, we did not consider the effect of the damping rate and kept it constant at $1Nsm^{-1}$. However, it was clear to see that this was causing discrepancies between our results and the outcome we had expected, since the max amplitude of the system, despite decreasing initially, rapidly increased as the number of vibration absorbers deployed increased. This was not what we had expected. With optimal parameters every additional vibration absorber should have led to a reduction (or no noticeable change) in max amplitude. To reveal the impact of the damping rate we ran simulations with different, damping rates in the range $0.25Nsm^{-1}$ – $1.25Nsm^{-1}$ for all the absorbers.

Figure 1 shows 4 graphs of a family of curves (the case where damping rate = 1.0 has been removed as it didn't reveal any significant information). The graphs have a horizontal line plotted at 10% of the maximum amplitude in the undamped condition. Each graph has a unique damping rate value which is constant across every vibration absorber that we deploy. Figure 1 demonstrates that for optimal absorber behaviour we need to tune the damping rate to the mass of the absorbers being deployed. This explains the discrepancy in our graphs compared to the outcomes we had expected. As we add vibration absorbers, we see they are not well tuned to the system due to the damping rate we have assigned to them. Therefore, their effectiveness decreases and the max amplitude drifts upwards for large numbers of vibration absorbers. To improve our model, we need to find a general expression for the optimal damping rate of an absorber as a function of its mass. This would allow us to deploy a recursive method for assigning vibration absorbers with optimal damping rates.

The curves where the damping rate was larger, like $1.25Nsm^{-1}$ reach their minima much earlier than the curves where the damping rate is lower, like $0.25Nsm^{-1}$ which reach their minima when a very large number of vibration absorbers are deployed. As well as this, the figure shows that for larger damping rate values the minima are narrower and remain under the 10% line across only a small range of values (approximately 5 – 40 absorbers of total mass 1.9kg and $\lambda = 1.25Nsm^{-1}$). In comparison, for lower damping rates, there is a much wider range of values where the max amplitude is kept below 10% of the undamped amplitude (approximately 10 – 100 absorbers of total mass 1.9kg and $\lambda = 1.25Nsm^{-1}$).



$Figure\ 1$

$Max\ amplitude\ by\ number\ of\ vibration\ absorbers\ for\ varying\ values\ of\ damping\ rate$

From figure 1 we can see that the curves show an ‘elbow’ type relationship between the number of absorbers and max amplitude. The ‘elbow’ appears to occur across a relatively narrow range of values (10 – 16 vibration absorbers). This demonstrates that most of the reduction in max amplitude is due to the first few vibration absorbers, beyond which, additional vibration absorbers provide only marginal benefit. This ‘elbow’ relationship also shows that there is a minimum total mass of absorbers required to reduce the max amplitude of vibration to 10% of the undamped condition. This is because there is a horizontal asymptote, set by the total mass of vibration absorbers, which the max amplitude tends towards for a very large number of vibration absorbers (when they are all tuned optimally). Analysing the graphs reveals that, for the values we have used, the minimum mass of total absorbers needed to reduce the amplitude of vibration to less than 10% of the undamped case is 0.5kg.

Figure 2 shows the frequency response of the system for the parameter values which returned the lowest max amplitude. The vibration absorbers provide comprehensive coverage across the range of frequencies where the max amplitude would have been greater than 10% of the undamped max amplitude. The figure also shows varying amplitudes of vibration between the absorbers. The absorber tuned to the resonant frequency of the system has a much larger amplitude than any of the other absorbers. This is because at resonant frequency the system produces the largest vibrations and has the best transfer of energy between the system and the absorber. Comparatively, the other absorbers are tuned to frequencies where the response is problematic but not resonant. Therefore, there is less energy transfer, and the absorbers vibrate with much smaller amplitude. It is interesting to note that numerous local minima and maxima have developed. This is because the vibration absorbers we have deployed are effective, to varying degrees, across a range of frequencies. As a result, when we superimpose the effect of the different vibration absorbers there are some frequencies which get coverage from many absorbers and so we get local minima at these points. Equally, there are frequencies which get less coverage from absorbers and so we get local maxima at these points.

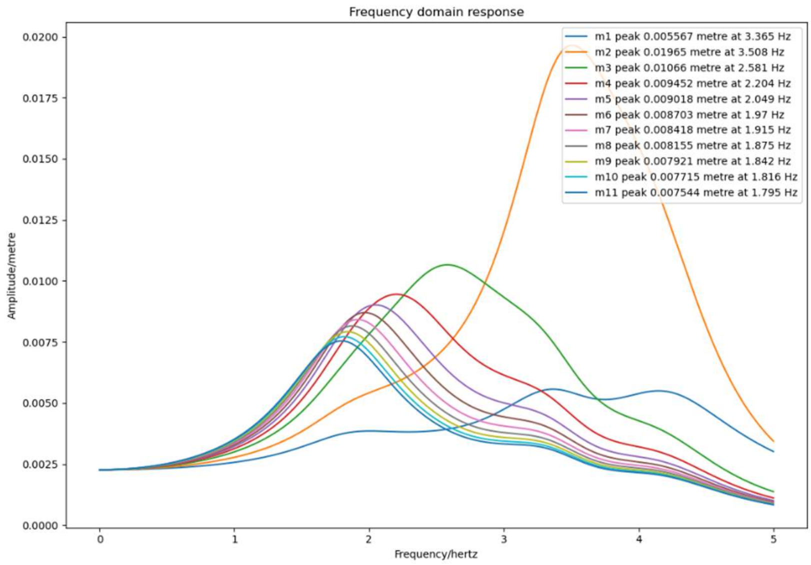


Figure 2

Frequency response for a system with 10 vibration absorbers with $\lambda = 1.25 Nsm^{-1}$

Conclusion

Using multiple vibration absorbers can be an effective solution for reducing the maximum amplitude in the frequency domain response of a system. The effectiveness of this solution is very sensitive to several factors. Chiefly, these are the total mass of the vibration absorbers and the damping rate of the dashpot in the vibration absorber assembly. Our results demonstrate that our model produced severe errors when trying to recreate the true behaviour of an optimised multiple vibration absorber system, especially when considering systems with a large number of absorbers. Our model could be greatly improved by using a recursive method for assigning damping rate values to absorbers. We found the critical total mass of absorbers for reducing vibration amplitude to 10% of the undamped case to be 0.5kg. I question the validity of this result greatly. If we had a more accurate model, I believe we would see smaller values of total mass successfully limiting the max amplitude in the frequency domain response to 10% of the undamped case. Investigating the frequency domain response of our best performing parameter values shows that the absorbers have the effect of flattening out the peak response. With the resulting characteristic shaped approximately like a plateau. When it comes to finding the optimal damping parameters, there is no such thing. Depending on the design constraints we can create successful vibration absorbing systems in various configurations of total mass of absorbers and number of absorbers.

```python
f = open('C:/Users/tokoa/Downloads/Coursework Labs/a1simulation.csv', 'w',
newline ='')

    writer = csv.writer(f)

    writer.writerow(['Number of absorbers', 'Total absorber mass', 'Max
amplitude', 'VA lambda value'])


    # Generate matrices describing the system
    for m in np.arange(0.1, 2, 0.2):
        m = round(m, 2)
        for l in np.arange(0.25, 1.5, 0.25):
            l = round(l, 3)
            freq_list = [3.67]
            for i in range(1, 101):
                m_list = [3.94] + [m/i] * i
                k_list = [2100] + [(j*2*np.pi)**2 * m/i for j in freq_list]
                l_list = [2.47] + [l] * i
                f_list = [4.757] + [0] * i
                #print(m_list, k_list, l_list, f_list)

                M, L, K, F = MLKF_ndof(
                    m_list, k_list, l_list, f_list
                )

                # Generate frequency and time arrays

                hz = np.linspace(args.hz[0], args.hz[1], 10001)
                sec = np.linspace(0, args.sec, 10001)

                # Plot results

                max_amp, new_freq = plot(hz, sec, M, L, K, F)
                freq_list.append(new_freq)
                print(max_amp)
                writer.writerow([i, m, max_amp, l])
```

Appendix 1

Python code for frequency domain response simulations

The code shown in appendix 1 begins by running an undamped simulation. For every iteration after that we return the frequency at which the maximum amplitude occurs and tune the next vibration absorber to it. This process is repeated until we have simulated a 100 vibration absorbers system. We then run the same simulations for varying total absorber mass values and absorber damping rate values. Simultaneously, we write the results to a csv file, which we can then use to visualise the relationships between our variables and the max amplitude of the frequency domain response.