Exercise 0

Pen-and-Paper

$$\mathcal{L} = \begin{cases} 0.5(\hat{y}-y)^2/\beta & |\hat{y}-y| \le \beta \\ |\hat{y}-y| - 0.5\beta & \text{else} \end{cases}$$

$$\hat{y} = W^T x = \sum_{i=0}^{n-1} w_i x_i \quad , \boxed{\beta > 0}$$

$$\mathcal{L} = \left[0.5\left(\sum_{i=0}^{n-1} w_i x_i - y\right)^2/\beta\right] \mathbb{1}\left(\left|\sum_{i=0}^{n-1} w_i x_i - y\right| \le \beta\right) +$$

$$\left[\sum_{n=0}^{n-1} w_i x_i - y - (0.5\beta)\right] \mathbb{1}\left(\left|\sum_{i=0}^{n-1} w_i x_i - y\right| > \beta\right)$$

$$|\hat{y}-y| > \beta \Rightarrow |\hat{y}-y| > 0 \Rightarrow |\hat{y}-y| = \hat{y}-y$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \left[\frac{0.5}{\beta} \cdot 2 \cdot \left(\sum_{i=0}^{n-1} w_i x_i - y\right)\left(\sum_{i=0}^{n-1} x_i\right)\right] \mathbb{1}\left(|\hat{y}-y| \le \beta\right) +$$

$$\left[\sum_{i=0}^{n-1} x_i\right] \mathbb{1}\left(|\hat{y}-y| > \beta\right)$$

$$= \begin{cases} \frac{1}{\beta}\left(\sum_{i=0}^{n-1} w_i x_i - y\right)\left(\sum_{i=0}^{n-1} x_i\right) & |\hat{y}-y| \le \beta \\ \sum_{i=0}^{n-1} x_i & \text{else} \end{cases}$$