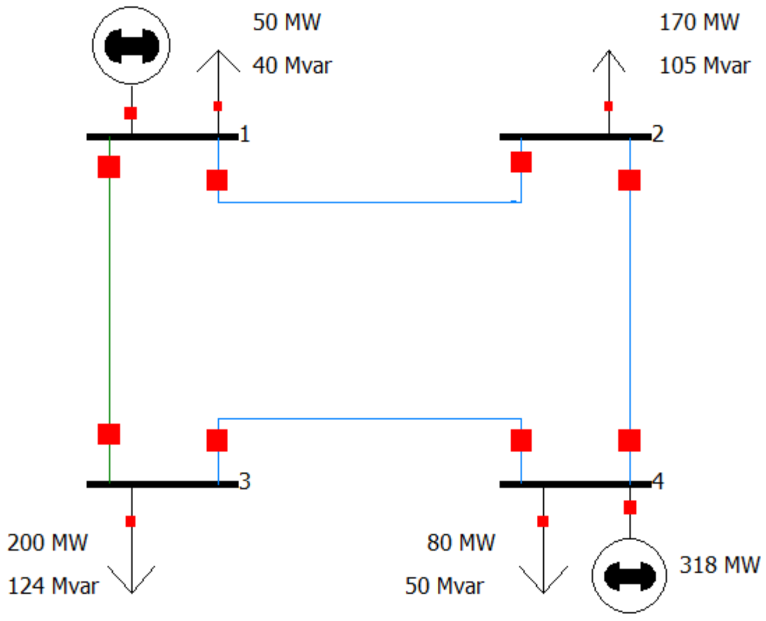
Exemplo

- Resolver o seguinte sistema pelo método de NR:



Dados dos circuitos:

	R (pu)	X (pu)	MVAR
1-2	0,010	0,050	10
1-3	0,0074	0,037	8,0
2-4	0,0074	0,037	8,0
3-4	0,0127	0,064	13

Dados das barras:
Bases:

	P_G (MW)	Q_G (MVAR)	P_D (MW)	Q_D (MVAR)	Tipo
1	-	-	50	40	Slack
2	0	0	170	105	PQ
3	0	0	200	124	PQ
4	318	-	80	50	PV

$S_B = 100\ MVA; V_B = 230\ KV$

1º Passo: Obter a matriz de admitâncias

- $Y_{11} = y_{12} + y_{13} + j0,05 + j0,04 = 9,11 - \mathrm{j}45,10 = 46,01 < -78,6°$
- $Y_{12} = -y_{12} = -3,85 + j19,23 = 19,61 < 101,3°$
- $Y_{22} = y_{12} + y_{24} + j0,05 + j0,04 = 9,11 - j45,10 = 46,01 < -78,6°$
- $Y_{24} = Y_{13} = -y_{13} = -5,26 + j25,96 = 26,48 < 101,5°$
- $Y_{33} = y_{13} + y_{34} + j0,04 + j0,065 = 8,31 - j40,86 = 41,7 < -78,5°$
- $Y_{34} = -y_{34} = -3,04 + j15,11 = 15,31 < 101,5°$
- $Y_{44} = y_{24} + y_{34} + j0,04 + j0,065 = 8,31 - j40,86 = 41,7 < -78,5°$

Matriz de Admitâncias

$$Y_{BUS} = \begin{bmatrix} 9{,}11 - j45{,}10 & -3{,}85 + j19{,}23 & -5{,}26 + j25{,}96 & 0 + j0 \\ -3{,}85 + j19{,}23 & 9{,}11 - j45{,}10 & 0 + j0 & -5{,}26 + j25{,}96 \\ -5{,}26 + j25{,}96 & 0 + j0 & 8{,}31 - j40{,}86 & -3{,}04 + j15{,}11 \\ 0 + j0 & -5{,}26 + j25{,}96 & -3{,}04 + j15{,}11 & 8{,}31 - j40{,}86 \end{bmatrix}$$

$$Y_{BUS} = \begin{bmatrix} 46{,}01\sphericalangle - 78{,}6° & 19{,}61\sphericalangle 101{,}3° & 26{,}48\sphericalangle 101{,}5° & 0\sphericalangle 0° \\ 19{,}61\sphericalangle 101{,}3° & 46{,}01\sphericalangle - 78{,}6° & 0\sphericalangle 0° & 26{,}48\sphericalangle 101{,}5° \\ 26{,}48\sphericalangle 101{,}5° & 0\sphericalangle 0° & 41{,}7\sphericalangle - 78{,}5° & 15{,}31\sphericalangle 101{,}5° \\ 0\sphericalangle 0° & 26{,}48\sphericalangle 101{,}5° & 15{,}31\sphericalangle 101{,}5° & 41{,}7\sphericalangle - 78{,}5° \end{bmatrix}$$

2 ° Passo: Cálculo das injeções de potência

- $P_2^0 = |V_2|^2 \cdot G_{22} + |V_2 \cdot V_1 \ \cdot Y_{21}| \cdot \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2 \ \cdot V_4 \cdot Y_{24}| \ \cdot \cos(\theta_{24} + \delta_4 - \delta_2) = -0{,}011953789 \text{ pu}$
- $P_3^0 = |V_3|^2 \cdot G_{33} + |V_3 \cdot V_1 \ \cdot Y_{31}| \cdot \cos(\theta_{32} + \delta_1 - \delta_3) + |V_3 \ \cdot V_4 \cdot Y_{34}| \ \cdot \cos(\theta_{34} + \delta_4 - \delta_3) = -0{,}021755561 \text{ pu}$
- $P_4^0 = -0{,}021755621 \text{ pu}$
- $Q_2^0 = -|V_2|^2 B_{22} - |V_2 \cdot V_1 \cdot Y_{12}| \cdot sen(\theta_{21} + \delta_1 - \delta_2) - \left|\dot{V}_2 \cdot V_4 \ \cdot Y_{24}\right| \cdot sen(\theta_{24} + \delta_4 - \delta_2) = -0{,}078221876 \, pu$
- $Q_3^0 = -0{,}091018892$ pu

3º Passo: Calcular os Resíduos

- $\Delta P_2^0 = (P_{G2} - P_{D2}) - P_2^0 = \left(0 - \frac{170}{100}\right) - (-0{,}011953789)$=−1,688046211 pu
- $\Delta P_3^0 = (P_{G3} - P_{D3}) - P_3^0 = \left(0 - \frac{200}{100}\right) - (-0{,}021755561)$=-1,9978244379 pu
- $\Delta P_4^0 = (P_{G4} - P_{D4}) - P_4^0 = 2{,}401755621\, pu$
- $\Delta Q_2^0 = (Q_{G2} - Q_{D2}) - Q_2^0 = -0{,}971778124\, pu$
- $\Delta Q_3^0 = (Q_{G3} - Q_{D3}) - Q_3^0 = -1{,}148981108\, pu$

4° Passo: Cálculo da matriz Jacobiana

Submatriz H

- $H_{22}^{0} = \frac{\partial P_2^0}{\partial \delta_2^0} = -Q_2^0 - \left|V_2^0\right|^2 \cdot B_{22} = -(-0{,}078221876) - 1^2 \cdot 45{,}10 = 45{,}178$
- $H_{23}^{0} = \frac{\partial P_2^0}{\partial \delta_3^0} = -\left|V_2^0 \cdot V_3^0 \cdot Y_{23}\right| \cdot sen(\theta_{23} + \delta_3 - \delta_2) = -|1 \cdot 1 \cdot 0| \cdot sen(0) = 0$
- $H_{24}^{0} = \frac{\partial P_2^0}{\partial \delta_4^0} = -\left|V_2^0 \cdot V_4^0 \cdot Y_{24}\right| \cdot sen(\theta_{24} + \delta_4 - \delta_2) = -|1 \cdot 1 \cdot 26{,}48| \cdot sen(101{,}5° + 0 - 0) = -25{,}94838421$
- $H_{32}^{0} = \frac{\partial P_3^0}{\partial \delta_2^0} = -\left|V_3^0 \cdot V_2^0 \cdot Y_{32}\right| \cdot sen(\theta_{32} + \delta_2 - \delta_3) = 0$
- $H_{33}^{0} = \frac{\partial P_3^0}{\partial \delta_3^0} = -Q_3^0 - \left|V_3^0\right|^2 \cdot B_{33} = 40{,}95101889$
- $H_{34}^{0} = \frac{\partial P_3^0}{\partial \delta_4^0} = -\left|V_3^0 \cdot V_4^0 \cdot Y_{34}\right| \cdot sen(\theta_{34} + \delta_4 - \delta_3) = -15{,}00263458$
- $H_{44}^{0} = \frac{\partial P_4^0}{\partial \delta_4^0} = -Q_4^0 - \left|V_4^0\right|^2 \cdot B_{44} = 40{,}95101889$
- $H_{42}^{0} = \frac{\partial P_4^0}{\partial \delta_2^0} = -\left|V_4^0 \cdot V_2^0 \cdot Y_{42}\right| \cdot sen(\theta_{42} + \delta_2 - \delta_4) = -25{,}94838431$
- $H_{43}^{0} = \frac{\partial P_4^0}{\partial \delta_3^0} = -\left|V_4^0 \cdot V_3^0 \cdot Y_{43}\right| \cdot sen(\theta_{43} + \delta_3 - \delta_4) = -15{,}00263458$

4° Passo: Cálculo da Matriz Jacobiana (cont.)

- $J_{22}^{0} = \frac{\partial Q_2^0}{\partial \delta_2^0} = P_2^0 - \left|V_2^0\right|^2 \cdot G_{22} = (-0{,}011953789) - 1^2 \cdot 9{,}11 = -9{,}121953$
- $J_{23}^{0} = \frac{\partial Q_2^0}{\partial \delta_3^0} = 0$
- $J_{24}^{0} = \frac{\partial Q_2^0}{\partial \delta_4^0} = -\left|V_2^0 \cdot V_4^0 \cdot Y_{24}\right| \cdot cos(\theta_{24} + \delta_4 - \delta_2) = -|1 \cdot 1 \cdot 26{,}48| \cdot cos(101{,}5° + 0 - 0) = 5{,}279370396$
- $J_{32}^{0} = \frac{\partial Q_3^0}{\partial \delta_2^0} = 0$
- $J_{33}^{0} = \frac{\partial Q_3^0}{\partial \delta_3^0} = P_3^0 - \left|V_3^0\right|^2 \cdot G_{33} = (-0{,}021755561) - 1^2 \cdot 8{,}31 = -8{,}331755621$
- $J_{34}^{0} = \frac{\partial Q_3^0}{\partial \delta_4^0} = -\left|V_3^0 \cdot V_4^0 \cdot Y_{34}\right| \cdot cos(\theta_{34} + \delta_4 - \delta_3) = 3{,}052385225$

4° Passo: Cálculo da Matriz Jacobiana (cont.)

- $N_{22}^{0} = |V_2| \frac{\partial P_2^0}{\partial |V_2|^0} = P_2^0 + \left|V_2^0\right|^2 \cdot G_{22} = (-0{,}011953789) + 1^2 \cdot 9{,}11 = 9{,}098046211$
- $N_{23}^{0} = 0$
- $N_{32}^{0} = 0$
- $N_{33}^{0} = |V_3| \frac{\partial P_3^0}{\partial |V_3|^0} = P_3^0 + \left|V_3^0\right|^2 \cdot G_{33} = (-0{,}021755561) + 1^2 \cdot 8{,}31 = 8{,}288244379$
- $N_{42} = |V_2| \frac{\partial P_4^0}{\partial |V_2|^0} = \left|V_4^0 \cdot V_2^0 \cdot Y_{42}\right| \cdot cos(\theta_{42} + \delta_2 - \delta_4) = -5{,}279370396$
- $N_{43} = |V_3| \frac{\partial P_4^0}{\partial |V_3|^0} = \left|V_4^0 \cdot V_3^0 \cdot Y_{43}\right| \cdot cos(\theta_{43} + \delta_3 - \delta_4) = -3{,}0523855225$

4° Passo: Cálculo da Matriz Jacobiana (cont.)

- $L_{22}^{0} = |V_2| \frac{\partial Q_2^0}{\partial |V_2|^0} = Q_2^0 - \left|V_2^0\right|^2 \cdot B_{22} = (-0{,}078221876) - 1^2 \cdot 45{,}10 = 45{,}02177812$
- $L_{23}^{0} = 0$
- $L_{32}^{0} = 0$
- $L_{33}^{0} = |V_3| \frac{\partial Q_3^0}{\partial |V_3|^0} = Q_3^0 + \left|V_3^0\right|^2 \cdot B_{33} = 40{,}76898111$

5° Passo: Montagem e Solução do Sistema de Equações

$$\begin{bmatrix} 45,178 & 0 & -25,948 & | & 9,0980 & 0 \\ 0 & 40,951 & -15,003 & | & 0 & 8,288 \\ -25,948 & -15,001 & 40,951 & | & -5,279 & -3,052 \\ -- & -- & -- & & -- & \\ -9,121 & 0 & 5,279 & | & 45,022 & 0 \\ 0 & -8,332 & 3,052 & & 0 & 40,769 \end{bmatrix} \cdot \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ -- \\ \dfrac{\Delta|V_2|}{|V_2|} \\ \dfrac{\Delta|V_3|}{|V_3|} \end{bmatrix} = \begin{bmatrix} -1,688 \\ -1,978 \\ 2,402 \\ -- \\ -9,972 \\ -1,149 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ -- \\ \dfrac{\Delta|V_2|}{|V_2|} \\ \dfrac{\Delta|V_3|}{|V_3|} \end{bmatrix} = \begin{bmatrix} -0,01191\ rad \\ -0,02834\ rad \\ 0,03432\ \mathrm{rad} \\ -- \\ -0,02847 \\ -0,03654 \end{bmatrix}$$

6º Passo: Cálculo das tensões para a segunda iteração:

$$\delta_i^{k+1} = \delta_i^k + \Delta\delta_i^k$$

$$|V_i|^{k+1} = |V_i|^k \cdot \left[1 + \frac{\Delta|V_i|^k}{|V_i|^k}\right]$$

$$V_2^1 = 0{,}9715 < 0{,}68° \ pu$$

$$V_3^1 = 0{,}9635 < -1{,}62° \, pu$$

$$V_4^1 = 1{,}00 < 1{,}97 \, pu$$

- Devem ser calculados novamente os $P_i, Qi, \Delta P_i, \Delta Q_i$ e o Jacobiano
- O processo continua até $\Delta P_i, \Delta Q_i < \epsilon$ (Critério de Convergência)
- Com os valores calculados de $V_i \ \ e \ \ \delta_i$ são calculados P_1, Q_1 na barra Slack e Q_4 na barra PV.
- O fluxo de potência em cada linha pode ser calculado por:

$$I_{ij} = \left(V_i - V_j\right) \cdot Y_{ij} \qquad\qquad S_{ij} = V_i \cdot I_{jk}^*$$