visual navigation exercise 03

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1 Exercise 2

1.1 Derive the computation formula of essential matrix

First we assume that there are two camera poses with name 0 and 1. We define rotation matrix and translation matrix between two poses as below:

t_{01}: the translation from 0 to 1(or 1's origin expressed in 0).

R_{01}: the rotation from 0 to 1(or 1's orientation expressed in 0).

X_0: vector from a 3D point A in the world to camera pose 0's origin.

X_1: vector from a 3D point A in the world to camera pose 1's origin.

Because A, 0 and 1 are in the same plane forming a triangle. We can use three vectors: t_{01}, X_0 and X_1 to represent three sides of the triangle. Any two of the three vectors' cross product is perpendicular to the third vector. So we have the equation:

$$(t_{01} \times X_0)^T X_0 = 0 \tag{1}$$

according to the relation between these three vectors, we have:

$$-X_0 = -R_{01}X_1 + t_{01} \tag{2}$$

also we want to express cross product in a matrix form:

$$[t_{01}]_\times X_0 = t_{01} \times X_0 \tag{3}$$

we substitute the X_0 on the right in (1) with (2):

$$([t_{01}]_\times X_0)^T (R_{01}X_1 - t_{01}) = 0 \tag{4}$$

expand (4):

$$X_0^T [t_{01}]_\times^T R_{01} X_1 - X_0^T [t_{01}]_\times^T t_{01} = 0 \tag{5}$$

because $[t_{01}]_\times$ is skew symmetric, it has the property of:

$$-[t_{01}]_\times = [t_{01}]_\times^T \tag{6}$$

so (5) can be rewrite as:

$$X_0^T(-[t_{01}]_\times) R_{01} X_1 - X_0^T(-[t_{01}]_\times t_{01}) = 0 \tag{7}$$

because vector's cross product with its own is zero, the second term of (7) is zero. So (7) can be rewrite as:

$$X_0^T(-[t_{01}]_\times)R_{01}X_1 = 0 \tag{8}$$

the essential matrix has the relationship with X_0 and X_1 as below:

$$X_0^T E X_1 = 0 \tag{9}$$

corresponding to matrix in (8), we can derive that E is as below:

$$E = (-[t_{01}]_\times)R_{01} \tag{10}$$