$- \vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{L} = I \vec{\omega}$

a) $\vec{L} = \left(\frac{d}{2} Mv + \frac{d}{2} Mv \right) \hat{z} = Mvd \, \hat{z}$

b) $K = \frac{Mv^2}{2} + \frac{Mv^2}{2} = Mv^2$

c) $\vec{L}_i = \vec{L}_f \Rightarrow \vec{L}_f = Mvd \, \hat{z}$

d) $2 \cancel{M} \frac{\cancel{d}}{4} \cdot v_f = \cancel{M} \cancel{d} v$

$v_f = 2v$

e) $K_f = \frac{M}{2} (2v)^2 + \frac{M}{2} (2v)^2 = 2Mv^2 + 2Mv^2 = 4Mv^2$

f) $\Delta K = W \Rightarrow W = 4Mv^2 - Mv^2 = 3Mv^2$

2- a) $L_i = L_f$

$L_i = m v_i \frac{d}{2} = L_f = I_B \omega + m \left(\frac{d}{2} \right)^2 \omega$

$$I_B = \int_{-\frac{d}{2}}^{\frac{d}{2}} x^2 \lambda \, dx = \lambda \frac{x^3}{3} \Big|_{-d/2}^{d/2} = \frac{\lambda}{3} \left(\frac{d^3}{8} + \frac{d^3}{8} \right) = \frac{\lambda d^3}{12} = \frac{Md^2}{12}$$

$$\Rightarrow \frac{Md^2}{12} \omega + m \frac{d^2}{4} \omega = m v_i \frac{d}{2}$$

$$\frac{Md}{6} \omega + m \frac{d}{2} \omega = m v_i$$

$$\omega = \frac{2 m v_i}{d(m + M/3)} = \frac{2 v_i}{d} \left(1 + \frac{M}{3m} \right)^{-1}$$

) $K = \frac{I_B \omega^2}{2} + \frac{m}{2}\frac{d^2}{4}\omega^2 = \frac{1}{2}\left(\frac{M d^2}{12} + \frac{m d^2}{4}\right) \frac{4 m^2 v_i^2}{d^2 (m + M/3)^2}$

$= \frac{1}{2} \frac{m^2 v_i^2}{m + M/3} = \frac{m v_i^2}{2}\left(1 + \frac{M}{3m}\right)^{-1}$

i- $mgh = \frac{m v^2}{2}$

$v^2 = 2gh$

$v = \sqrt{2gh}$

a) $L = D p = m D \sqrt{2gh}$

) $I_B = \int_0^D x^2 \lambda \, dx = \lambda \frac{x^3}{3}\Big|_0^D = \lambda \frac{D^3}{3} = \frac{M D^2}{3}$

$L_i = L_f$

$I_B \omega + m D^2 \omega = m D \sqrt{2gh}$

$\frac{M D^2}{3}\omega + m D^2 \omega = m D \sqrt{2gh}$

$\omega = \frac{m \sqrt{2gh}}{D(m + M/3)} = \frac{\sqrt{2gh}}{D}\left(1 + \frac{M}{3m}\right)^{-1}$

:) $Mg\left[D - \frac{D}{2}\cos\theta - \frac{D}{2}\right] + mg\left[D - D\cos\theta\right] = (I_B + m D^2)\frac{\omega^2}{2}$

$Mg\frac{D}{2}(1 - \cos\theta) + mgD(1 - \cos\theta) = \left(\frac{M}{3}D^2 + m D^2\right)\frac{\omega^2}{2}$

$1 - \cos\theta = \left(\frac{M}{3}D + m D\right)\frac{\omega^2}{2g\left(\frac{M}{2} + m\right)} =$

$$\ldots \cos\theta = \frac{M/3 + m}{M/2 + m} \frac{D}{\cancel{2}g} \frac{m^2}{D^2(m + M/3)^2} \cancel{2}g h$$

$$= \frac{m^2}{(M/2 + m)(M/3 + m)} \frac{h}{D} = \frac{h}{D}\left(1 + \frac{M}{2m}\right)^{-1}\left(1 + \frac{M}{3m}\right)^{-1}$$

l) Não. O pivo em torno do qual a barra gira exerce uma força externa.