Measuring the Solar Diameter with a Michelson Radio Interferometer

1. Scientific Background

The radius of a star is a fundamental property that is estimated in all models of stellar evolution. However, it is a very elusive property to determine empirically. With the exception of very few of the nearest or largest known stars (e.g., Betelgeuse), most stars appear as unresolved point sources eve in the largest filled-aperture telescopes. The fundamental limitation is that of Fourier optics: for a telescope of diameter D observing at wavelength λ, it is not possible to attain images sharper than the telescope diffraction limit, $1.2\lambda/D$. For example, one of the nearest and brightest stars in the sky, Vega, has an angular size of 3.28 milli-arcseconds (mas; Ciardi et al. 2001). For comparison, the theoretical diffraction limit of the largest ground-based optical telescopes—the 10 m twin Kecks–is 13 mas at $\lambda = 550\ nm$. In practice, the diffraction limit of the Kecks can not be attained at visible wavelengths because of turbulence in the atmosphere ("seeing") that smears astronomical images on scales of $\approx$500 mas.
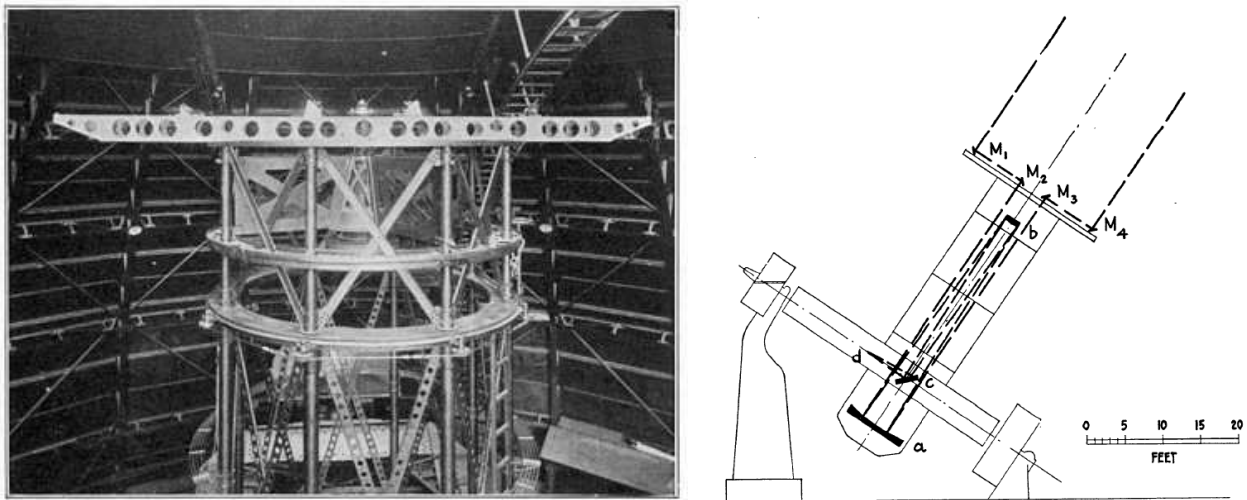

Fig. 1.— *Left:* Twenty-foot Michelson interferometer for measuring star diameters, attached to upper end of the skeleton tube of the 100-inch Hooker telescope. This instrument is now on display at the Rose Center of the American Museum of Natural History in New York. *Right:* Diagram of optical path of interferometer. M_1, M_2, M_3, M_4, mirrors; a, 100-inch paraboloidal primary mirror; b convex secondary mirror; c, coudé flat pick-off mirror; d, focus. Figures from Michelson & Pease (1921).

A work-around the diffraction limitation can be implemented when using two (or more) telescope apertures, where the distance B between the apertures is larger than each telescope diameter. The angular resolution limit of such an interferometer is then defined by λ/B, rather than λ/D. By using small-aperture telescopes, with diameters comparable to the sizes of the coherent patches of air in the turbulent atmosphere ($d \sim 10$ cm), one can overcome the effect of seeing. The small aperture sizes ensure that the radiation wavefronts detected by each of the two telescopes are coherent, i.e., have approximately the same phase across each of the telescope apertures. The only

difference between the two detected wavefronts is then a difference in phase. By adjusting the distance between the apertures, the phase difference can be changed to produces series constructive or destructive interference: interferometric fringes.

This technique was first applied by Michelson & Pease (1921) to measure the size of Betelgeuse at visible wavelengths. They used two 15 cm mirrors, and varied the spacing between them from 1.8 m to 5.5 m, along a beam attached on top of the skeleton tube of the 100-inch Hooker telescope on Mt. Wilson (Fig. 1). The same technique is now implemented in much more advanced optical, infrared, and radio interferometers to measure much smaller angular sizes of galactic and extragalactic objects, with telescopes spaced as far as thousands of kilometers apart (e.g., Very Long Baseline Interferometry; http://lupus.gsfc.nasa.gov/).

Our Stony Brook radio interferometer (Fig.2) functions on the same principle as the Michelson & Pease optical inteferometer. At radio wavelengths, $\lambda \approx 2.7$ cm in our case, the requirements for the surface accuracy δ of the reflective elements are much more relaxed ($\delta \lesssim \lambda/10$), hence a small radio interferometer can be built from off-the shelf components: aluminum mirrors and a $D = 1.0$ m commercial satellite dish antenna. The drawback is that the sensitivity of our small interferometer to celestial objects is very limited. However, the Sun and geostationary commercial TV satellites are detected well. Moreover, because the angular size of the Sun ($\theta \approx 30'$) is smaller than the resolution limit of single-dish (non-interferometric) mode observations ($\lambda/D \approx 90'$), alternating the two modes of operation effectively demonstrates the much greater resolving power attained by employing baselines $B > D$. The geostationary satellites remain unresolved in both modes of operation, and are hence appropriate as point-source calibrators for the telescope response function.

The mathematical details of interferometry are given in the accompanying manuscript "Michelson-type Radio Interferometer for University Education" by Koda et al. (2016), linked off the course
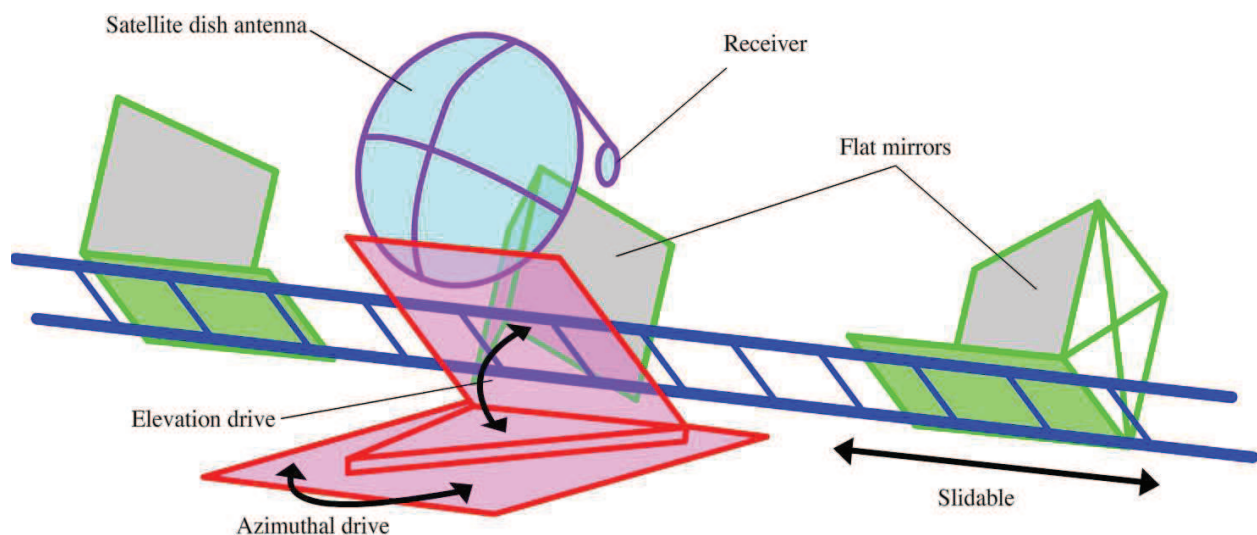

Fig. 2.— Conceptual sketch of the Stony Brook Michelson radio interferometer (Koda et al. 2016).

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website. *You are required to read this manuscript, as you will need the mathematical background to interpret your measurements with the interferometer.*

2. Experiment Goals and Plan

In this experiment you will measure the angular size of the Sun with the Stony Brook radio telescope and interferometer. You will use the telescope in both single-dish and interferometric modes of operation. The experiment demonstrates the greater resolving power of long-baseline (baseline B) interferometry compared to single-dish telescope (diameter D) observations, when $B > D$.

2.1. Experiment Goals

- measure the angular profile of the Sun with a single-dish radio antenna ($D = 1.0$ m);

- measure the angular fringe profile of the Sun with a two-telescope radio interferometer over a range of baselines (2 m $\lesssim B \lesssim$ 5 m);

- determine the angular size of the Sun in both cases through comparisons to unresolved point sources: commercial TV satellites;

- perform a fit to the interferometric visibility function of the Sun as a function of baseline B, and obtain an accurate estimate of the errors of your measurement.

2.2. Experiment Plan

1. Acquaint yourself with the "Mt. Stony Brook Radio Observatory Operations Manual" (Metchev 2013) and the "Michelson-type Radio Interferometer for University Education" (Koda et al. 2016) linked off the course website. Make sure to read these ahead of time.

2. Decide when you will conduct your observations. The radio signal from the Sun or the TV satellites is not affected by cloud cover. However, having enough sunlight so that you can see shadows will help greatly with the calibration of the telescope pointing on the Sun. You will also need the Sun to be at least 10° away from the nearest obstacle: building or a tall tree. From the location where we set up the radio interferometer in front of the Physics building, the main obstacles are the Earth and Space Sciences building and the Math tower. In February, you will be able to obtain data only between about 10am–2pm.

3. Make sure to start setting up at least an hour before your intended start of observations. You will need this time to collect the radio telescope and interferometer components from the S level of the Physics building to assemble them in the space in front of the building.

4. Come prepared with a list of altitude and azimuth coordinates for the Sun, at 5 min intervals, for the date of observation. You can find these on the US Naval Observatory website (http://aa.usno.navy.mil/data/docs/AltAz.php).

5. You will also need the positions of geosynchronous TV satellites that you will use as reference point sources. A plot showing their distribution in altitude and azimuth for Stony Brook's location is linked off the course website. The individual satellites are spaced $<1°$ apart along this linear locus, so when observing you will have to select a satellite that is *not* along the portion of the curve parallel to the azimuth direction. Otherwise, when slewing across the satellite in azimuth, you will get signal contamination from neighboring satellites. (Recall that the angular resolution of the telescope in single-dish mode is $\approx 90'$.)

6. Retrieve the radio telescope components from the Physics S level and the ESS 4th floor telescope storage room, and assemble them in front of the Physics building. Refer to the "Mt. Stony Brook Radio Observatory Operations Manual" for details.

7. With the dish first pointed to the sky, locate the Sun, and calibrated the azimuth and altitude readings. Take a constant slew speed scan across the Sun in azimuth. Repeat the procedure for a nearby TV satellite. You may need to adjust the receiver attenuation to use the same set of attenuators on the Sun and the satellite.

8. With the dish pointed toward the interferometer centerpiece with the two aluminum mirrors at $90°$ to each other, once again locate the Sun and re-calibrate the altitude reading. Take a constant slew speed scan of the Sun in the azimuth direction, and repeat on that on a TV satellite. Make sure to apply any necessary changes to the attenuation. Change the interferometer baseline (the distance between the two side mirrors), and repeat the measurements on the Sun and the satellite. Do that for at least 5 baselines.

9. You will be recording all data with the provided laptop an the Logger Pro software. When you are done, make sure to convert the data from Logger Pro format to another format, and to transfer it from the laptop to a convenient carrier.

10. Shut down the laptop, lower the telescope mount, and disassemble the interferometer. Bring all components back in place in the S level of Physics.

3. Data Acquisition

The recommended procedures for radio telescope calibration and data acquisition are described in the 'Mt. Stony Brook Radio Observatory Operations Manual" (§§4–5). Please, read the Manual, and obtain data according to the prescriptions outlined in §5.1 (single-dish) and §5.2 (interferometry).

Ideally, your data will bear some resemblance to the fringe patterns in the bottom panels of Figure 3, which show the infinite fringe patter of the source n the sky convolved with the telescope beam.

Make sure to keep a log of the times of your observations, as you will need these to infer the elevation of the Sun for each observation. Also, record the elevations of all observed satellites.
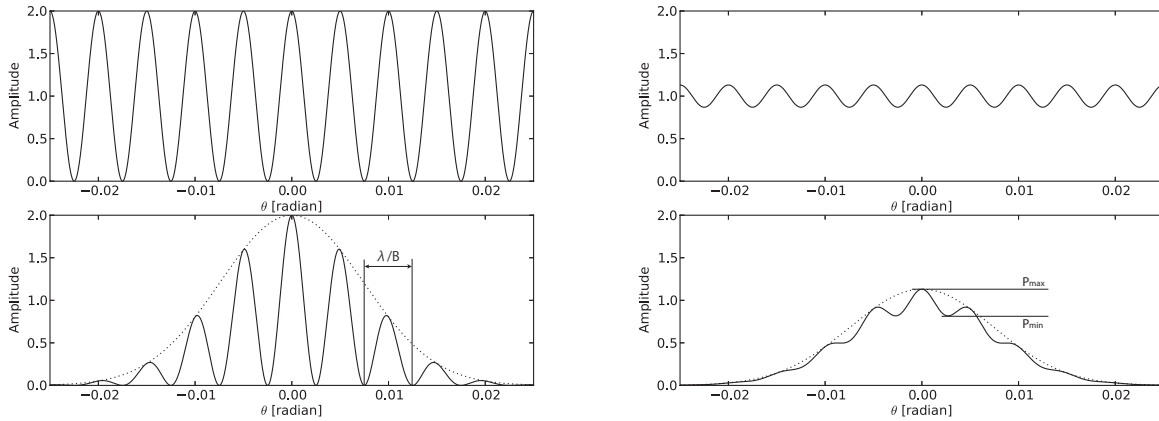
Fig. 3.— Example plot of the total power as a function of telescope pointing θ in cases of a point source (*left* panels) or of a resolved disk (the Sun; *right* panels). *Top:* Respective fringe patterns on the sky (Eq. 11 of Koda et al. 2016). *Bottom:* Fringe patterns attenuated by the telescope beam pattern. The dotted lines are Gaussian beam profiles with FWHM of $1°$. Figures from Koda et al. (2016).

Keep a record of the slew rate for each scan, so that you can later confirm that the Sun and the satellites were scanned at the same rate. Also note the approximate baseline B of the interferometer, as read off the scale on the ladder.

The data are saved by Logger Pro on the laptop in a directory of your choosing. The Logger Pro file format is not readable in text editors or Excel. Hence, when you finish all data taking, make sure to export all data as CSV files, and to transfer them on a data carrier.

4. Data Reduction

You are free to use your favorite programming language or mathematical software. The `python` primer on the webpage should cover all of the tools that you need to complete the lab in `python`.

4.1. Single-dish Observation

Compare the azimuthal profiles of the Sun and of the TV satellite in single-dish mode by overplotting them on the same graph, normalized to the same peak value. If the object elevations during the two observations were different, scale the width of the solar profile by the ratio of cosines of the elevations. (The higher the elevation above the horizon, the more of an azimuthal slew is needed to cover the same angular size on the sky.)

A useful script that simulates a Gaussian profile (the PSF) and an observation of a uniform disk (the cross-section of which is a tophat), and then plots them on the same graph is linked off the course webpage (`gauss_tophat.py`).

4.2. Interferometric Observations

Calculate the visibility V_0 at each baseline B_λ, and plot $V_0(B_\lambda)$.

$$|V_0(B_\lambda)| = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} \tag{1}$$

$P_{\max}$ and $P_{\min}$ are the amplitudes of the peaks and the troughs of the fringes (Fig. 3). You should estimate an average $V_0(B_\lambda)$ from several fringes in your observation. B_λ is the baseline B expressed in multiples of λ: $B_\lambda = B/\lambda$, which equals the angular frequency $1/\delta\theta$ of the fringes on the sky. That is, the angular spacing of the fringes is $\delta\theta = 1/B_\lambda = \lambda/B$.

Your visibility measurements should scatter around the `sinc` function profile plotted in Figure 4. Note that if the Sun and the satellite were observed at different elevations, you will need to scale the θ direction (i.e., the inverse of the baseline B) by the ratio of the cosines when comparing fringe patterns and visibilities.

In some cases the fringe patterns on the Sun and the satellite are not symmetrical around their peaks. The cause for this may have to do with a decrease of the signal from one of the reflectors of the interferometer—potentially because of a slight mis-pointing. Regardless, the data are still usable. Measure the visibility at the location of the deepest fringes. Make sure to use the same respective location for measuring the visibilities of both the Sun and the satellite.
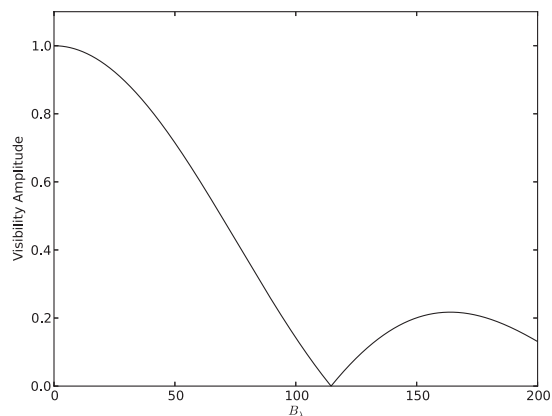

Fig. 4.— Visibility amplitude as a function of baseline length in case of a disk. Figure from Koda et al. (2016).

5. Analysis and Discussion

5.1. Single-dish Observations

Compare the profiles of the Sun and the satellite. Is the Sun resolved? Is this result expected? What is the signal-to-noise ratio of your detection of the Sun and of the satellite?

Given that you can determine the width (or full-width at half-maximum: FWHM) of the solar and satellite profiles to an accuracy of about FWHM / SNR, how much higher SNR do you need to confidently (at the 3σ level) conclude that you have resolved the solar disk with the single-dish observation? How many more scans across the Sun and the satellite would you need?

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5.2. Interferometric Observations

You recorded only approximate baselines in inches for all interferometric measurements. What are the actual baselines, based on the fringe spacing? Make sure to correct by the cosine of the solar elevation angle.

At what baseline B_λ is the visibility of the Sun closest to zero? This gives an approximate measurement of the size of the Sun. What is it?

Can your plot of visibilities $V_0(B_\lambda)$ be adequately fit with a `sinc` function (the Fourier transform of a tophat)? Such a fit will give a more accurate estimate of the width of the tophat function—the solar angular diameter. Estimate the diameter from the fit. You can use the `curve_fit` routine in `scipy` to fit the data.

You can use the visibility of the satellite at each baseline to better calibrate your visibility measurements of the Sun. Plot both sets of visibilities on the same visibility amplitude vs. B_λ plot. Since theoretically the satellite visibilities should all be identically unity, if one or more are smaller than 1.0 that indicates a systematic loss of throughput in the interferometer for that observation. Normalize all of your solar visibility measurements to those of the satellite, re-plot the solar visibilities as a function of B_λ, and re-fit the measurements with a `sinc` function. Did the quality (χ^2) of the fit improve? Did your estimate of the angular diameter of the Sun change?

6. Lab report

The timeline for the lab report, and intermediate check-ins is the following:

+1 week: Send in your analysis of the single-dish observations. Make sure to compare the measured profiles of the Sun and of the satellite (make sure to correct for the elevation!), as well as simulated profiles, as appropriate.

+2 weeks: Calculate the visibilities from your interferometric observations of the Sun, and of the satellite. Send in your plot of the visibility amplitude as function of baseline.

+3 weeks: Fit the visibilities for the Sun with a `sinc` function. Compare to the expected value. Calibrate the measurements based on the observed visibilities of the satellite, and re-fit the results. Report both results.

+4 weeks: Hand in your lab report. Make sure that your analysis codes are attached to your report, or available on github.

REFERENCES

Ciardi, D. R., van Belle, G. T., Akeson, R. L., Thompson, R. R., Lada, E. A., & Howell, S. B. 2001, ApJ, 559, 1147

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