PHY517 / AST443: Observational Techniques

Homework 4

Radio interferometry with two mirrors (see Koda et al. 2016):

1. The separation between the two mirrors, called baseline length B, causes a time delay, τ, in the arrival of the signal at the second detector. Using the angular direction of the telescope pointing θ and to an object in the sky θ_0, show that

$$\tau = \frac{B \sin(\theta - \theta_0)}{c} \quad .$$

2. Consider now the field due to a source of frequency ν observed at each detector at time t:

$$\begin{aligned} E_1(t) &= E_0(\theta_0) \cos(2\pi\nu t) \quad , \\ E_2(t) &= E_0(\theta_0) \cos[2\pi\nu(t - \tau)] \quad . \end{aligned}$$

Show that the total power detected by the receiver is

$$P(\theta) = E^2(\theta_0)\{1 + \cos[2\pi(\theta - \theta_0)]\} \tag{1}$$

for a point source. What do you need to assume to arrive at this expression? Plot this function for a source located at $\theta_0 = 0$.

3. For an extended source, Eq. 1 can be generalized to

$$P(\theta) = \int \mathcal{E}(\theta_0)\{1 + \cos[2\pi(\theta - \theta_0)]\}\mathrm{d}\theta_0$$

Show that this can be written as

$$P(\theta) = S_0[1 + V(\theta, B_\lambda)] = S_0\{1 + V_0(B_\lambda)\cos[2\pi B_\lambda(\theta - \Delta\theta)]\} \tag{2}$$

with the introduction of the total source flux S_0:

$$S_0 = \int \mathcal{E}(\theta_0)\mathrm{d}\theta_0$$

and:

$$V(\theta, B_\lambda) = \frac{1}{S_0}\int \mathcal{E}_0(\theta_0)\cos[2\pi B_\lambda(\theta - \theta_0)]\mathrm{d}\theta_0 \quad . \tag{3}$$

Make sure to show all your steps (the paper leaves some out). Plot Eq. 2 for a visibility $0 < |V_0(B_\lambda)| < 1$.

4. Use Eq. 3 to show that the visibility of the Sun is

$$|V_0(B_\lambda)| = \left|\frac{\sin(\pi B_\lambda \alpha)}{\pi B_\lambda \alpha}\right| = |\mathrm{sinc}(\pi B_\lambda \alpha)| \quad ,$$

   where α is the diameter of the Sun.

5. Show how the visibility can be calculated from the maximum and minimum power near the peak. What is the visibility of the data shown in Fig. 8a?