# CS 425 Homework 3

Mukai Yu

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1. (a)

| Process | Vote For |
|---|---|
| P2 | P2 (self) |
| P3 | P3 (self) |
| P4 | P3 |
| P5 | P2 |

No leader will be elected for term 2.

(b)

| Process | Vote For |
|---|---|
| P2 | P2 (self) |
| P3 | P3 (self) |
| P4 | P2 |
| P5 | P2 |

P2 will be elected for term 2.

(c) i) $(160, 500]$ ms

ii) $[100, 140)$ ms

iii) $(140, 160)$ ms

(d)

$$
\begin{aligned}
& P(\textit{another process also calls election}) \\
&= 1 - P(P_3 \textit{ doesn't call } \cup\ P_4 \textit{ doesn't call } \cup\ P_5 \textit{ doesn't call}) \\
&= 1 - P(P_3 \textit{ doesn't call})P(P_4 \textit{ doesn't call})P(P_5 \textit{ doesn't call}) \\
&= 1 - \frac{500 - (105 + 30)}{400} \times \frac{500 - (105 + 25)}{500} \times \frac{400 - (105 + 10)}{400} \\
&\approx 0.1876
\end{aligned}
$$

2. (a) Yes, scenario as follows:

Term 1:
$S_1$: 1, 1, 1
$S_2$: 1
$S_3$: 1
$S_1$ is the leader, it is initialized to be the leader or it gets votes from all three processes, and it replicates the 1s for $S_2$ and $S_3$.

Term 2:
$S_1$: 1, 1, 1
$S_2$: 1, 2, 2
$S_3$: 1
$S_2$ is the leader, it gets vote from itself and $S_3$, only appends 2s to its own log.

Term 3:
$S_1$: 1, 1, 1
$S_2$: 1, 2, 2
$S_3$: 1, 1
$S_1$ is the leader, it gets vote from itself and $S_3$, and replicates a 1 to $S_3$.

(b) Yes, scenario as follows:

Term 1:
$S_1$: 1, 1, 1
$S_2$: 1, 1
$S_3$: 1, 1
$S_1$ is the leader, it is initialized to be the leader or it gets votes from all three processes, and it replicates the 1s for $S_2$ and $S_3$.

Term 2:
$S_1$: 1, 1, 1
$S_2$: 1, 1, 2
$S_3$: 1, 1
$S_2$ is the leader, it gets votes from all three processes, only appends a 2 to its own log.

(c) No.
$S_3$ must be the leader of term 3 since it's the only process with an entry in term 3.
Similarly $S_2$ must be the leader of term 2.
$S_1$ must be the leader of term 1 since it's has the most up-to-date log of term 1.

Thus, by the time $S_3$ initiates new election for term 3, the system must have the following log:

$S_1$: 1, 1, 1
$S_2$: 1, 1, 2
$S_3$: 1, 1
But in this situation $S_3$ will be rejected by both $S_1$ and $S_2$, thus the final entries are invalid.

(d) Yes, scenario as follows:

Term 1:
$S_1$: 1, 1, 1
$S_2$: 1
$S_3$: 1
$S_1$ is the leader, it is initialized to be the leader or it gets votes from all three processes, and it replicates the 1s for $S_2$ and $S_3$.

Term 2:
$S_1$: 1, 1, 1
$S_2$: 1, 2, 2
$S_3$: 1, 2
$S_2$ is the leader, it gets votes from all three processes, appends 2s to its own log and replicate a 2 to $S_3$.

Term 3:
$S_1$: 1, 1, 1
$S_2$: 1, 2, 2
$S_3$: 1, 2, 3
$S_3$ is the leader, it gets votes from itself and $S_1$, and only append a 3 to its own log.

(e) Yes, similar to (a):

Term 1:
$S_1$: 1, 1
$S_2$: 1
$S_3$: 1
$S_1$ is the leader, it is initialized to be the leader or it gets votes from all three processes, and it replicates the 1s for $S_2$ and $S_3$.

Term 2:
$S_1$: 1, 1
$S_2$: 1, 2, 2
$S_3$: 1
$S_2$ is the leader, it gets votes from all three processes, appends 2s to its own log.

$S_1$ can start new election after it voted $S_2$ as the leader of term 2, updated currentTerm to 2, and starts a new election (due to timeout, etc.) without appending any entries from term 2.

Term 3:
$S_1$: 1, 1, 3, 3
$S_2$: 1, 2, 2
$S_3$: 1, 1, 3
$S_1$ is the leader, it gets votes from itself and $S_3$, appends 3s to its own log, and replicate 1 and 3 to $S_3$.

3. (a)

$$P(H(x||seed) < T) = \frac{2^{226}}{2^{256}} = 2^{-30} \approx 0.0000000009313$$

(b) Probability of finding a winning solution is the same since sample with replacement is adopted, thus:

$$\begin{aligned}
&P(finding\ a\ solution\ in\ 10\ hours) \\
&= 1 - P(finding\ no\ solution\ in\ 10\ hours) \\
&= 1 - (1 - P(H(x||seed) < T))^{10\times 60\times 60\times 2^5} \\
&= 1 - (1 - 2^{-30})^{10\times 60\times 60\times 2^5} \\
&\approx 0.00107
\end{aligned}$$

(c) Similar to (b), probability of 1 machine finding a winning solution in 5 hour is:

$$\begin{aligned}
&P(finding\ a\ solution\ in\ 5\ hours) \\
&= 1 - P(finding\ no\ solution\ in\ 5\ hours) \\
&= 1 - (1 - P(H(x||seed) < T))^{5\times 60\times 60\times 2^5} \\
&= 1 - (1 - 2^{-30})^{5\times 60\times 60\times 2^5} \\
&\approx 0.000536297944049366083497744009227752
\end{aligned}$$

Thus the probability of 5000 machine finding a winning solution in 5 hour is:

$$\begin{aligned}
&P(1\ machine\ finding\ a\ solution\ in\ 5\ hours) \\
&= 1 - P(\ no\ machine\ finding\ a\ solution\ in\ 5\ hours) \\
&= 1 - (1 - P(finding\ a\ solution\ in\ 5\ hours))^{5000} \\
&= 1 - (1 - 2^{-30})^{(5\times 60\times 60\times 2^5)^{5000}} \\
&\approx 0.9316
\end{aligned}$$