

CS 425 Homework 1

Mukai Yu

February 8, 2022

1. (a) In worst case, In worst case, assuming process a crashes immediately after it sends out the last heartbeat (died right after SOS) :

$$\begin{aligned}
 e_{a \rightarrow b} &= \max\{a \rightarrow b\} + \Delta_{a \rightarrow b} = 2 \max\{a \rightarrow b\} - \min\{a \rightarrow b\} \\
 T_b &= T + e_{a \rightarrow b} = 50 + 10 = 60(s) \\
 T_c &= T + e_{a \rightarrow c} = 50 + 26 = 76(s) \\
 T_d &= T + e_{a \rightarrow d} = 50 + 19 = 69(s)
 \end{aligned}$$

- (b) Need to check all the path from **a** to other processes

$$\begin{aligned}
 T'_b &= T + \min_{path} \{e_{a \rightarrow b}, \\
 &\quad e_{a \rightarrow c} + \max\{b \rightarrow b\}, \\
 &\quad e_{a \rightarrow d} + \max\{d \rightarrow b\}, \\
 &\quad e_{a \rightarrow c} + \max\{c \rightarrow d\} + \max\{d \rightarrow b\}, \\
 &\quad e_{a \rightarrow d} + \max\{d \rightarrow c\} + \max\{c \rightarrow b\}\} \\
 &= 50 + \min\{10, 26 + 18, 19 + 13, 26 + 4 + 13, 19 + 4 + 18\} \\
 &= 50 + 10 \\
 &= 60(s)
 \end{aligned}$$

$$\begin{aligned}
 T'_c &= T + \min_{path} \{e_{a \rightarrow c}, \\
 &\quad e_{a \rightarrow b} + \max\{c \rightarrow c\}, \\
 &\quad e_{a \rightarrow d} + \max\{d \rightarrow c\}, \\
 &\quad e_{a \rightarrow b} + \max\{b \rightarrow d\} + \max\{d \rightarrow c\}, \\
 &\quad e_{a \rightarrow d} + \max\{d \rightarrow b\} + \max\{b \rightarrow c\}\} \\
 &= 50 + \min\{26, 10 + 18, 19 + 4, 10 + 13 + 4, 19 + 13 + 18\} \\
 &= 50 + 23 \\
 &= 73(s)
 \end{aligned}$$

$$\begin{aligned}
T'_d &= T + \min_{path} \{ e_{a \rightarrow d}, \\
&\quad e_{a \rightarrow b} + \max\{b \rightarrow d\}, \\
&\quad e_{a \rightarrow c} + \max\{c \rightarrow d\}, \\
&\quad e_{a \rightarrow b} + \max\{b \rightarrow c\} + \max\{c \rightarrow d\}, \\
&\quad e_{a \rightarrow c} + \max\{c \rightarrow b\} + \max\{b \rightarrow d\} \} \\
&= 50 + \min\{19, 10 + 13, 26 + 4, 10 + 18 + 4, 26 + 18 + 13\} \\
&= 50 + 19 \\
&= 69(s)
\end{aligned}$$

- (c) Protocol: each process sends heartbeat to 2 other nearest processes, where nearest process means a process that has the least worst case failure detection time of the current process.
- (d) the minimal set of processes **a** must send heartbeats to $= \{b, d\}$

2. (a) i. The client should choose S_2 to achieve the lowest accuracy at:

$$skew(client, S_2) \leq \frac{RTT(S_2)}{2} + \min < \frac{RTT(S_2)}{2} = 12(ms)$$

- ii. The longest time-period = $\frac{(90-12) \times 10^{-3}}{3 \times 10^{-6}} = 26,000(s)$

- (b) i. The lowest synchronization bound is:

$$\frac{(39 - 12) + (52 - 71)}{2} = 4$$

The corresponding estimated offset value of **B relative to A** is:

$$\frac{(39 - 12) - (52 - 71)}{2} = 23$$

So the estimated offset value of **A relative to B** is **-23**

- ii. The tightest synchronization bound is:

$$\frac{75 - 60}{2} = 7.5$$

3. (a)

A	1
B	2
C	7
D	8
E	2
F	5
G	6
H	7
I	1
J	2
K	5
L	7
M	2
N	3
O	4
P	7

(b)

A	1	0	0	0
B	2	0	1	0
C	3	3	1	3
D	4	3	1	3
E	0	1	1	0
F	1	2	1	3
G	1	3	1	3
H	1	4	1	3
I	0	0	1	0
J	1	0	2	0
K	1	0	3	3
L	1	3	4	3
M	0	0	1	1
N	1	0	1	2
O	1	0	1	3
P	1	3	1	4

- (c)
- i. $A \parallel e \in \{E, I, M\}$
 - ii. $F \parallel e \in \{B, J, K\}$
 - iii. $K \parallel e \in \{B, C, D, E, F, G, H, P\}$
 - iv. $N \parallel e \in \{B, E, J\}$

4. (a) $E \parallel e \in \{A, B, J, K, M, N, O\}$

So removing the events happened too early (events that happened before the last event right before real time 8, we have the following possible frontier events combination of cuts:

$\{A, E, J, O\}$

$\{A, E, K, O\}$

$\{B, E, J, O\}$

$\{B, E, K, O\}$

(b)

Frontier Cuts	Incoming channel of P_1	Incomming Channel of P_3
A, E, J, O	IB, OD	OK
A, E, K, O	IB, OD	
B, E, J, O	OD	OK
B, E, K, O	OD	