

Evaluation of Functions

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Evaluation of Functions: Problems

For small h ,

$$\sin(x+h) - \sin(x); \quad (1)$$

for small x ,

$$\frac{1 - \cos x}{\sin x}; \quad (2)$$

for large N ,

$$\int_N^{N+1} \frac{dx}{1+x^2}; \quad (3)$$

for small x ,

$$\exp x - 1; \quad (4)$$

for large N ,

$$\int_N^{N+1} dx \log(1+x). \quad (5)$$

Evaluation of Functions: Solutions

$$(1) = \sin(x + h) - \sin(x) \quad (6)$$

$$\approx \sin x + h \cos x - \sin x \quad (7)$$

$$= h \cos x. \quad (8)$$

$$(2) = \frac{1 - \cos x}{\sin x} \quad (9)$$

$$= \frac{(\cos^2 t + \sin^2 t) - (\cos^2 t - \sin^2 t)}{2 \sin t \cos t} \quad (10)$$

$$= \frac{\sin^2 t}{\sin t \cos t} \quad (11)$$

$$= \tan t \approx t = \frac{x}{2} \quad (12)$$

$$(3) = \int_N^{N+1} \frac{dx}{1+x^2} \quad (13)$$

$$= [\arctan x]_N^{N+1} \quad (14)$$

$$= \arctan(N+1) - \arctan N \quad (15)$$

$$= \arctan\left(\frac{1}{1+N(N+1)}\right) \quad (16)$$

$$\approx \arctan\left(\frac{1}{N^2}\right) \approx \frac{1}{N^2} \quad (17)$$

$$(4) = \exp x - 1 \quad (18)$$

$$\approx 1 + x - 1 = x \quad (19)$$

$$(5) = \int_N^{N+1} dx \log(1+x) \quad (20)$$

$$= [x \log x - x]_N^{N+1} \quad (21)$$

$$= (N+1) \log(N+1) - (N) \log(N) \quad (22)$$

$$= \log(N+1) + N \log\left(\frac{N+1}{N}\right) \quad (23)$$

$$= \log(N+1) + N \log\left(1 + \frac{1}{N}\right) \quad (24)$$

$$\approx \log(N) + \frac{N}{N} = \log(N) + 1 \quad (25)$$

We could further simplify 25 to $\log(N)$ in the limit of large N , but note that in double precision, the largest signed float is $\Omega = 2^{1023} \times (2 - 2 * -52) \approx 2 \times 10^{308}$, and $\log \Omega \approx 700 \gg 1$.